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**Abstract**

**Full Text**

**Physics**

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**On the Statistical Theory of Particle Acceleration by Magnetohydrodynamic Waves in a Weakly Turbulent Rarefied Plasma**

*(Presented by Academician G. I. Budker, 26 X 1964)*

The statistical mechanism of particle acceleration was first proposed by E. Fermi <sup>(1)</sup> to explain the origin of cosmic rays. He considered, specifically, the acceleration of a charged particle in a medium consisting of a multitude of chaotically moving magnetic clouds. The choice of such a model is convenient in that the interaction of particles with magnetic clouds can be represented as ordinary “collisions” of particles of small and very large mass. Calculation of the energy balance in a system of such particles of different mass shows that the energy initially stored in the magnetic clouds of large mass tends to be distributed equally among the individual particles and clouds. Under real conditions, turbulent motion is not necessarily the motion of separate clumps of matter as a whole. Very often it may be thought of as a set of weakly interacting collective motions—oscillations of the medium. Thus, in particular, we shall consider the acceleration of a particle by a weakly turbulent plasma in which an enormous number of natural oscillations of small amplitude have been excited. The acceleration of a particle in this model can no longer be represented as vividly as in Fermi’s model <sup>(1)</sup>; however, owing to the smallness of the amplitudes of the oscillations, their weak interaction with one another and with the plasma particles can be studied by means of perturbation theory <sup>(2,3)</sup>, and an exact quantitative result can be obtained.

Particle acceleration as a result of inverse Cherenkov absorption of plasma oscillations that are in resonance with the accelerated particles appears in the first order of perturbation theory in the weak interaction of the waves with the particles and has been studied in detail in <sup>(4)</sup>. In <sup>(4)</sup> it is shown that the resonance condition ( $\omega = \mathbf{k}\mathbf{v}$ ) very often imposes restrictions on the particle velocity and, consequently, on its energy. In addition, the very presence of the acceleration effect then depends substantially on the distribution of wave energy in phase space ( $\omega, \mathbf{k}$ ).

Here we shall consider in detail the case of weak stationary turbulence, which is a set of a large number of low-frequency ( $\omega \ll \Omega_H = eH/Mc$ ), long-wavelength ( $k_z < v_{Tj}/\Omega_H$ ,  $v_{Tj} = \sqrt{T_j/m_j}$ ) Alfvén oscillations propagating almost along the unperturbed magnetic field  $H_{0z}^*$  ( $k_\perp \ll k_z$ ). The fluctuating magnetic

and electric fields can then be represented as a sum of the fields of individual oscillations of constant amplitude

$$\delta\mathbf{H} = \sum_{\mathbf{k}} \mathbf{H}_{\mathbf{k}} e^{-i\omega_{\mathbf{k}}t + i\mathbf{k}\cdot\mathbf{r}} + c. c. \quad (1)$$

For convenience in determining the polarization of the waves, we assume the presence of a small component of the wave vector transverse to the field,  $k_{\perp}$ , and shall take the vector of the electric field  $\mathbf{E}_{\mathbf{k}}$  to be perpendicular to the plane  $(\mathbf{k}, \mathbf{H}_0)$ :

[[unclear: next formula begins on following page]]

\* Such a turbulent state may arise, for example, as a result of the development of the “fire-hose” instability in a weak magnetic field <sup>(5)</sup> and small temperature anisotropy.

$$\mathbf{E}_{\mathbf{k}} = [\mathbf{k} \times \mathbf{h}] E_k / k_{\perp}, \quad \mathbf{h} \simeq \mathbf{H}_0 / H_0. \quad (2)$$

The relation between the electric and magnetic fields is determined by Maxwell’s equation

$$\mathbf{H}_{\mathbf{k}} = \frac{c}{\omega_{\mathbf{k}}} [\mathbf{k} \times \mathbf{E}_{\mathbf{k}}]. \quad (3)$$

In contrast to (4), we shall consider the acceleration of very fast particles moving with velocities  $v_{0z}$  considerably greater than the phase velocities of Alfvén waves  $\omega/k_z$  and the thermal velocities of particles  $v_{Tj} = \sqrt{T_j/m_j}$ . For such particles, the law of conservation of energy permits only the process of simultaneous absorption of two oscillations with “momenta”  $k_z, k'_z$  close to one another in magnitude and opposite in sign:

$$\omega_{\mathbf{k}} + \omega_{\mathbf{k}'} = (k_z + k'_z)v_{0z}, \quad \omega_{\mathbf{k}}, \omega_{\mathbf{k}'} > 0. \quad (4)$$

For classical wave fields of small amplitude ( $|\mathbf{H}_{\mathbf{k}}|^2/4\pi\hbar\omega_{\mathbf{k}} \gg 1$ ,  $\sum_{\mathbf{k}} |\mathbf{H}_{\mathbf{k}}|^2 \ll H_0^2$ ), process (4) may be considered within the framework of classical perturbation theory <sup>(2,3)</sup>. To this end, iterating the Boltzmann equation in the absence of collisions

$$\left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \frac{e_j}{m_j c} [\mathbf{v} \times \mathbf{H}_0] \frac{\partial}{\partial \mathbf{v}} \right) f_j = -\frac{e_j}{m_j} \sum_{\mathbf{k}} \left( \mathbf{E}_{\mathbf{k}} + \frac{1}{c} [\mathbf{v} \times \mathbf{H}_{\mathbf{k}}] \right) \frac{\partial f_j}{\partial \mathbf{v}};$$

$$j = i, e,$$

we obtain an expansion of the particle distribution function  $f_j(\mathbf{r}, \mathbf{v}, t)$  in a series in powers of the wave amplitudes:

$$f_j = f_{0j} + f^{j(1)} + f^{j(2)} + \dots, \quad (5)$$

where the successive approximations for  $f^{j(n)}$  are obtained with the aid of the iteration formula for the corresponding Fourier components <sup>(6)</sup>:

$$f_{\mathbf{k}, \omega + \omega''}^{j(n+1)} = -\frac{e_j}{m_j} \int_{-\infty}^t \sum_{\mathbf{k}' + \mathbf{k}'' = \mathbf{k}} \left( E_{\mathbf{k}'\omega'} + \frac{1}{c} [\mathbf{v} \times \mathbf{H}_{\mathbf{k}'\omega'}] \right) \frac{\partial f_{\mathbf{k}''\omega''}^{j(n)}}{\partial \mathbf{v}} dt'. \quad (6)$$

The integral (6) is taken along the unperturbed trajectory of the particles in the magnetic field:

$$\frac{d\mathbf{r}_j}{dt} = \mathbf{v}_j(t), \quad \mathbf{v}_j(t) \simeq \{v_{\perp} \cos(\theta_j - \omega_{Hj}t), v_{\perp} \sin(\theta_j - \omega_{Hj}t), v_z\}. \quad (7)$$

The unperturbed distribution of particles over velocities is chosen in the form of the sum of the Maxwellian distribution of the main plasma particles and a monochromatic-in-velocity distribution of the accelerated particles

$$f_{0j}(\mathbf{v}, t) \simeq n_0 \left( \frac{m_j}{2\pi T_j} \right)^{3/2} e^{-m_j v^2 / 2T_j} + \delta n_j \delta(\mathbf{v} - \mathbf{v}_{0j}(t)). \quad (8)$$

Then, with the aid of (5)–(8), we calculate the particle current  $\mathbf{j}$  to terms of third order in the wave amplitudes and calculate the work of the electric field on the particles  $\mathbf{j} \cdot \mathbf{E}$  under the assumption of chaotic phases of the amplitudes  $\mathbf{E}_{\mathbf{k}}$ . By means of the indicated procedure we obtain a correct mathematical description of effects quadratic in the wave energy, such as process (4), and find the complete change of the particle energy per unit time due to these processes.

We now proceed to the direct calculation. From (6)–(8), with the aid of (2), (3), we obtain the expansion (5), neglecting terms with  $\lambda \ll 1$ ,

$$f_{\mathbf{k}\omega}^{j(1)} \simeq -\frac{e_j}{m_j} \sum_{n=\pm 1} \frac{J'_n(\lambda) \left[ \frac{\partial}{\partial v_{\perp}} \left( 1 - \frac{k_{zv} z}{\omega} \right) + \frac{k_{zv}}{\omega} \frac{\partial}{\partial v_z} \right]}{\omega - k_{zv} z - n\omega_{Hj}} f_{0j} e^{-in(\theta_j + \pi/2 - \delta_{\mathbf{k}}) + i[\mathbf{k} \times \mathbf{v}]_z / \omega_{Hj}} E_{\mathbf{k}},$$

$$\begin{aligned}
 f_{\mathbf{k}+\mathbf{k}',\omega+\omega'}^{j(2)} &\simeq \frac{e_j^2}{m_j^2} \sum_{n=\pm 1} \frac{J'_n(\lambda') J_n(\lambda) \left( \frac{k_z}{\omega} - \frac{k'_z}{\omega'} \right) \frac{\partial}{\partial v_z} f_{0j}}{\omega - k_{zv} z - n\omega_{Hj}} \times \\
 &\times \frac{e^{-in(\delta_{\mathbf{k}} - \delta_{\mathbf{k}'}) + i[\mathbf{k} \times \mathbf{v}]_z / \omega_{Hj}}}{\omega + \omega' - (k_z + k'_z)v_z + i0} E_{\mathbf{k}'} E_{\mathbf{k}}, \\
 f_{\mathbf{k},\omega}^{j(3)} &\simeq -\frac{e_j^3}{m_j^3} \sum_{l,n=\pm 1} \sum_{\mathbf{k}'} \frac{J'_l(\lambda') \left[ \frac{\partial}{\partial v_{\perp}} \left( 1 - \frac{k_{zv} z}{\omega} \right) + \frac{k_{zv_{\perp}}}{\omega} \frac{\partial}{\partial v_z} \right]}{\omega - k_{zv} z - l\omega_{Hj}} \times \\
 &\times \frac{e^{-il(\theta + \pi/2 - \delta_{\mathbf{k}'})}}{\omega + \omega' - (k_z + k'_z)v_z + i0} \frac{J'_n(\lambda) J'_n(\lambda') \left( \frac{k_z}{\omega} - \frac{k'_z}{\omega'} \right) \frac{\partial}{\partial v_z} f_{0j}}{\omega - k_{zv} z - n\omega_{Hj}} \times \\
 &\times e^{-in(\delta_{\mathbf{k}} - \delta_{\mathbf{k}'}) + i[\mathbf{k} \times \mathbf{v}]_z / \omega_{Hj}} |E_{\mathbf{k}'}|^2 E_{\mathbf{k}},
 \end{aligned} \tag{9}$$

where  $\lambda = k_{\perp} v_{\perp} / \omega_{Hj}$ ,  $\mathbf{k} = \{-k_{\perp} \sin \delta_{\mathbf{k}}, k_{\perp} \cos \delta_{\mathbf{k}}, k_z\}$ , and  $J_n(\lambda)$  is the Bessel function.

The current of first order in the amplitude  $\mathbf{j}^{(1)}$  describes only the processes of emission (absorption) of one quantum of oscillation and is of no interest to us. The current of second order in the amplitudes of transverse oscillations  $E_{\mathbf{k}'}, E_{\mathbf{k}''}$  tends to zero as  $k_{\perp} \rightarrow 0$ . However, two transverse waves give a substantial contribution to the charge-density oscillations  $\rho^{(2)}$  and therefore are able to excite a longitudinal oscillation with potential  $\varphi^{(2)}$ , equal to

$$\varphi_{\mathbf{k}'+\mathbf{k}'',\omega'+\omega''}^{(2)} \simeq \sum_j \frac{4\pi e_j}{(\mathbf{k}'' + \mathbf{k}')^2} \int f_{\mathbf{k}'\mathbf{k}'',\omega'+\omega''}^{j(2)} d\mathbf{v} / \varepsilon_{\mathbf{k}'+\mathbf{k}''}(\omega' + \omega''), \tag{10}$$

where

$$\varepsilon_{\mathbf{k}'+\mathbf{k}''}(\omega' + \omega'') \simeq 1 + \sum_j \frac{4\pi e_j^2}{(\mathbf{k}'' + \mathbf{k}')^2} \int \frac{k_z (\partial f_{0j} / \partial v_z) J_0^2(\lambda)}{\omega' + \omega'' - (k'_z + k''_z)v_z + i0} d\mathbf{v}$$

is the dielectric permittivity of the plasma.

In turn, the interference of the forced longitudinal oscillations (10) with the transverse ones gives a contribution to the current  $\sim E_{\mathbf{k}}^3$ , comparable with the current (9) owing to the interference of three transverse oscillations directly. With the aid of (6) we have, for this case, a correction to  $f_{\mathbf{k},\omega+\omega''}^{j(3)'}$

$$\begin{aligned}
 f_{\mathbf{k}, \omega + \omega''}^{j(3)'} &\simeq \frac{e_j^2}{m_j^2} \sum_{\substack{n=\pm 1 \\ \mathbf{k}' + \mathbf{k}'' = \mathbf{k}}} J_0(\lambda'') J_n'(\lambda') \left\{ \frac{k_z'' \frac{\partial}{\partial v_z}}{\omega' + \omega'' - k_z'' v_z - n\omega_{Hj}} \times \right. \\
 &\times \frac{\frac{\partial}{\partial v_\perp} \left(1 - \frac{k_z' v_z}{\omega'}\right) + \frac{k_z' v_\perp}{\omega'} \frac{\partial}{\partial v_z}}{\omega' - k_z' v_z - n\omega_{Hj}} + \frac{\frac{\partial}{\partial v_\perp} \left(1 - \frac{k_z' v_z}{\omega'}\right) + \frac{k_z' v_\perp}{\omega'} \frac{\partial}{\partial v_z}}{\omega' + \omega'' - k_z v_z - n\omega_{Hj}} \times \\
 &\left. \times \frac{k_z'' \frac{\partial}{\partial v_z}}{\omega'' - k_z'' v_z + i0} \right\} f_{0j} E_{\mathbf{k}' \omega'} \varphi_{\mathbf{k}'', \omega''}^{(2)} e^{-in(\theta_j + \pi/2 - \delta_{\mathbf{k}'}) + i[\mathbf{k} \times \mathbf{v}]_z / \omega_{Hj}}.
 \end{aligned} \tag{11}$$

Calculating, with the aid of (12)–(14), the current  $\mathbf{j}$  and averaging the work over the phases of the amplitudes, we obtain the rate of change of the particle energy.

In the case of not very large particle velocities,  $v_{Tj} < v_{0j} < \omega_{Hj}/k_z$ , a considerable cancellation of the contributions from the currents (9)–(11) occurs, and the final-

the final expression for the work  $j \cdot E$  has the form:

$$\begin{aligned}
 \frac{d}{dt} \int \frac{1}{2} m_j v^2 \delta f_{0j} dv &= -\frac{1}{4} \sum_{\mathbf{k}, \mathbf{k}'} \left( \frac{\omega}{k_z} - \frac{\omega'}{k_z'} \right)^2 \cdot \pi \cos^2(\delta_{\mathbf{k}} - \delta_{\mathbf{k}'}) k_z^2 \frac{|H_{\mathbf{k}}|^2 |H_{\mathbf{k}'}|^2}{H_0^4} \times \\
 &\times \int \frac{m_j v_z^2}{2} v_z \frac{\partial \delta f_{0j}}{\partial v_z} \delta(\omega + \omega' - (k_z + k_z') v_z) dv_z.
 \end{aligned}$$

Substituting (6) here, neglecting terms of order

$$\sim \frac{\omega}{v_{0z} |H_{\mathbf{k}}|^2} \frac{d|H_{\mathbf{k}}|^2}{dk_z}$$

and using the equality of the phase velocities of the interacting waves that follows from (4), we obtain

$$\begin{aligned}
 \frac{d}{dt} \frac{m_j v_{0j}^2}{2} &= \sum_{\mathbf{k}, \mathbf{k}'} \frac{\omega^2}{k_z^2 v_{0z}^2} \cdot 2\pi \cdot \cos^2(\delta_{\mathbf{k}} - \delta_{\mathbf{k}'}) k_z |v_{0z}| \frac{|H_{\mathbf{k}'}|^2 |H_{\mathbf{k}}|^2}{H_0^4} \times \\
 &\times k_z \delta \left( k_z + k_z' - \frac{\omega + \omega'}{v_{0z}} \right) \frac{m_j v_{0z}^2}{2}.
 \end{aligned} \tag{12}$$

This expression is very reminiscent of E. Fermi' s formula <sup>1</sup>

$$\frac{d}{dt} \frac{mv^2}{2} = \left(\frac{V}{v}\right)^2 \frac{v}{l} \frac{mv^2}{2},$$

if one substitutes here, instead of the mean velocity of the clouds, the mean phase velocity of the magnetohydrodynamic pulsations  $V \sim \omega/k_z$ ; instead of the velocity  $v$ , their free-motion velocity  $v_{0z}$  along the field lines of the unperturbed field  $H_0$ ; and, finally, instead of the mean free path  $l$ , a quantity of the order of the wavelength of the pulsations, increased by a factor  $(H_0^2/\sum_{\mathbf{k}} |H_{\mathbf{k}}|^2)^2$

$$\left( l^{-1} \simeq 2\pi \cdot \sum_{\mathbf{k}, \mathbf{k}'} k_z |H_{\mathbf{k}}|^2 |H_{\mathbf{k}'}|^2 H_0^{-4} \cos^2(\delta_{\mathbf{k}} - \delta_{\mathbf{k}'}) k_z \delta \left( k_z + k'_z - \frac{\omega + \omega'}{v_{0z}} \right) \right).$$

But at velocities greater than  $v_{0z} > \omega_{Hj}/k_z$ , the acceleration obeys another law:

$$\begin{aligned} \frac{d}{dt} \frac{m_j v_{0j}^2}{2} &= \sum_{\mathbf{k}, \mathbf{k}'} \frac{\omega^2}{k_z^2 v_{0z}^2} \pi \sin^2(\delta_{\mathbf{k}} - \delta_{\mathbf{k}'}) k_z |v_{0z}| \frac{|H_{\mathbf{k}'}|^2 |H_{\mathbf{k}}|^2}{H_0^4} \times \\ &\times k_z \delta(k_z + k'_z - (\omega + \omega')/v_{0z}) m_j \omega_{Hj}^2 / 2k_z^2. \end{aligned} \quad (13)$$

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*Note: Figure translations are in progress. See original paper for figures.*

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