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Abstract

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ON SOME FEATURES OF THE ENERGY SPECTRUM OF OCEANIC TURBULENCE

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The well-known "power $-5/3$" law ([1,2]) for the distribution of energy density over the wave numbers k of turbulent "eddies" is obtained under the assumption that the supply of energy to the flow from external sources occurs only in the very long-wave part of the spectrum. The chaotic nature of the mechanism of the fragmentation of "eddies" makes it possible to regard them, beginning with some scale, as isotropic. Then, for these "eddies" (provided that the viscosity forces are immaterial for them), the "power $-5/3$" law is easily obtained from dimensional considerations: $f(k) = c\varepsilon^{2/3}k^{-5/3}$, where ε is the determining parameter, representing the energy flux along the cascade of eddies (equal to the rate of dissipation of turbulent energy in the viscous part of the spectrum), and c is a dimensionless universal constant.

In nature, however, there may exist turbulent motions in which the supply of energy occurs not only from the long-wave part of the spectrum but, generally speaking, over the entire range of wave numbers. Examples of such flows may be, in particular, sea currents, one of the sources of whose energy supply is the tangential stress of friction of a turbulent air flow (wind). Indeed, small turbulent formations of the wind can apparently, through tangential friction, give rise to equally small vortical formations in the liquid, directly transferring their energy to them.

Another important source of energy supply for sea currents is the nonuniformity of the processes of heating and cooling of seawater. The scales of these nonuniformities may also vary widely, from local disturbances caused by shielding of the Sun by clouds, up to the global difference in the heat supply of waters in the equatorial and polar regions of the ocean. Among the energy-supplying factors one should also mention tide-generating forces, which cause tidal motions of water of a periodic character in the ocean.

In studying the problem of the energy supply of ocean waters, it is very important to establish whether the transfer of energy from external sources over the entire spectrum of wave numbers is significant, or whether it is localized only in neighborhoods of individual values of the quantity k. Unfortunately, it is very difficult to answer this question because the statistical characteristics of

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marine turbulence have been poorly studied. The study of these characteristics is complicated by the need to design special apparatus suitable for operation at sea in different parts of the spectrum. Thus, to record turbulent pulsations with periods from hundredths of a second to tens of seconds, very low-inertia apparatus is required, examples of which have recently been created both in our country and abroad. Records of velocity fluctuations in the sea obtained with the aid of such apparatus have made it possible, in particular, to show that the energy spectrum of marine turbulence in the range of wave numbers $k = 1$—10^{-2} cm^{-1} is satisfactorily described by the "power $-5/3$" law ([3]). Consequently, in this range of values of k the direct transfer of energy to seawater from external sources may be regarded as immaterial. This pro-

process, in all likelihood, must be significant in phenomena of somewhat larger scales—of the order of ten meters ($k = 10^{-3}$ cm^{-1}). At such scales, as is known, the main part of the energy of waves generated on the ocean surface by the wind is contained. The inflow of energy from the wind to the waves is, of course, highly variable—it depends on the wind speed and duration, the fetch, etc.; therefore, in what follows, by the magnitude of such an energy inflow one should understand only a value averaged over a long time interval.

As for processes with scales of the order of 1–10–100 km ($k = 10^{-5}$–10^{-7} cm^{-1})*, information about their energy content is still more scanty. It is only clear that in such large-scale motions a considerable fraction of the total kinetic energy of ocean waters is concentrated. This conclusion makes it possible to arrive at the inference that, in studies of the energy content of large-scale vortices, coarser instruments of the type of ordinary hydrological current meters can be used. Such instruments (for example, the BPV-2 current meters now widely used) record the horizontal component of the current velocity with averaging over several minutes and are capable of operating continuously on buoy stations in the ocean for many days.

Fig. 1. Graphs of the energy-density function $f(\omega)$. 1—experimental curve calculated from current-meter data; 2—curve of the "power $-5/3$" law.

The calculation of the energy content of large vortices in the ocean was carried out by us from current-meter data by means of the method of analytical filtering of the current-velocity records under study. The frequency (spectral) characteristic of the filter core used for this purpose and the technique of the filtering process itself are analogous to those described in [4]. For the calculation, a 30-day record of velocity from a BPV-2 current meter was used (with intervals of 30 min between individual velocity readings), operating in the Atlantic Ocean at the point with coordinates 55°15′ N and 16°30′ W at the 200 m horizon. The calculation of the energy density of the velocity fluctuations was carried out for fluctuations with periods T lying in the intervals 3–6–12–24–48–96 h. The error of the calculation, caused by the limited length of the record and its discreteness, is insignificant for harmonics of the indicated period [5]. The mutual transition of spatial and temporal variables in the energy-density function and in the "power $-5/3$" law was carried out with the help of the hypothesis of "frozen turbulence," according to which $2\pi k = \omega/V$, where $\omega = 2\pi/T$ is the angular frequency of the velocity oscillation in the given turbulent vortex as it passes through a fixed point in space with the mean velocity V, which during the period of observations was approximately 10 cm/sec.

The graph of the energy-density function $f(\omega)$, obtained as a result of the calculations, is presented in Fig. 1. The graph is constructed from 5 experimental values of the function $f(\omega)$, referred to the midpoints of the above-mentioned time intervals. It is seen from the figure that the experimental data agree satisfactorily with the curve of the "power $-5/3$" law down to periods

* Here, obviously, only processes with horizontal orientation can be meant.

pulsations T, approximately equal to 10 hours. For larger values of T, however, a substantial deviation of the experimental data from the theoretical curve is found; moreover, for values $T < 26$ hours the experimental points lie above the theoretical ones, while with a further increase of T a sharp decrease is observed in the values of the function $f(\omega)$. The presence of a sharply expressed maximum on the curve $f(\omega)$ indicates the existence in the velocity field under study of certain predominant oscillations containing considerable energy, which, in turn, points to an enhanced influx of energy from external sources in the frequency range of these oscillations.

The period of the detected predominant oscillations is close to the period of tidal currents and inertial oscillations (i.e., resonant motions of water caused only by the force of inertia and the Coriolis force). Such oscillations are generally typical of the velocity fields of the oceans, and they are usually easily detected directly in records of the velocity of motion of ocean waters. The spatial scale l corresponding to inertial (for middle latitudes) and tidal oscillations can be obtained by multiplying the periods of the oscillations by the mean transport velocity of the current, which is close, both in our case and on the average for the ocean, to 10 cm/sec. The quantity l for such oscillations then proves to be approximately equal to 10 km. In the range of scales from several kilometers to several tens of meters, the influx of energy from external sources is apparently insignificant, and here, as follows from Fig. 1, the distribution of energy density is close to that given by the "$-5/3$ power" law. In the zone of wave numbers close to 10^{-3} cm^{-1} ($l = 10$ m), the function $f(k)$ forms the already mentioned "overshoot" and in so doing "jumps" (because of the addition of energy from the wind) from one curve of the "$-5/3$ power" law to another curve of the analogous law, but with a different value of the quantity ε. The necessity of the existence of such a jump in the function $f(k)$ is also indicated by the unrealistically large values of the energy density that would be obtained for large-scale processes if the curve of the "$-5/3$ power" law were extrapolated to them with the value $\varepsilon = 0.61\ \text{cm}^2 \cdot \text{sec}^{-3}$, obtained in work ([3]) for small-scale processes. Thus the quantity ε, which, according to our data, has the order $10^{-3}\ \text{cm}^2 \cdot \text{sec}^{-3}$ in the expression of the "$-5/3$ power"law for medium-scale processes ($l \sim 50$ m—5 km), represents only the flux of energy along a cascade of such eddies, and the quantity ε here is not equal to the rate of dissipation of energy in the viscous interval, where the added energy received in the zone of 10-meter scales is also dissipated.

An enhanced influx of energy into ocean waters must also exist in the range of scales commensurate with the dimensions of the oceans themselves (i.e., of the order of 1000 km). This influx of energy must be caused by global differences in the heat supply of ocean waters and by the action of large-scale baric formations (permanent centers of atmospheric action, trade-wind systems, etc.). Therefore one should expect that in the vicinity of the scale $l = 1000$ km there should also exist a certain overshoot in the function $f(k)$, which, unfortunately, at present still cannot be confirmed by experimental data because of the absence of such data. Between this global maximum and the maximum in the range of scales $l = 10$ km, there may exist a small interval of wave numbers where the "$-5/3$ power" law is again applicable (again with a different value of the quantity ε). This interval may be located near scales $l \sim 100$ km, where the influx of external energy is again, apparently, relatively small.

The general scheme of the distribution of energy density over motions of ocean waters of different scales is presented in Fig. 2. According to the data given above, the figure shows three zones in which the function $f(k)$ is described by the "$-5/3$ power" law, and three intervals of intense influx of energy into ocean waters from external sources. The right-hand

the straight line (on a logarithmic scale) of the "power $-5/3$" law (with $c\varepsilon^{2/3} = 0.34\ \text{cm}^{4/3} \cdot \text{sec}^{-2}$) is borrowed from work (3); the middle straight line is the logarithmic representation (in spatial variables) of the curve of the "power $-5/3$" law in Fig. 1 ($c\varepsilon^{2/3} = 6.0 \cdot 10^{-3}\ \text{cm}^{4/3} \cdot \text{sec}^{-2}$), while the left-hand part of the scheme is, as has already been said, conjectural in character.

It is quite obvious, of course, that the proposed scheme for the distribution of energy density is greatly simplified and to a certain extent hypothetical. It may be justified to one degree or another only on the average over a large number of different hydrometeorological situations and geographical conditions. In specific regions of the World Ocean, in individual seasons and under different weather conditions, the form of the functions $f(k)$ may change substantially– individual maxima and portions where the "power $-5/3$" law holds may appear or, conversely, disappear. In addition, an increase in the accuracy and duration of measurements of velocities in the ocean, as well as improvement of practical methods for calculating the function $f(k)$, will undoubtedly make it possible to specify a number of details on the averaged curve $f(k)$ (for example, to separate

Fig. 2. Scheme of the distribution of energy density over multiscale motions of ocean waters. a—b–zones of validity of the "power $-5/3$" law

Figure 2: Fig. 2. Scheme of the distribution of energy density over multiscale motions of ocean waters. a—b–zones of validity of the "power $-5/3$" law

close maxima due to tidal and inertial oscillations). However, even now, with sufficient grounds, it apparently can be asserted that the principal features of the function $f(k)$ for the ocean, shown in Fig. 2, are quite objective in character and reflect the general regularities of the energy supply of the ocean and the redistribution of energy in ocean waters.

Fig. 2. Scheme of the distribution of energy density over multiscale motions of ocean waters.
a—b–zones of validity of the "power $-5/3$" law.

In conclusion I take this opportunity to express my sincere gratitude to Academician A. N. Kolmogorov and Prof. V. B. Shtokman for a number of suggestions and indications on the questions touched upon in the present note.

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Note: Figure translations are in progress. See original paper for figures.