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ELASTICITY THEORY

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Abstract

Full Text

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ON THE FORMULATION OF THE PROBLEM OF BUCKLING OF A SHELL UNDER CREEP

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ELASTICITY THEORY

1. There are several formulations of the problem of stability under creep conditions ($^{1-6}$). One of them consists in studying the buckling in time of a rod, plate, or shell under load in the presence of certain initial imperfections. For rods this concept was developed in ($^{7-9}$), and for shells in ($^{10-12}$). In the present work two possible formulations are considered for the problem of buckling of a shell under creep conditions, as the problem of the development of initial imperfections on the basis of equations composed with allowance for geometrically nonlinear relations. In deriving the equations, the assumption is introduced that, in the process of creep, the stresses and strains in the shell differ little from the stresses and strains of the basic momentless state. This assumption makes it possible to linearize the physical relations relative to the basic state and, in both variants, to reduce the problem of determining the deflections and stresses of a shallow shell under creep conditions to a corresponding system of two nonlinear integro-differential equations in the deflection and the stress function, depending on the coordinates on the middle surface and on time.

2. Let the equation of state under creep have the form

$$\dot{p}_i = g(p_i; \sigma_i)\sigma_i \tag{1}$$

and let the relations of the flow theory hold between the components of the creep strain-rate tensor $\dot{p}_{ij}$ and the stress deviator σ_{ij}^{**}:

$$\dot{p}_{ij} = {}^3/_2 g(p_i, \sigma_i)\sigma_{ij}^{**}, \qquad p_{ij} = \varepsilon_{ij} - \frac{1}{2G}\sigma_{ij}^{**}. \tag{2}$$

For the intensities of stresses and creep strain rates we have

$$\sigma_i^2 = {}^3/_2 \sigma_{ij}^{**}\sigma_{ij}^{**}, \qquad \dot{p}_i^2 = {}^2/_3 \dot{p}_{ij}\dot{p}_{ij}, \qquad p_i = \int_0^t \dot{p}_i \, dt. \tag{3}$$

We shall consider a shell which, under the action of an external load in the absence of disturbances, is in a momentless stressed state (the basic state). If there is some disturbance (initial deflection), as a result of which a disturbed motion occurs, then the redistribution of stresses and strains in the middle surface and through the thickness of the shell will be regarded as small, so that for the quantities characterizing the deviation of the state under consideration from the basic one, the relations obtained by linearizing (1) and (2) will be valid:

$$\delta\dot{p}_i = \sigma_i \frac{\partial g}{\partial p_i}\delta p_i + \sigma_i \frac{\partial g}{\partial \sigma_i}\delta\sigma_i + g\delta\sigma_i,$$

$$\delta\dot{\varepsilon}_{ij} - \frac{1}{2G}\delta\dot{\sigma}_{ij}^{**} = {}^3/{}_2 g\delta\sigma_{ij}^{**} + {}^3/{}_2\sigma_{ij}^{**}\left(\frac{\partial g}{\partial p_i}\delta p_i + \frac{\partial g}{\partial \sigma_i}\delta\sigma_i\right). \tag{4}$$

Here the quantities p_i, σ_i refer to the basic state, while $\delta p_i, \delta\sigma_i$ characterize the deviation of the actual state from the basic one.

Passing, with the aid of (1), from the variable t to the variable p_i and integrating equations (4) under the initial condition

$$\delta p_{ij} = \delta\varepsilon_{ij} - \frac{1}{2G}\delta\sigma_{ij}^{**} = 0 \quad \text{for } p_i = p_i^*,$$

we find

$$\delta p_i = \frac{g}{E}\int_{\xi^*}^{\xi} \frac{1+b}{g}\,\delta\sigma_i\,d\xi,$$

$$\delta\sigma_{ij}^{**} = \frac{2}{3}E\delta\varepsilon_{ij} - \frac{2}{3}Ee^{-\xi}\int_{\xi^*}^{\xi} e^{\xi}\delta\varepsilon_{ij}\,d\xi - \alpha_{ij}^{**}e^{-\xi}\int_{\xi^*}^{\xi} e^{\xi}S(\delta\sigma_i)\,d\xi, \tag{5}$$

where

$$a = \sigma_i\frac{\partial g}{\partial p_i}, \quad b = \frac{\sigma_i}{g}\frac{\partial g}{\partial \sigma_i}, \quad \xi = \frac{Ep_i}{\sigma_i}, \quad \xi^* = \frac{Ep_i^*}{\sigma_i}, \quad \alpha_{ij}^{**} = \frac{\sigma_{ij}^{**}}{\sigma_i}, \quad \alpha_{ij} = \frac{\sigma_{ij}}{\sigma_i}$$

$$S(\varphi) = \frac{a}{E}\int_{\xi^*}^{\xi}\frac{1+b}{g}\varphi\,d\xi + b\varphi.$$

Let the shell at the time $t = t^*$ acquire some initial deflection w. For the deformations of a shallow shell associated with deviation from the basic state, taking the initial deflection into account, we use the expressions

$$\delta\varepsilon_{11} = u_x - z(w_{xx} - w_{xx}^0) + \frac{1}{R_1}(w - w^0) + \frac{1}{2}(w_x^2 - w_x^{02}). \tag{6}$$

Here u, v, w are displacements associated with the deviation of the shell from the basic state; R_1, R_2 are the principal radii of curvature of the shell midsurface.

For the moments and additional forces in the midsurface of the shell associated with its bending during motion, taking (5) and (6) into account, we obtain

$$N_{11} = \frac{2}{3}B\left[2u_x + v_y + \frac{2(w-w^0)}{R_1} + \frac{w-w_0}{R_2} + w_x^2 - w_x^{02} + \frac{1}{2}(w_y^2 - w_y^{02})\right] -$$

$$-\frac{2}{3}Be^{-\xi}\int_{\xi^*}^{\xi} e^{\xi}\left[2u_x + v_y + \frac{2(w-w^0)}{R_1} + \frac{w-w_0}{R_2} + w_x^2 - w_x^{02} + \frac{1}{2}(w_y^2 - w_y^{02})\right]d\xi - \alpha_{11}e^{-\xi}\int_{\xi^*}^{\xi} e^{\xi}S(N_i)\,d\xi, \ldots \tag{7}$$

$$\ldots, \quad G_{11} = -\frac{D}{2}\left[2(w_{xx} - w_{xx}^0) + (w_{yy} - w_{yy}^0)\right] +$$

$$+\frac{D}{2}e^{-\xi}\int_{\xi^*}^{\xi} e^{\xi}\left[2(w_{xx} - w_{xx}^0) + (w_{yy} - w_{yy}^0)\right]d\xi - \alpha_{11}e^{-\xi}\int_{\xi^*}^{\xi} e^{\xi}S(M_i)\,d\xi, \ldots$$

Here

$$M_i = (\alpha_{11} - \tfrac{1}{2}\alpha_{22})G_{11} + (\alpha_{22} - \tfrac{1}{2}\alpha_{11})G_{22} + 3\alpha_{12}G_{12},$$

$$N_i = (\alpha_{11} - \tfrac{1}{2}\alpha_{22})N_{11} + (\alpha_{22} - \tfrac{1}{2}\alpha_{11})N_{22} + 3\alpha_{12}N_{12}, \quad D = \frac{8}{9}Eh^3, \quad B = 2Eh.$$

In the equilibrium equations of a shallow shell

$$N_{11,x} + N_{12,y} = 0, \qquad N_{22,y} + N_{12,x} = 0,$$

$$G_{11,xx} + G_{22,yy} + 2G_{12,xy} - \frac{1}{R_1}(N_{11}^0 + N_{11}) - \frac{1}{R_2}(N_{22}^0 + N_{22}) +$$

$$+(N_{11}^0 + N_{11})w_{xx} + (N_{22}^0 + N_{22})w_{yy} + 2(N_{12}^0 + N_{12})w_{xy} = 0, \tag{8}$$

where $N_{11}^0, N_{22}^0, N_{12}^0$ refer to the basic state, we introduce the function of stresses ...

stresses Φ, so that

$$\Phi_{xx} = N_{22}, \qquad \Phi_{yy} = N_{11}, \quad \Phi_{xy} = -N_{12},$$

and carry out transformations of the same type as in paper (⁵), associated with eliminating N_i and M_i from the equations. We arrive at two equations of the same kind as the equations in paper (⁵), with the difference that they will contain nonlinear terms associated with deflections. After some transformations the system is written in the form

$$\Delta\Delta\Phi + e^{-\xi}\int_{\xi^*}^{\xi} e^{\xi} S(\Lambda_1\Lambda_1\Phi)\,d\xi - B\left[\Gamma(w,w^0) - e^{-\xi}\int_{\xi^*}^{\xi} e^{\xi}\Gamma(w,w^0)\,d\xi\right] = 0, \quad (9)$$

$$\begin{aligned} U(w,w^0,\Phi) + e^{-\xi}\int_{\xi^*}^{\xi} e^{\xi} D\Delta\Delta(w-w^0)\,d\xi - e^{-\xi}\int_{\xi^*}^{\xi} e^{\xi} S\Big[-U(w,w^0,\Phi) \\ - \frac{3}{4}D\Delta\Delta(w-w^0) - e^{-\xi}D\int_{\xi^*}^{\xi} e^{\xi}(\Delta\Delta - \frac{3}{4}\Lambda\Lambda)(w-w^0)\,d\xi\Big]\,d\xi = 0. \end{aligned} \quad (10)$$

Here

$$\Gamma(w,w^0) = w_{xy}^2 - w_{xx}w_{yy} - (w_{xy}^{0\,2} - w_{xx}^0 w_{yy}^0) + \left(\frac{1}{R_1}\frac{\partial^2}{\partial y^2} + \frac{1}{R_2}\frac{\partial^2}{\partial x^2}\right)(w-w^0); \quad (11)$$

$$\begin{aligned} U(w,w^0,\Phi) = &- D\Delta\Delta(w-w^0) - \frac{1}{R_1}(N_{11}^0 + \Phi_{yy}) - \frac{1}{R_2}(N_{22}^0 + \Phi_{xx}) \\ &+ 2h\sigma_i\Delta w + \Phi_{yy}w_{xx} + \Phi_{xx}w_{yy} - 2\Phi_{xy}w_{xy}; \end{aligned} \quad (12)$$

$$\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2, \qquad \Lambda = a_{11}\partial^2/\partial x^2 + 2a_{12}\partial^2/\partial x\partial y + a_{22}\partial^2/\partial y^2;$$

$$\Lambda_1 = a_{11}(\partial^2/\partial y^2 - \tfrac{1}{2}\partial^2/\partial x^2) + a_{22}(\partial^2/\partial x^2 - \tfrac{1}{2}\partial^2/\partial y^2) - 3a_{12}\partial^2/\partial x\partial y.$$

Equations (9) and (10), with the corresponding boundary conditions, determine the deflections and the stressed state of the shell, to which at the time $t = t^*$ the initial deflection $w = w^0$ is prescribed.

3. Let the equation of state have the form (1), and let the components of the strain tensor ε_{ij} and of the stress deviator σ_{ij}^{**} be related by the deformation theory relations

$$\varepsilon_{ij} = \frac{3}{2}\frac{\varepsilon_i}{\sigma_i}\sigma_{ij}^{**}, \tag{13}$$

where

$$\varepsilon_i^2 = \frac{2}{3}\varepsilon_{ij}\varepsilon_{ij}, \qquad \sigma_i^2 = \frac{3}{2}\sigma_{ij}^{**}\sigma_{ij}^{**}, \qquad p_i = \varepsilon_i - \sigma_i/E. \tag{14}$$

For the quantities characterizing the deviation of the state under consideration from the basic one, taking into account the initial condition $\delta p_i = 0$ for $p_i = p_i^*$, we find

$$\delta\varepsilon_i = \frac{1}{E}\delta\sigma_i + \frac{g}{E}\int_{\xi^*}^{\xi}\frac{b+1}{g}\,\delta\sigma_i\,d\xi. \tag{15}$$

Linearizing (13), taking (15) into account, we obtain instead of (5)

$$\delta\sigma_{ij}^{**} = \frac{2E}{3(1+\xi)}\delta\varepsilon_{ij} + \frac{\xi}{1+\xi}\alpha_{ij}^{**}\delta\sigma_i - \frac{a_{ij}^{**}}{1+\xi}g\int_{\xi^*}^{\xi}\frac{b+1}{g}\delta\sigma_i\,d\xi. \tag{16}$$

For the moments and additional forces we shall have

$$\begin{gathered} G_{11} = -\frac{D}{2(1+\xi)}\left[2(w_{xx} - w_{xx}^0) + w_{yy} - w_{yy}^0\right] + \\ + \frac{\xi}{1+\xi}\alpha_{11}M_i - \frac{\alpha_{11}}{1+\xi}\,g\int_{\xi^*}^{\xi}\frac{b+1}{g}\,M_i\,d\xi, \ldots \end{gathered} \tag{17}$$

$$\begin{gathered} \ldots, \quad N_{11} = \frac{2}{3}\frac{B}{(1+\xi)}\left[2u_x + v_y + \frac{2(w-w^0)}{R_1} + \frac{w-w^0}{R_2} + w_x^2 - w_x^{0\,2} + \right. \\ \left. + \frac{1}{2}\left(w_y^2 - w_y^{0\,2}\right)\right] + \alpha_{11}\left[\frac{\xi}{1+\xi}N_i - \frac{1}{1+\xi}\,g\int_{\xi^*}^{\xi}\frac{b+1}{g}\,N_i\,d\xi\right], \ldots \end{gathered}$$

We introduce the stress function. After transformations we obtain the compatibility equation and the equilibrium equation in the form

$$\Delta\Delta\Phi = \frac{B}{1+\xi}\Gamma(w, w^0) + \frac{\xi}{1+\xi}\Delta_1\Delta_1\Phi - \frac{1}{1+\xi}\,g\int_{\xi^*}^{\xi}\frac{b+1}{g}\Delta_1\Delta_1\Phi\,d\xi, \tag{18}$$

$$U(w, w^0, \Phi) + \frac{\xi}{1+\xi} D \left(\Delta\Delta - \frac{3}{4}\Lambda\Lambda \right) (w - w^0) + \\ + g \int_{\xi^*}^{\xi} \frac{b+1}{g} \left[U(w, w^0, \Phi) + \frac{D}{1+\xi} \left(\frac{3}{4}\Lambda\Lambda + \xi\Delta\Delta \right) (w - w^0) \right] d\xi = 0. \tag{19}$$

On the basis of equations (9), (10) or (18), (19), one can obtain the dependence of the deflection of the shell on time for a prescribed value of the initial deflection and, from the character of this dependence, judge the value of the critical time.

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Note: Figure translations are in progress. See original paper for figures.