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Abstract

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FOCUSING OF COLD QUASINEUTRAL BEAMS IN ELECTROMAGNETIC FIELDS

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If an ion beam is in a focusing magnetic field, then its space charge is, as a rule, compensated by electrons arising from ionization of the residual gas or from secondary emission. This circumstance makes it possible to obtain dense beams in magnetic separators without fear of the influence of the space charge (1).

The situation is different with systems in which focusing is produced simultaneously by electric and magnetic fields. In these devices **charged** beams of low density are used. Moreover, there exists the opinion that in such systems one cannot in general work with compensated beams. In fact this is not quite so. Certainly, practically existing systems cannot be brought into a compensated-space-charge regime by any simple method, for example by increasing the pressure of the residual gas. However, in principle, systems with compensated beams can be created. For this it is first of all necessary to provide conditions under which an electric field will exist inside the beam.

The principal reason hindering the existence of an electric field in a plasma is the large mobility of the electrons. Therefore let us consider, in a simplified hydrodynamic approximation, the equation of motion of the electron component of the plasma (2), neglecting collisions of electrons with neutral atoms:

$$m_e n_e d\mathbf{v}_e/dt = -\nabla p_e - en_e\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}_e, \mathbf{H}]\right) + \frac{en_e}{\sigma}\mathbf{j}. \qquad (1)$$

Here σ is the conductivity of the compensated beam; $\mathbf{j}$ is the current density of the beam; n_e is the electron density, which, by virtue of quasineutrality, we take equal to the ion density. The remaining notation is standard. Let:

1. The thermal energy of the electrons be small in comparison with the characteristic potential difference U in the system, i.e.

$$kT_e/eU \ll 1. \qquad (2a)$$

2. The current density in the beam be small, so that

$$j/\sigma \ll E^*. \qquad (2\ )$$

Fig. 1

Figure 1: Fig. 1

3. The directed velocities of the electrons be less than or of the order of the ion velocities,

$$v_e \lesssim v_i. \quad (2\ )$$

Then equation (1) takes the form

$$\mathbf{E} + \frac{1}{c}[\mathbf{v}_e, \mathbf{H}] = 0.^{**} \quad (3)$$

* For example, for $T_e \sim 1$ eV and $E_{\text{char}} \sim 100$ V/cm, one must have $j \ll 300$ A/cm^2.

** If conditions (2a)–(2) are satisfied, then, taking instead of equation (1) a more exact equation, we would still obtain equation (3).

It is seen from this that in a rarefied plasma an electric field can exist only in the presence of a magnetic field and electron drift. In this case the electric field is perpendicular both to $\mathbf{H}$ and to $\mathbf{v}_e$. Consequently, the magnetic lines of force must be electric equipotentials. This conclusion is valid to within the accuracy of assumptions (2a)–(2c). The most serious of these is the neglect of the electron pressure. If the electron temperature T_e in the volume of the system may be regarded as constant, then the distribution of potential along a magnetic line of force, as follows from (1), (2b), (2c), will have the form

$$\varphi - \varphi_0 = \frac{kT_e}{e} \ln \frac{n}{n_0}.$$

Here φ_0 and n_0 are the potential and the electron density at some point of the beam. If the beam is not surrounded by at least a very rarefied "coat," then the boundary jump of the potential will be infinitely large for any T_e. However, if, for example, $T_e \sim 1$ eV, and $n_{\min}/n_{\max} \sim 10^{-3}$, then the potential jump will be ~ 7 V.

Fig. 1

In the practical realization of systems satisfying condition (3), it is necessary to provide electron drift. In principle, two types of systems are possible: a) systems in which the electron drift is open, i.e., it begins near one wall (the "cathode") and ends near another wall (the "anode") (Fig. 1a); b) systems in which the electron drift is closed (Fig. 1b) ([5]).

In the first case we must provide continuous emission of electrons from the "cathode." This encounters a number of difficulties, the main one being not so much the provision of emission itself as the necessity of conducting the electron flux from the "cathode" to the beam in a very high vacuum.* In doing so, of course, we assume that the beam is detached from the walls.

This problem does not arise in systems of the second type. True, here too certain electron emitters may be required in order to maintain definite parameters of the cloud of rotating electrons. It is very probable, however, that the exchange of electrons between the cloud and the walls can be made comparatively weak.

Let us consider the conditions for realizing axially symmetric systems with closed drift. Obviously, in such systems the electric and magnetic fields must not have azimuthal components. Therefore the magnetic field can be specified by one component of the vector potential $A_\theta(r, z)$, and the electric field by a scalar potential φ, depending on two coordinates (r, z). The equation of the lines of force of the magnetic field of interest to us has the form ([3])

$$\psi \equiv r A_\theta(r, z) = \text{const}. \tag{4}$$

At the same time, since a magnetic line of force is an equipotential (see (3)),

$$\varphi = \varphi(\psi). \tag{5}$$

This relation is the basic requirement imposed on the electric and magnetic fields of a focusing system operating in the regime of compensated space charge. In what follows we shall assume that the intrinsic–

* The creation within the volume of the system of a sufficiently dense, slowly moving plasma appears undesirable.

the beam' s own magnetic field is small,* and therefore ψ satisfies the equation for a vortex-free field

$$\Delta^* \psi \equiv r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{\partial^2 \psi}{\partial z^2} = 0. \tag{6}$$

On the other hand, φ satisfies Poisson' s equation

$$\Delta \varphi \equiv \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial \varphi}{\partial r} + \frac{\partial^2 \psi}{\partial z^2} = -4\pi q_e. \tag{7}$$

Here ρ_e is the density of uncompensated charge. Substituting (5) and (7), and taking (6) into account, we obtain

$$\varphi'' \left[\left(\frac{\partial \psi}{\partial r} \right)^2 + \left(\frac{\partial \psi}{\partial z} \right)^2 \right] + \varphi' \frac{2}{r} \frac{\partial \psi}{\partial r} = -4\pi q_e. \tag{8}$$

It is seen from this that, in the general case, $\rho_e \neq 0$. In particular, if

$$\varphi = K\psi, \tag{5a}$$

then

$$4\pi q_e = -\frac{2}{r}\frac{\partial\varphi}{\partial r} = \frac{2E_r}{r}, \tag{9}$$

and, consequently, the electric field produced by $q_e \equiv e\delta n$ turns out to be sufficiently large. Therefore the beam density $n_0 = n_i \approx n_e$ must be such that

$$\varkappa \equiv |\delta n|/n_0 = |E_r|/2\pi r n_0 \ll 1^{**}. \tag{10}$$

Thus, systems satisfying conditions (5) and (5a) are essentially plasma systems.

If we wish to create a focusing system whose properties are preserved as $n \to 0$, it is necessary that, in addition to (5), Laplace' s equation be satisfied,

$$\Delta\varphi = 0. \tag{11}$$

For this there must exist a field $\psi(r, z)$ that satisfies not only equation (6), but also the equation (see (8))

$$\frac{1}{r}\frac{\partial\psi}{\partial r}\Big/\left[\left(\frac{\partial\psi}{\partial r}\right)^2 + \left(\frac{\partial\psi}{\partial z}\right)^2\right] = G(\psi) = -\frac{1}{2}\frac{d}{d\psi}\ln\varphi'(\psi). \tag{12}$$

However, it is not clear whether the system (6), (12) has solutions different from the trivial ones

$$\psi = \psi(r), \qquad \psi = \psi(z). \tag{13}$$

As the simplest example of a system with closed drift, let us consider an ordinary cylindrical capacitor with an additional magnetic field, whose lines of force coincide with the equipotentials. Such a magnetic field can be obtained by passing a current through the inner cylinder (Fig. 2). In the system under consideration the fields are plane and,*** as (9) shows, the quantity q_e will be equal to zero if one takes

$$\varphi = E_0 a \ln\frac{\rho}{a}, \qquad A_z = -H_0 a \ln\frac{\rho}{a}. \tag{14}$$

Fig. 2

Fig. 2

Figure 2: Fig. 2

Fig. 3

Figure 3: Fig. 3

* The beam magnetic field $\mathbf{H}_n$ has all three components $H_{nr}, H_{n\theta}, H_{nz}$. The appearance of the field $H_{n\theta}$ will lead to radial drift of the electrons. This drift is comparatively small if $|H_{n\theta}|/|\mathbf{H}| \ll 1$.

** One can verify that, in order of magnitude, $\varkappa \sim (D/L)^2$, where D is the Debye radius calculated from the ion energy, and L is the characteristic scale of the system.

*** In formula (9), r is the radius measured from the symmetry axis z of the system (Fig. 1b); therefore, in the plane case under consideration, $r \to \infty$.

The drift of electrons in such fields occurs perpendicular to the plane of the drawing.

Simple calculations of the motion of ions in the system shown in Fig. 2 show that narrow monoenergetic beams of identical ions emerging from the line $\rho = a,\ \theta = 0$ are focused on the plane

Fig. 3

$$\theta_* = \frac{\pi}{\sqrt{2 + (\omega_H/\dot{\theta})^2}}. \tag{15}$$

Here $\omega_H = eH_0/Mc,\ \dot{\theta} = P/Ma^2,\ M$ is the ion mass, and P is the ion angular momentum. In deriving formula (15), it was assumed that the velocity of the ions emerging from the source lies in the plane of the drawing.

In order to close the electron drift, the system of Fig. 2 should be rolled into a torus of large radius R (Fig. 3).

Bending a magnetic field into a torus, as is known ([3]), is not a unique procedure. It can be shown that, under the condition $\varphi = K\psi$, it is possible to choose additions to ψ of order $1/R$ in such a way that, to accuracy including terms $\sim 1/R$, the focusing of paraxial beams in the toroidal system is the same as in the straight system (see Fig. 2). Then formula (15) also remains valid. However, in this case the density of uncompensated charges is $q \neq 0$ and is determined by formula (9), where R may be substituted for r, and E_{r0}, calculated for the "straight" system, may be substituted for E_r.

Of course, the class of systems with closed electron drift is not limited to the simple example considered. Since in these systems the magnetic field is specified by one component $A_\theta(r, z)$, in the general case the problem reduces to investigating the motion of ions in the r, z plane under the action of an effective force with potential

$$U = \frac{1}{2M}\left(\frac{D - \frac{e}{c}\psi}{r}\right)^2 + e\varphi(\psi), \qquad \psi \equiv rA_\theta. \tag{16}$$

Here D is the conserved generalized angular momentum. The properties of the focusing systems sought can conveniently be found with the aid of the generalized Greenberg scheme ([4]), in which it is taken into account that $\Delta U \neq 0$.

In conclusion, let us note that in all the preceding discussion we have not touched upon dissipative processes, nor upon questions of stability. In practice these are very important questions that require detailed study.

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Note: Figure translations are in progress. See original paper for figures.