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**Abstract**

**Full Text**

**Physics**

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## **KINETIC CURRENT INSTABILITY OF ALFVÉN WAVES IN A PLASMA WITH A FINITE ION LARMOR RADIUS**

*(Presented by Academician M. A. Leontovich, October 7, 1964)*

**1.** Studies in recent years have shown the important role of taking into account the finiteness of the ion Larmor radius  $\rho$  in comparison with the wavelength  $\lambda$  of perturbations for the problem of plasma stability in a magnetic field <sup>(1,2)</sup>. However, up to now the main attention has been devoted to the investigation of the stability of an inhomogeneous plasma, in which, for a finite ratio  $\rho/\lambda$ , oscillations are built up at the expense of the Larmor drift of particles (drift instability). Nevertheless, effects of a finite ion Larmor radius may also appear in a homogeneous but nonequilibrium plasma (in particular, in the presence of particle flows). An example of this is the kinetic current instability of Alfvén waves considered below, which arises in a plasma with a finite Larmor radius at relative electron and ion velocities  $u_0$  exceeding the Alfvén velocity  $c_A = H_0/(4\pi n_0 m_i)^{1/2}$  ( $H_0$  is the magnetic-field strength,  $n_0$  is the particle concentration in the plasma, and  $m_i$  is the ion mass).

**2.** Let an electric current with density  $j_0 = en_0 u_0$  ( $-e$  is the electron charge) flow through a homogeneous low-pressure plasma ( $\beta = 8\pi p/H_0^2 \ll 1$ ) along the magnetic field  $\mathbf{H}_0$ . We shall assume that the condition  $(\omega_0 a/c)(u_0/c_A) < 1$  is satisfied ( $\omega_0 = (4\pi e^2 n_0/m_i)^{1/2}$  is the ion plasma frequency,  $a$  is the radius of the plasma column, and  $c$  is the speed of light). This means that the self-magnetic field of the current is small in comparison with the external field. For  $u_0 \sim c_A$  this condition may be written in the form  $\sqrt{\beta} < \rho_i/a$  ( $\rho_i$  is the ion Larmor radius). On the other hand, let us suppose that  $\sqrt{\beta} \gg (m_e/m_i)^{1/2}$  ( $m_e$  is the electron mass), i.e., that  $c_A \ll v_e$  ( $v_e = (2T_e/m_e)^{1/2}$  is the mean thermal velocity and  $T_e$  is the electron temperature). Obviously, both conditions can be compatible only for sufficiently large ion Larmor radii.

Let us consider plasma oscillations with frequencies  $\omega \ll \omega_{Hi}$  ( $\omega_{Hi} = eH_0/m_i c$  is the ion cyclotron frequency), propagating almost perpendicular to the magnetic field  $\mathbf{H}_0$ , so that  $k_\perp \gg k_\parallel$ , where  $k_\perp$  is the transverse and  $k_\parallel$  the longitudinal (with respect to  $\mathbf{H}_0$ ) component of the wave vector  $\mathbf{k}$ . Let us note here that for a cylindrically symmetric plasma column, according to <sup>(2)</sup>,  $k_\parallel = mH_\varphi/rH_0 + k_{zH}z/H_0$ , where  $H_\varphi$  is the magnetic field of the current,  $H_z$  the

external longitudinal field,  $H_0 = \sqrt{H_\varphi^2 + H_z^2}$ , and  $m = 0, 1, 2, 3, \dots$ . However, for  $j_0 = \text{const}$  ( $H_\varphi \sim r$ ) and  $H_\varphi^2 \ll H_z^2$  one may take  $k_\parallel = \text{const}$ . In this case the dispersion equation of the oscillations has the form (cf. with (2))

$$\varepsilon_\parallel \left( 1 - \frac{\omega^2}{k_\parallel^2 c^2} \varepsilon_\perp \right) + \frac{k_\perp^2}{k_\parallel^2} \varepsilon_\perp = 0, \quad (1)$$

where

$$\varepsilon_\perp = \frac{\omega_{0i}^2}{\omega_{Hi}^2} \frac{1 - I_0(z_i) e^{-z_i}}{z_i}; \quad (2)$$

$$\varepsilon_\parallel = \frac{2\omega_{0e}^2}{k_\parallel^2 v_e^2} [1 - x_e Y(x_e)] + \frac{2\omega_{0i}^2}{k_\parallel^2 v_i^2} [1 - x_i Y(x_i)] I_0(z_i) e^{-z_i}. \quad (3)$$

Here  $\omega_{0e} = (4\pi e^2 n_0 / m_e)^{1/2}$  is the electron Langmuir frequency;  $x_e = (\omega - k_\parallel u_0) / k_\parallel v_e$ ;  $x_i = \omega / k_\parallel v_i$ ;  $z_i = k_\perp^2 \rho_i^2 = k_\perp^2 T_i / m_i \omega_{Hi}^2$ ;  $v_i$  is the mean thermal velocity;  $T_i$  is the ion temperature;  $I_0$  is the modified Bessel function of zero order;  $Y(x) = i\sqrt{\pi} W(x)$ ;  $W(x)$  is the Kramp function (2).

In an isothermal plasma ( $T_e \sim T_i$ ), for oscillations with longitudinal phase velocity lying in the interval  $v_i \ll \omega / k_\parallel \ll v_e$ , and under the condition that the electron-stream velocity  $u_0 \ll v_e$ , dispersion equation (1) takes the form:

$$\left( 1 + i\sqrt{\pi} \frac{\omega - k_\parallel u_0}{k_\parallel v_e} \right) \left[ 1 - \frac{\omega^2}{k_\parallel^2 c_A^2} \frac{1 - I_0(z_i) e^{-z_i}}{z_i} \right] + \frac{T_e}{T_i} [1 - I_0(z_i) e^{-z_i}] = 0. \quad (4)$$

From (4) it follows that

$$\text{Re } \omega = \omega_A(k) = k_\parallel c_A \left[ \frac{z_i}{1 - I_0(z_i) e^{-z_i}} + \frac{T_e}{T_i} z_i \right]^{1/2}; \quad (5)$$

$$\text{Im } \omega = \gamma_A(k) = -\frac{\sqrt{\pi}}{2} \frac{T_e}{T_i} z_i \frac{k_\parallel^2 c_A^2}{\omega_A(k)} \frac{\omega_A(k) - k_\parallel u_0}{k_\parallel v_e}, \quad (6)$$

where  $\gamma_A \ll \omega_A$ . For  $z_i = k_\perp^2 \rho_i^2 \rightarrow 0$ , formula (5) goes over into the well-known expression for the frequency of Alfvén waves,  $\omega_A = k_\parallel c_A$ , and the decrement (increment)  $\gamma_A \rightarrow 0$ . At finite ion Larmor radii ( $z_i \sim 1$ ), and for sufficiently large velocities of relative motion of electrons and ions ( $u_0 > c_A$ ), the Alfvén oscillations become unstable ( $\gamma_A > 0$ ).

3. Let us briefly discuss the physical nature of this instability. In the usual hydrodynamic treatment of Alfvén oscillations, when  $\rho/\lambda \rightarrow 0$ , it turns out that their electric field is directed across the constant magnetic field ( $\mathbf{E} \perp \mathbf{H}_0$ ). As a result, there is no interaction of the wave with the particles ( $E_{\parallel} j_{\parallel} = 0$ ), so that in this approximation Alfvén oscillations are undamped, in contrast to other types of oscillations (for example, ion or magnetic sound <sup>(3)</sup>).

The situation, however, changes substantially if the finiteness of the ion Larmor radius is taken into account. In this case, the Alfvén oscillations acquire a nonzero longitudinal component of the electric field,  $E_{\parallel} = -(k_{\perp}/k_{\parallel})(\varepsilon_{\perp}/\varepsilon_{\parallel})E_{\perp} \approx -(k_{\parallel}/k_{\perp})(T_e/T_i)z_i E_{\perp} \neq 0$ . As a result, Alfvén waves, interacting with resonant particles (electrons), can either be damped (Landau damping in an equilibrium plasma or at  $u_0 < c_A$ ) or be amplified (current instability,  $\gamma_A > 0$  at  $u_0 > c_A$ ). It is interesting to note that in the present case, owing to an instability of kinetic type, magnetohydrodynamic (macroscopic) oscillations are amplified, which may affect the confinement of the plasma by the magnetic field.

4. Let us note that for  $z_i = k_{\perp}^2 \rho_i^2 \gtrsim 1$ , and for sufficiently large  $k_{\parallel}$ , the frequency of Alfvén oscillations can become of the order of the ion cyclotron frequency ( $\omega_A \sim \omega_{Hi}$ ). In this case, in equation (1) one must substitute, instead of (2), the following expression for  $\varepsilon_{\perp}$ :

$$\varepsilon_{\perp} = \frac{c^2}{c_A^2} \frac{1}{z_i} \left\{ 1 - I_0(z_i) e^{-z_i} - \sum_{n=-\infty}^{\infty} \int \frac{\omega I_n(z_i) e^{-z_i} f_{\parallel}^i(v_{\parallel}) dv_{\parallel}}{\omega - n\omega_{Hi} - k_{\parallel} v_{\parallel}} \right\}, \quad (7)$$

where  $f_{\parallel}^i$  is the “longitudinal” ion distribution function. (The ion distribution is assumed to be isotropic and Maxwellian.)

Then the dispersion equation (1), under the condition that  $k_{\parallel} v_i \ll |\omega - n\omega_{Hi}|$ , near the  $n$ -th cyclotron harmonic takes the form:

$$1 - I_0(z_i) e^{-z_i} + \frac{T_i}{T_e} \left[ 1 + i\sqrt{\pi} \frac{\omega - k_{\parallel} u_0}{k_{\parallel} v_e} \right] \Bigg/ \left[ 1 - \frac{\omega^2}{(T_e/T_i) z_i k_{\parallel}^2 c_A^2} \left( 1 + i\sqrt{\pi} \frac{\omega - k_{\parallel} u_0}{k_{\parallel} v_e} \right) \right] = \frac{\omega I_n(z_i) e^{-z_i}}{\omega - n\omega_{Hi}}; \quad (8)$$

$I_n$  is the modified Bessel function of index  $n$  ( $n = 1, 2, 3, \dots$ ).

If it is assumed that  $\omega^2 \sim (n\omega_{Hi})^2 \ll (T_e/T_i) z_i k_{\parallel}^2 c_A^2$ , then equation (8) becomes the dispersion equation of work <sup>(4)</sup>, where the current instability of potential cyclotron oscillations was considered. For  $k_{\parallel} u_0 \sim n\omega_{Hi}$ ,  $T_e \sim T_i$ , and  $z_i \sim 1$ , it follows from this that  $u_0^2 \ll c_A^2$ . But, on the other hand, from the condition of small ion damping there follows the inequality <sup>(4)</sup>  $u_0^2/c_A^2 > 10^2 \beta$ . Thus, the assumption of the potential character of the oscillations made in <sup>(4)</sup> is valid only in the case of a sufficiently rarefied plasma, when  $\beta \ll 10^{-2}$ .

The growth rate of nonpotential ( $\beta \gg m_e/m_i$ ) cyclotron oscillations ( $\omega \approx n\omega_{Hi}$ ), according to (8), is equal to:

$$\gamma_n \simeq \sqrt{\pi} n\omega_{Hi} \left( \frac{u_0}{v_e} - \frac{n\omega_{Hi}}{k_{\parallel}v_e} \right) \frac{T_e}{T_i} I_n(z_i) e^{-z_i} \times \left\{ 1 + \frac{T_e}{T_i} [1 - I_0(z_i) e^{-z_i}] \left( 1 - \frac{n^2 \omega_{Hi}^2}{(T_e/T_i) z_i k_{\parallel}^2 c_A^2} \right) \right\}^{-2}. \quad (9)$$

It tends to a maximum, formally infinite, precisely under the condition that the ion cyclotron frequency  $\omega_{Hi}$  (or one of its harmonics,  $n\omega_{Hi}$ ) is equal to the frequency of the Alfvén oscillations  $\omega_A(\mathbf{k})$ . A joint consideration of the cyclotron and Alfvén branches of the oscillations near their point of intersection gives a finite maximum value of the growth rate for the cyclotron oscillations:

$$\gamma_{n\max}^{(1,2)} \simeq \frac{\sqrt{\pi}}{2} n\omega_{Hi} \left( \frac{u_0}{v_e} - \frac{n\omega_{Hi}}{k_{\parallel}v_e} \right) \left\{ \frac{1}{2} \frac{1 - I_0(z_i) e^{-z_i}}{2 - I_0(z_i) e^{-z_i}} + \left[ \frac{1}{2} \frac{I_n(z_i) e^{-z_i}}{1 - I_0(z_i) e^{-z_i}} \right]^{1/2} [2 - I_0(z_i) e^{-z_i}]^{-1/2} \right\} \quad (10)$$

(for simplicity we take  $T_e = T_i$ ). Hence, in particular, it follows that, as a result of the interaction of Alfvén oscillations with cyclotron harmonics, the growth rate of the current instability of oscillations with  $\omega \approx n\omega_{Hi}$  can be substantially greater than the growth rate of potential cyclotron oscillations <sup>(4)</sup>.

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*Note: Figure translations are in progress. See original paper for figures.*

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