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Abstract

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*CYBERNETICS
AND CONTROL THEORY*

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MULTIPOLES CONTAINING MAKE CONTACTS OF INTERNAL RELAYS

(Presented by Academician A. N. Kolmogorov on 28 III 1965)

In the proposed work the method of characteristic functions of valve-contact multipoles ⁽¹⁾ is generalized to the case in which the multipole, in addition to valves and contacts of external relays (i.e., relays on which the circuit itself does not act), contains make contacts of internal relays (i.e., relays on which the circuit itself acts). The presence of feedback (contacts of internal relays) causes a transient process (multiple-operation behavior of the circuit). Because of the absence in the circuit of break contacts of internal relays, after a certain interval of time, depending on the operating time of the internal relays, the transient process stabilizes; the circuit reaches a stable state from which only an external action can remove it (a change in the states of external relays or another application of potentials to the poles of the multipole). This stable state depends only on the states of the external relays and on the potentials applied to the poles, and does not depend on the operating time of the internal relays (Theorem 1). Multipoles for which the indicated dependence is the same will be called equivalent. As in ⁽¹⁾, any transformation of a multipole into an equivalent multipole can be carried out by an algebraic transformation of its characteristic form (Theorems 2 and 4).

Consider a contact ⁽¹⁾ (or, more precisely, a valve-contact) n -pole A . M_1, \dots, M_n are its poles; a_{β}^{α} is the direct, and χ_{β}^{α} the complete, conductivity from pole M_{α} to pole M_{β} ; $a_{\alpha}^{\alpha} = 1$; $\chi_{\alpha}^{\alpha} = 1$ ($\alpha, \beta = 1, 2, \dots, n$). The potential u_{α} at pole M_{α} ($u_{\alpha} = 0$ or 1) will be called the state of this pole, and the vector $u = (u_1, \dots, u_n)$ the state of the n -pole A . Suppose that to each pole M_{α} one end of the winding of a relay Z_{α} is connected, the other end of which is grounded; then a current y_{α} will pass through the winding of this relay. By z_{α} we shall denote the make contact (and its conductivity) of the relay Z_{α} . The quantity z_{α} is also called the state of the relay Z_{α} . The relays Z_1, \dots, Z_n will be called internal relays of the multipole A , and the vector $z = (z_1, \dots, z_n)$ the state of the internal relays.

Fig. 1

Figure 1: Fig. 1

Assume that the valve-contact multipole A , in addition to valves and contacts of external relays, contains make contacts of internal relays. In this case the direct conductivities a_β^α will be increasing (monotone) functions of the state z of the internal relays; i.e., from $z' \leq z''$ ($z' = (z'_1, \dots, z'_n)$, $z'' = (z''_1, \dots, z''_n)$, $z'_k \leq z''_k$, $k = 1, \dots, n$) it will follow that $a_\beta^\alpha(z') \leq a_\beta^\alpha(z'')$. Let us note that $a_\beta^\alpha(z)$ will also depend on the states of the external relays, and the character of this dependence is arbitrary. We do not indicate this dependence explicitly, since the process of operation of the multipole (the transient process) is being considered during which the state of the external relays remains unchanged. Let

throughout the entire process under consideration an unchanging potential x_α is applied to each pole M_α ($\alpha = 1, \dots, n$) ($x_\alpha = 1$ if the pole M_α is directly connected to an external voltage source; $x_\alpha = 0$ if there is no such connection). The input potential $x = (x_1, \dots, x_n)$, like the state of the external relays, plays the role of an input signal.

In Fig. 1 the quantities x_α are represented as conductances of contacts (switches); the multi-pole network itself is represented by a dashed rectangle; the connections that exist between the poles of the multi-pole network are not shown in the figure. Throughout the article, by a **multi-pole network** we shall mean a multi-pole network containing no break contacts of internal relays, and by a **transient process** we shall mean the process of operation of the multi-pole network for a fixed state of the external relays, a fixed input potential x , and the zero (nonoperating) initial state of the internal relays; more precisely, a process described in discrete time $t = 0, 1, 2, \dots$ by the system of equations

Fig. 1

$$z_\beta(t) = 0, \quad 0 \leq t < \tau_\beta; \quad z_\beta(t + \tau_\beta) = y_\beta(t);$$

$$y_\beta(t) = \sum_{\alpha=1}^n \chi_\beta^\alpha(z(t)) x_\alpha, \quad t \geq 0, \quad \beta = 1, \dots, n, \quad (1)$$

where the sum and the products are logical, and the natural number τ_β is the operating time of the relay Z_β .

How the transient process proceeds is described by

Theorem 1. *No later than at the τ -th step ($\tau = \tau_1 + \dots + \tau_n$), the state y of the multi-pole network and the state z of the internal relays stabilize, i.e. $y(t) = z(t) = z(\tau)$ for $t \geq \tau$. This final state of the multi-pole network (we shall call*

it the output potential) does not depend on the magnitudes of the delays of the internal relays.

We shall denote the output potential of the n -pole network A by $y(A, x) = (y_1(A, x), \dots, y_n(A, x))$, or, more briefly, by $y(x) = (y_1(x), \dots, y_n(x))$. It is clear that the output potential $y = y(x)$ will satisfy the system of equations

$$y_\beta = \sum_{\alpha=1}^n \chi_\beta^\alpha(y) x_\alpha \quad (\beta = 1, \dots, n).$$

Applying tensor notation to the sum (which we shall do throughout the article), i.e. omitting the summation sign if the summation index occurs twice in the term, once as a superscript and once as a subscript, the last system can be rewritten in the form

$$y_\beta = \chi_\beta^\alpha(y) x_\alpha.$$

Let us also note, incidentally, the equations

$$y_\beta = a_\beta^\alpha(y) y_\alpha, \quad y^\alpha = a_\beta^\alpha(y) y^\beta$$

(where $y^k = \bar{y}_k$), which are satisfied by the output potential.

In what follows it will be convenient for us to denote the poles of the multi-pole network by the same symbols as the closing contacts of the corresponding internal relays: z_1, \dots, z_n , and to write $A(z_1, \dots, z_n)$, thereby indicating the designations of the poles of the multi-pole network A . Two n -pole networks $A(z_1, \dots, z_n)$ and $B(z_1, \dots, z_n)$, in which identically denoted poles are put into correspondence with one another, will be called **equivalent** if, for any fixed state of the external relays, equal input potentials cause equal output potentials in the multi-pole networks A and B , i.e. $y(A, x) = y(B, x)$. For the case of multi-pole networks not containing contacts of internal-

of internal relays, this definition is equivalent to the definition in (1). The function

$$f(A, z) = a_\beta^\alpha(z) z_\alpha z^\beta, \tag{2}$$

where $z^\beta = \bar{z}_\beta$, will be called the **characteristic function**, and the double sum on the right-hand side of (2) the **characteristic form** of the multi-terminal network $A(z_1, \dots, z_n)$.

The problem of analyzing an n -terminal network (finding the output potential $y(x)$ for a given n -terminal network A) is solved by the formulas

Fig. 2

Figure 2: Fig. 2

$$y_\beta(x) = (Ez_1) \cdots (Ez_n)(f(z) + x_\alpha z^\alpha + z_\beta) \quad (\beta = 1, \dots, n), \quad (3)$$

where $f(z)$ is the characteristic function of the multi-terminal network A , and E is the operation of eliminating a variable $(^1)^*$. A particular case of this formula,

$$y_2(1, 0, \dots, 0) = (Ez_3) \cdots (Ez_n)f(1, 0, z_3, \dots, z_n),$$

may be regarded as a generalization of formula (28) from $(^1)$ to the case of multi-terminal networks with closing contacts of internal relays.

Fig. 2

The algebraic method for transforming an n -terminal network into an equivalent n -terminal network gives

Theorem 2. *In order that two n -terminal networks $A(z_1, \dots, z_n)$ and $B(z_1, \dots, z_n)$ be equivalent, it is necessary and sufficient that their characteristic functions be equal: $f(A, z) = f(B, z)$.*

For example, the three-terminal networks $A(z_1, z_2, z_3)$ (Fig. 2a) and $B(z_1, z_2, z_3)$ (Fig. 2b) are equivalent, since $f(A, z) = z_1 \cdot z_2 \bar{z}_3$, $f(B, z) = z_2 \cdot z_1 \bar{z}_3$, and $f(A, z) = f(B, z)$.

The method of synthesizing an n -terminal network (finding an n -terminal network A from a given output potential $y(x)$) follows from Theorem 3 with subsequent use of Theorem 2.

Theorem 3. *In order that a function $y(x)$ ($x, y(x)$ are n -dimensional vectors) be the output potential of some n -terminal network, it is necessary and sufficient that $y(x)$ satisfy the following four conditions: 1) $y(x)$ is an increasing function of x ; 2) $y(0) = 0$; 3) $y(x) \geq x$; 4) $y(y(x)) = y(x)$. When these conditions are fulfilled, the function $f(z) = y_\beta(\bar{z})z^\beta$ will be the characteristic function of an n -terminal network whose output potential is the function $y(x)$.*

Consider two multi-terminal networks: an $(n + p)$ -terminal network $A(z_1, \dots, z_n, u_1, \dots, u_p)$ and an $(n + q)$ -terminal network $B(z_1, \dots, z_n, v_1, \dots, v_q)$, in which identically denoted poles are matched with one another. We shall say that the multi-terminal networks A and B are **equivalent with respect to the poles** z_1, \dots, z_n if the same application of potentials to these poles (for any fixed state of the external relays) produces in the multi-terminal networks A and B the same output potentials at these same poles, i.e.

$$y_\beta(A, x_1, \dots, x_n, 0, \dots, 0) = y_\beta(B, x_1, \dots, x_n, 0, \dots, 0)$$

Fig. 3

Figure 3: Fig. 3

for $\beta = 1, \dots, n$.

Theorem 4. *In order that the multi-terminal networks $A(z_1, \dots, z_n, u_1, \dots, u_p)$ and $B(z_1, \dots, z_n, v_1, \dots, v_q)$ be equivalent with respect to the poles z_1, \dots, z_n , it is necessary and sufficient that*

$$(Eu_1) \cdots (Eu_p)f(A, z_1, \dots, u_p) = (Ev_1) \cdots (Ev_q)f(B, z_1, \dots, v_q). \quad (4)$$

Thus, the transformation of a multi-terminal network A into any other multi-terminal network B equivalent to A with respect to the selected poles z_1, \dots, z_n can be carried out by transforming the characteristic form of the multi-terminal network, using the operations of eliminating variables and introducing new variables.

* $(Ez_0)\varphi(z_0) = \varphi(0)\varphi(1)$ for a function $\varphi(z_0)$ of the (scalar) variable z_0 .

The theory presented can be applied to the construction of valve-contact circuits (with multicycle operation) realizing Boolean functions. The internal relays in such circuits play an auxiliary role and are introduced in order to reduce the number of contacts in the circuit ((²), p. 84). For example, the construction of a two-terminal circuit with a prescribed conductance from pole z_1 to pole z_2 reduces, as in (¹), to the construction of a multipole $B(z_1, \dots, z_n)$, whose characteristic function satisfies the equality

$$(Ez_3) \dots (Ez_n)f(B, 1, 0, z_3, \dots, z_n) = f(A, 1, 0), \quad (5)$$

where $f(A, 1, 0)$ is the prescribed conductance. But in comparison with (¹) we have more possibilities for transformations, since now we are not bound by the requirement that the coefficients be constant. The only thing we should avoid is terms containing as factors two inverted variables $\bar{z}_i\bar{z}_j$, since we shall not be able to realize such terms. In constructing a contact realization (without valve elements), the requirement of symmetry of the coefficients is added: $b_\beta^\alpha = b_\alpha^\beta$.

Fig. 3

Example. Let us construct a contact two-terminal circuit whose conductance between poles z_1 and z_2 is equal to

$$f(1, 0) = a\bar{b}(c + h) + d(a + hc + h\bar{b}).$$

Successive transformations give:

$$\begin{aligned}
 f(1, 0) &= \bar{a}\bar{b}c + (d + \bar{a}\bar{b}h)(a + hc + h\bar{b}), \\
 f(1, 0, z_3) &= \bar{a}\bar{b}c + (d + \bar{a}\bar{b}h)\bar{z}_3 + (a + hc + h\bar{b})z_3 \\
 &= d\bar{z}_3 + az_3 + (hz_3 + \bar{a}\bar{b})(h\bar{z}_3 + \bar{b}z_3 + c), \\
 f(1, 0, z_3, z_4) &= d\bar{z}_3 + az_3 + (hz_3 + \bar{a}\bar{b})\bar{z}_4 + (h\bar{z}_3 + \bar{b}z_3 + c)z_4 \\
 &= d\bar{z}_3 + h(z_3\bar{z}_4 + \bar{z}_4z_3) + cz_4 + (\bar{b}z_4 + a)(\bar{b}z_4 + z_3), \\
 f(1, 0, z_3, z_4, z_5) &= d\bar{z}_3 + h(z_3 \circ z_4) + cz_4 + (\bar{b}z_4 + a)\bar{z}_5 + (\bar{b}z_4 + z_3)z_5 \\
 &= d\bar{z}_3 + h(z_3 \circ z_4) + cz_4 + a\bar{z}_5 + z_3z_5 + \bar{b}(z_4 \circ z_5)^*, \\
 f(z_1, z_2, z_3, z_4, z_5) &= d(z_1 \circ z_3) + h(z_3 \circ z_4) + c(z_4 \circ z_2) + a(z_5 \circ z_1) \\
 &\quad + \bar{b}(z_4 \circ z_5) + z_5(z_3 \circ z_2)
 \end{aligned}$$

(Fig. 3a). If instead of the last term we take $z_3(z_5 \circ z_2)$, we obtain another realization (Fig. 3b).

For the applicability of the theory presented to a process with a changing input signal (input potential and the state of external relays), it should be assumed that the input signal changes rarely (in comparison with the quantity τ) and that every change of the input signal is accompanied by the return of the internal relays to the nonoperating state, for example by de-energizing the circuit.

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$$* x \circ y = x\bar{y} + y\bar{x} \quad \text{—sum modulo 2.}$$

Note: Figure translations are in progress. See original paper for figures.

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