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DETERMINATION OF THE CRITICAL TEMPERATURE OF A SUPERCONDUCTOR WITH A PARAMAGNETIC IMPURITY IN A TWO-BAND MODEL

V. A. MOSKALENKO

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In paper ([1]) the influence of a nonmagnetic impurity on the critical temperature T_c of a superconductor in the two-band model was investigated. It was shown that, owing to transitions of electrons between overlapping energy bands, under the influence of a nonmagnetic impurity a substantial change in the value of T_c occurs: an increase in the impurity concentration is accompanied by a decrease in the critical temperature.

In the present note we give the main results of an investigation of the influence of a paramagnetic impurity on the value of T_c. The operator of interaction of an electron with spin $\vec{\sigma}$ with a paramagnetic impurity atom has the form

$$V(\mathbf{r}) = U(\mathbf{r}) + S\vec{\sigma}\,U'(\mathbf{r}). \tag{1}$$

The positions of the impurity atoms and the orientations of their spins S are assumed to be chaotic, and therefore an averaging over them is subsequently performed (denoted by an overbar).

Using the notation of paper ([1]) and the results of paper ([2]), it is not difficult to see that the electronic one-particle Green function in our case differs from the case of a nonmagnetic impurity by replacing the quantity $|U(\mathbf{q})|^2$ in the definition of the relaxation times τ_{ij} by

$$|U(\mathbf{q})|^2 + \frac{S(S+1)}{4}|U'(\mathbf{q})|^2,$$

i.e., in the present case:

$$\frac{\hbar}{2\tau_{ij}} = \frac{c\pi}{(2\pi\hbar)^3}\int\frac{dS_j}{|\nabla E_j|}\left[|U(\mathbf{k}_i^F-\mathbf{k}_j^F)|^2 + \frac{S(S+1)}{4}|U'(\mathbf{k}_i^F-\mathbf{k}_j^F)|^2\right]\times$$

$$\times|\chi(i\mathbf{k}_i^F, j\mathbf{k}_j^F)|^2. \tag{2}$$

Along with the averages of the one-particle Green functions, let us consider the average of their product:

$$K_{\alpha\beta\alpha'\beta'}(xy;x'y') = \overline{G_{\alpha\alpha'}(x,x')G_{\beta\beta'}(y,y')}. \tag{3}$$

This quantity enters the equation determining the critical temperature T_c:

$$F(x\alpha, y\beta) = \int \cdots \int dx_1 \dots dx_4 \sum_{\sigma_1 \dots \sigma_4} K_{\alpha\beta\sigma_1\sigma_2}(xy, x_1x_2)\times$$

$$\times\sigma(x_1\sigma_1x_2\sigma_2;\ x_3\sigma_3x_4\sigma_4)F(x_3\sigma_3, x_4\sigma_4). \tag{4}$$

The solution of this equation must possess the property

$$F(x\alpha, y\beta) = -F(y\beta, x\alpha) \neq 0 \quad \text{for } \beta = -\alpha. \tag{5}$$

The function F can be represented in the form

$$F(x\alpha, y\beta) = g_{\alpha\beta}F(xy); \qquad F(xy) = F(yx); \qquad \hat{g} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{6}$$

(6) entails the introduction of the function

$$K_{\alpha\beta}(xy;x'y') = \sum_{\alpha'\beta'} K_{\alpha\beta\alpha'\beta'}(xy;x'y')g_{\alpha'\beta'}. \tag{7}$$

As is readily seen from (2), the following equation holds:

$$K_{\alpha\beta}(xy;x'y') = g_{\alpha\beta}K(xy;x'y'); \tag{8}$$

$$K(xy;x'y') = \widetilde{G}(xx')\widetilde{G}(yy') + \int_0^\beta \int dx_1\, dx_2\, \widetilde{G}(xx_1)\widetilde{G}(yx_2)\times$$

$$\times\, \frac{c}{V}\sum_q e^{-i\mathbf{q}(\mathbf{x}_1-\mathbf{x}_2)} \left[|U(\mathbf{q})|^2 - \frac{S(S+1)}{4}|U'(\mathbf{q})|^2\right] K(x_1x_2;x'y'). \tag{9}$$

Thus the equation determining the value of T_c takes the form

$$F(x,y)=\int_0^\beta\int dx_1\,dx_2\,K(xy,x_1x_2)\,B(x_1-x_2)\,F(x_1,x_2), \tag{10}$$

where, as in (1), the quantity σ has been replaced by its zero approximation—the Green' s B-function of free phonons.

The further solution of the system of equations (3) and (10) can obviously be carried out in the same way as in paper ([1]). It should only be noted that in the definition of the function K, unlike in paper ([1]), instead of the quantity $|U(\mathbf{q})|^2$ there appears

$$|U(\mathbf{q})|^2-\frac{S(S+1)}{4}|U'(\mathbf{q})|^2,$$

which will lead to the appearance of new relaxation times χ_{ij}:

$$\frac{\hbar}{2\chi_{ij}}=\frac{c\pi}{(2\pi\hbar)^3}\int_{S_j}\frac{dS_j}{|\nabla E_j|}\left[\left|U(\mathbf{k}_i^F-\mathbf{k}_j^F)\right|^2-\right.$$

$$\left.-\frac{S(S+1)}{4}\left|U'(\mathbf{k}_i^F-\mathbf{k}_j^F)\right|^2\right]\left|\chi(i\mathbf{k}_i^F,j\mathbf{k}_j^F)\right|^2. \tag{11}$$

Taking into account the above-mentioned modifications in comparison with paper ([1]), for determining the critical temperature of the superconductor we obtain the following final system of equations $(r,m=1,2)$.

$$f_r(\Omega)=\frac{\pi}{\beta}\sum_{\Omega'}\sum_m\mathcal{L}_{rm}(\Omega,\Omega')\,N_m\Lambda_m(\Omega')f_m(\Omega'); \tag{12}$$

$$N_m=\frac{1}{2\pi^2}\left(\frac{k^2}{|\nabla E_m|}\right)_{k=k_m^F};\qquad \Lambda_m(\Omega)=\frac{1}{\pi\eta_m(\Omega)}; \tag{13}$$

$$\eta_m(\Omega)=1+\hbar/2\tau_m|\Omega|;\qquad 1/\tau_m=1/\tau_{m1}+1/\tau_{m2}; \tag{14}$$

$$\mathcal{L}_{rm}(\Omega,\Omega')=\Delta_{rm}(\Omega,\Omega')/\Delta(\Omega); \tag{15}$$

$$\Delta_{1m}(\Omega,\Omega')=4\pi\left[V_{1m}(\Omega-\Omega')\left(1-\Lambda_2(\Omega)\frac{\hbar}{2\chi_{22}}\right)+V_{2m}(\Omega-\Omega')\frac{\hbar\Lambda_2(\Omega)}{2\chi_{12}}\right]; \tag{16}$$

$$\Delta_{2m}(\Omega,\Omega') = 4\pi\left[V_{2m}(\Omega-\Omega')\left(1-\frac{\hbar\Lambda_1(\Omega)}{2\chi_{11}}\right)+V_{1m}(\Omega-\Omega')\frac{\hbar\Lambda_1(\Omega)}{2\chi_{21}}\right]. \quad (17)$$

$$\Delta(\Omega) = \left(1-\frac{\hbar\Lambda_1(\Omega)}{2\chi_{11}}\right)\left(1-\frac{\hbar\Lambda_2(\Omega)}{2\chi_{22}}\right)-\frac{\hbar\Lambda_1(\Omega)}{2\chi_{21}}\frac{\hbar\Lambda_2(\Omega)}{2\chi_{12}}; \qquad (18)$$

$$V_{rm}(\Omega-\Omega')N_m = \frac{1}{(2\pi)^3}\int_{S_m}\frac{dS_m}{|\nabla E_m|}\,B(\mathbf{k}_r^F-\mathbf{k}_m^F|\Omega-\Omega')\,|\chi(r\mathbf{k}_r^F, m\mathbf{k}_m^F)|^2. \quad (19)$$

Separating the logarithmically large terms in $\beta_c = (kT_c)^{-1}$, following the works ([3],[4]), in the system of equations (12) one can obtain the following relation between the quantity β_c and the relaxation times of impurity scattering:

$$\ln\frac{\beta_c}{\beta_{c0}} = \mp\frac{(p_1+p_2)p_1N_1}{(e_2-e_1)\sqrt{b_0^2-4a}}\left(I(\beta_c\sqrt{e_2})-I(\beta_c\sqrt{e_1})\right)\times$$
$$\times\left[V_{11}+V_{22}j-V_{21}j_1-V_{12}jj_2-\frac{N_2+jN_1}{2N_1N_2}\left(b_0\pm\sqrt{b_0^2-4a}\right)\right]-$$
$$-\frac{\sqrt{e_1}I(\beta_c\sqrt{e_2})-\sqrt{e_2}I(\beta\sqrt{e_1})}{\sqrt{e_2}-\sqrt{e_1}}, \qquad (20)$$

where

$$p_i = \frac{\hbar}{2\tau_i}-\frac{\hbar}{2\chi_{ii}}; \qquad a = N_1N_2(V_{11}V_{22}-V_{12}V_{21}) \quad b_0 = V_{11}N_1+V_{22}N_2;$$

$$e_{1,2} = \frac{p_1^2+p_2^2+2p_1p_2j_1j_2}{2}\mp\sqrt{\left[\frac{p_1^2+p_2^2+2p_1p_2j_1j_2}{2}\right]^2-p_1^2p_2^2(1-j_1j_2)^2},$$

$$j = \frac{N_2}{N_1}\frac{p_2}{p_1}; \qquad j_1 = \frac{\hbar}{2\chi_{12}p_1}; \qquad j_2 = \frac{\hbar}{2\chi_{21}p_2}; \qquad (21)$$

$$I(x) = \int_0^\infty\frac{dt}{t}\,\frac{\operatorname{th}xt/2}{1+t^2};$$

β_{c0} is the reciprocal critical temperature of the pure metal. In the right-hand side of (20) terms have been omitted that are due to the influence of the impurity on the characteristic frequencies of the system entering the definition of the critical temperature of the system.

In the case when $U' = 0$, i.e., the impurity is nonmagnetic, we have

$$p_1 = \frac{\hbar}{2\tau_{12}}; \qquad p_2 = \frac{\hbar}{2\tau_{21}}; \quad j_1 = j_2 = 1; \quad e_1 = 0; \quad e_2 = (p_1 + p_2)^2.$$

In this case expression (20) naturally goes over into the corresponding expression of work ([1]).

In the region of small impurity concentration $|\beta_c \sqrt{e_i}| \ll 1$, the last term of formula (20) is negligible. In this limiting case we obtain a linear change of the critical temperature with impurity concentration:

$$T_c \simeq T_{c0} - \frac{(p_1 + p_2) p_1 N_1 \pi}{4(\sqrt{e_2} + \sqrt{e_1})\sqrt{b_0^2 - 4a}} \left[V_{11} + V_{22} j - V_{21} j_1 - V_{12} j j_2 - \right.$$

$$\left. - \frac{N_2 + j N_1}{2 N_1 N_2} \left(b_0 \pm \sqrt{b_0^2 - 4a} \right) \right].$$

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Moscow State University
named after M. V. Lomonosov

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Note: Figure translations are in progress. See original paper for figures.