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Academician of the Academy of Sciences of the Ukrainian SSR A. S. Davydov, V. I. Ovcharenko

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Abstract

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Electric Quadrupole Transitions Between Rotational States with Large Spins in Even Nuclei

In experiments recently carried out at the University of California, Stephens, Lark, and Diamond ([1]) investigated the energy spectra of rotational states with large spin (up to $I = 16$) in 9 nonspherical even atomic nuclei. In these works it was shown that the energies of the rotational states agree well (discrepancy less than 1.5%) with the theoretical predictions of the work of Davydov and Chaban ([2]), in which a theory of rotational states was developed with allowance for deformation of the nuclear shape (without a change in volume). The deformability of nuclei during rotation was determined in ([2]) (see also ([3])) by the nonadiabaticity parameter μ.

In an absolutely rigid ($\mu = 0$) axially symmetric even nucleus, the ratios of the energies of successive rotational states must satisfy the interval rule

$$1 : {}^{10}/_3 : 7 : 12 : {}^{55}/_3 : 26 : 35 : \dots .$$

In "soft" nuclei this interval rule changes substantially. For example, at $\mu = 1$ it is replaced by the ratios

$$1 : 2.32 : 3.77 : 5.27 : 6.80 : 8.34 : \dots .$$

Simultaneously with the change in the ratios of the energies of the rotational states, the probabilities of cascade $E2$ transitions also change, by means of which nuclei pass from an excited state to the ground state. Experimental study of the probabilities of $E2$ transitions may serve as an additional independent method for determining the deformability of the shape of atomic nuclei upon their transition to a rotational state. In connection with this, we have carried out calculations of the probabilities of $E2$ transitions between neighboring rotational states in even axial nuclei without using the adiabatic approximation.

The wave functions of the ground rotational band of even axial nuclei for spin values $I = 0, 2, 4, \dots$ may be written, according to ([2]), in the form

$$\Psi_{IM}(\beta, \theta_i) = \sqrt{\frac{2I+1}{8\pi^2}}\, D^I_{M0}(\theta_i)\, \varphi_I(\beta), \tag{1}$$

where $D^I_{M0}(\theta_i)$ are generalized spherical functions ([4]), depending on the three Euler angles; β is a parameter characterizing the nonsphericity of the nuclear shape;

$$\varphi_I(\beta) = \frac{N(I)}{\beta^{3/2}\beta_0} \exp\left\{-\frac{1}{2}\left[\frac{\beta - p_I\beta_0}{\beta_0\mu_I}\right]^2\right\}. \tag{2}$$

Here β_0 is the parameter characterizing the equilibrium shape of the nucleus in the ground–

state;

$$N(I) = \frac{\sqrt{2}}{\mu_I}\left\{e^{-x_I^2} + x_I\sqrt{\pi}\left[1 + \Phi(x_I)\right]\right\}^{-1/2};$$

$$x_I = \frac{p_I}{\mu_I}, \qquad \Phi(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt.$$

The real quantities $p_I \geq 1$ and $\mu_I \geq 0$ are uniquely determined for each value of the nonadiabaticity parameter μ and spin I by the equalities

$$p_I^4 - p_I^3 = \frac{1}{3}I(I+1)\mu^4, \qquad \mu_I^4 = \frac{\mu^4}{4 - 3/p_I}.$$

Using (1), one can show that the reduced probability of an $E2$ transition between neighboring rotational states is

$$B(E2; I+2 \to I) =$$

$$= e^2 Q_{20}^2 \frac{15(I+1)(I+2)}{32\pi(2I+3)(2I+5)} S(I,\mu), \tag{3}$$

$$Q_{20} = 3ZR_0^2\beta_0/5\pi; \qquad S(I,\mu) = \beta_0^{-2}\left|\langle\varphi_{I+2}|\beta|\varphi_I\rangle\right|^2$$

is a correction factor taking into account the change in the shape of the nucleus during rotation.

With the aid of (2) we find

$$S(I,\mu) = c^{-8}(I)N^2(I+2)N^2(I)e^{-a(I)}\{b(I)+$$

$$+\frac{c^2(I) + b^2(I)}{c(I)} e^{\frac{1}{2}[b(I)/c(I)]^2}\sqrt{\frac{\pi}{2}}$$

Fig. 1

Figure 1: Fig. 1

$$\times \left[1 + \Phi\left(\frac{b(I)}{\sqrt{2}c(I)}\right)\right]\Bigg\}^2. \quad (4)$$

In this expression the abbreviated notation

$$a(I) = x_{I+2}^2 + x_I^2, \qquad b(I) = x_{I+2}\mu_{I+2}^{-1} + x_I\mu_I^{-1},$$

$$c(I) = \left[\mu_{I+2}^{-2} + \mu_I^{-2}\right]^{1/2}.$$

has been used.

From (4) it follows that in the adiabatic approximation ($\mu = 0$) the correction factor $S(I, 0) = 1$ for any values of I. In deformed nuclei ($\mu \neq 0$) the factor $S(I, \mu)$ is greater than unity, and the larger the spin of the rotational state I, the more significant it is. Table 1 gives, as functions of I and μ, the values of $S(I, \mu)$ and the quantity p_I, which determines the ratio of the equilibrium value of the nuclear deformation β_I during rotation to its value β_0 in the ground state. Fig. 1 shows the dependence of the ratios of the reduced probabilities

Fig. 1

Table 1

I	$\mu =$ 0.2	$\mu =$ 0.2	$\mu =$ 0.4	$\mu =$ 0.4	$\mu =$ 0.6	$\mu =$ 0.6	$\mu =$ 0.8	$\mu =$ 0.8	$\mu =$ 1.0	$\mu =$ 1.0
I	p_I	$S(I,\mu)$	p_I	$S(I,\mu)$	p_I	$S(I,\mu)$	p_I	$S(I,\mu)$	p_I	$S(I,\mu)$
2	1.003	1.053	1.045	1.303	1.164	1.802	1.340	2.516	1.543	3.423
4	1.010	1.070	1.121	1.460	1.351	2.228	1.630	3.309	1.929	4.674
6	1.021	1.093	1.205	1.645	1.518	2.684	1.873	4.125	2.242	5.940
8	1.035	1.122	1.288	1.843	1.669	3.149	2.085	4.944	2.513	7.203
10	1.051	1.155	1.367	2.047	1.806	3.615	2.275	5.760	2.754	8.460
12	1.068	1.193	1.443	2.255	1.933	4.081	2.449	6.573	2.975	9.742
14	1.087	1.233	1.515	2.464	2.051	4.545	2.611	7.382	3.179	10.958
16	1.107		1.584		2.162		2.762		3.370	

probabilities of the $E2$ transitions.

$$\Lambda(E2; I+2 \to I) = B(E2; I+2 \to I)/B(E2; 2 \to 0)$$

of the parameter μ. In rigid nuclei ($\mu \approx 0$) the ratio Λ of the reduced probabilities never exceeds 2. In soft nuclei, at $\mu = 1$, this ratio is approximately equal to $\frac{1}{2}I + 1$.

The values of μ, determined in (1) for the nuclei Hf^{172}, Yb^{166}, W^{176}, Hf^{170}, W^{174}, Yb^{164}, Hf^{168}, W^{172}, Hf^{166}, are respectively 0.24; 0.27; 0.28; 0.29; 0.30; 0.31; 0.35; 0.33; 0.38. From Fig. 1 we see that, for these values of μ, the change in the relative probabilities of cascade $E2$ transitions caused by deformation of the nucleus during rotation should be very substantial. An experimental verification of this prediction of the theory is desirable.

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Institute of Physics
Academy of Sciences of the Ukrainian SSR

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Note: Figure translations are in progress. See original paper for figures.