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Abstract

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ON THE INFLUENCE OF PHOTODISSOCIATION ON THE DISTRIBUTION OF SURFACE BRIGHTNESS IN THE CARBON ATMOSPHERE OF COMETS

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Introduction. In the paper "Models of a Cometary Coma" F. Miller (^{1}), discussing the results of observations of Comet Burnham (1959 k) with the aid of an interference filter isolating the band λ5165 Å belonging to C_2, arrives at the following conclusions:

1) The isophotes in the coma were circular at distances up to 150,000 km from the photometric center.
2) During the period from 23 IV to 4 V 1959 the observed surface brightness as a function of distance from the nucleus changed little. An independent photoelectric scan made on 1 V by O' Dell (^{2}) is in brilliant agreement with the photographic photometry of 28 IV and 2 V.
3) For distances from the nucleus $R < 20{,}000$ km the changes in surface brightness are close to the law $1/R$, while for $R > 20{,}000$ km they correspond more nearly to the law $1/R^2$. A similar distribution was also observed in the comets 1955 e, 1955 g.
4) Because of the violation of the $1/R$ law, the stationary model of Mokhnach must be abandoned. An attempt to explain the observed distribution on the basis of Hazer' s work (^{3}), which attempted to take account of the effect of dissociation, was unsuccessful. None of Hazer' s models gave satisfactory agreement with the observations. For example, if the theoretical and observed brightnesses are taken to coincide at $R = 20{,}000$ km, then at $R = 3000$ km and $R = 150{,}000$ km theory and observations differ by about 80% of the quantity being determined.

In view of this failure, I have attempted to construct the simplest model of a carbon coma to explain the observed deviations from the $1/R$ law.

1. Derivation of the formula for the visible density with allowance for dissociation of the luminous molecules.

Assuming the ejection of luminous molecules from a point—the emission center, which is the origin of the cometocentric coordinate system—we obtained (^{4}) the formula for computing the visible density

$$N(x,y) = \frac{2}{v_0} \int_{t_1}^{t} \frac{n\,dt}{t\sqrt{-\frac{1}{4}g^2t^4 + (v_0^2 - gx)t^2 - (x^2+y^2)}}. \tag{1}$$

Formula (1) was derived under the assumption that the line of sight is perpendicular to the radius vector of the comet. The molecular velocity v_0 and the acceleration g are regarded as constant.

For a constant emission coefficient n, independent, moreover, of the angle of ejection, and for integration limits t_1 and t_2 that make the expression under the radical vanish, we obtain, for any angle of inclination of the line of sight to the radius vector, the law

$$N(R) \sim 1/R, \tag{2}$$

where $R^2 = x^2 + y^2$.

With a variable emission coefficient the integrand in (1) changes and the inverse-distance law is violated. In particular, n may become a function of t under the influence of photodissociation. Let us consider the pro-

the simplest case of dissociation under the action of the constant radiation field of the Sun.

Let n_0 be the number of particles ejected from the emission center into a steradian per second. If we begin to take the effect of photodissociation into account from the moment $t = t_0$, then between t and $t + \Delta t$ the number of molecules that have dissociated is determined by the equation

$$dn = -k_1 n\,dt, \tag{3}$$

where n is the number of molecules remaining by the time $t > t_0$ out of the total number of molecules n_0 at $t = t_0$.

As a result of integrating equation (3), we find

$$n = n_0 e^{-k(t/t_0 - 1)}. \tag{4}$$

The coefficient k in formula (4) characterizes the rate of the process and is equal to the ratio t_0/τ, if τ denotes the mean lifetime of a molecule under photodissociation. These quantities can be determined from observations.

Using formulas (1) and (4), we obtain an expression for calculating the visible density with allowance for the influence of molecular photodissociation

$$N(x,y) = \frac{2n_0}{v_0}\left[\int_{t_1}^{t_0} \frac{dt}{t\sqrt{-\frac{1}{4}g^2t^4+(v_0^2-gx)t^2-(x^2+y^2)}} + e^k\int_{t_0}^{t}\frac{e^{-kt/t_0}dt}{t\sqrt{-\frac{1}{4}g^2t^4+(v_0^2-gx)t^2-(x^2+y^2)}}\right]. \tag{5}$$

If $t_0 \leq t_1$, the first integral vanishes. In deriving formula (5) we assumed that the heliocentric distance of the comet changes insignificantly, i.e., that the acceleration g_0, the initial value of the emission coefficient n_0, and the coefficient k can be regarded as constant during the time the molecule remains in the coma. Therefore formula (5) should be regarded as a first approximation in studying the question of the influence of photodissociation on the distribution of surface brightness in the head of a comet.

2. Distribution of surface brightness in the head of the 1959 comet

In paper (1) F. Miller suggested that the cause of the deviation from the $1/R$ law in the distribution of surface brightness of the carbon coma of Burnham’ s 1959 comet is photodissociation.

This supposition is quite plausible. Outbursts of solar activity that could produce an effect close to the observed one were not noted in the second half of April and in May 1959. The $1/R$ law, observed many times in the cyanogen coma, does not contradict Miller’ s supposition. The dissociation energy of C_2 is 3.6 eV, and for CN it is 7 eV (5), which corresponds to $\lambda\,3440$ Å (C_2) and $\lambda\,1770$ Å (CN). The intensity of solar radiation at $\lambda\,3440$ Å is several orders of magnitude greater than at $\lambda\,1770$ Å. Therefore one can simultaneously observe the $1/R$ law in the cyanogen coma and deviations from it in the carbon coma.

Changes in the heliocentric distance of the comet during the period of observations are insignificant. For example, on 27 IV $r = 0.975$, on 1 V $r = 1.042$. Consequently, the line of sight may be regarded as perpendicular to the radius vector. Since Miller made measurements in the direction perpendicular to the projection of the radius vector, one may put $x = 0$ in formula (5) and calculate $N(0, y)$.

Table 1 gives the values of $N(0, y)$ for $k = 0.25$ and $k = 0.5$, with y varying from 10^8 to $14 \cdot 10^9$ cm. The initial velocity is $v_0 = 1.2 \cdot 10^5$ cm/sec, $g = 0.36$ cm/sec^2; the latter corresponds to $f = 0.02$, residual intensity $= 0.80$, $r = 1$ AU, and a solar temperature of 5750° K. For the calculation

formula (5) is transformed as follows:

$$N(0,y) = \frac{2n_0 e^k}{v_0 y} \left[\frac{\pi}{2} e^{-kt_1/t_0} - \frac{k}{t_0} \int_{t_1}^{t_2} e^{-kt/t_0} \operatorname{arc\,tg} \frac{t_1}{t_2} \sqrt{\frac{t_2^2 - t^2}{t^2 - t_1^2}}\, dt \right], \quad \text{if } t_0 \leqslant t_1; \tag{5a}$$

$$N(0,y) = \frac{2n_0}{v_0 y} \left[\frac{\pi}{2} - \frac{ke^k}{t_0} \int_{t_0}^{t_2} e^{-kt/t_0} \operatorname{arc\,tg} \frac{t_1}{t_2} \sqrt{\frac{t_2^2 - t^2}{t^2 - t_1^2}}\, dt \right], \quad \text{if } t_0 > t_1. \tag{5b}$$

The values of $N(0,y)$, given in Table 1 to within a constant factor, were calculated under the assumption that $t_0 = 1.667 \cdot 10^4$ sec. If, with our model and the adopted values $v_0 = 1.2 \cdot 10^5$ and $g = 0.36$ cm/sec^2, t_0 is taken smaller, then a noticeable deviation from the $1/R$ law will be detected at distances less than $2 \cdot 10^9$ cm from the photometric center. This contradicts the data of F. Miller. Therefore $t_0 = 1.667 \cdot 10^4$ sec may be regarded as an upper limit for the dissociation time of the C_2 molecule. The circular form of the observed isophotes permits two possible explanations: either the cloud of parent molecules from which the C_2 molecules are formed is small in comparison with the dimensions of the coma, or the $1/R$ law holds in it if its dimensions are of the order of 10^9 cm. This requires additional investigations.

Table 1*

y, 10^9 cm	$N_{0.25}$	$\frac{c_1}{y}$	$\frac{c_2}{y^{1.4}}$	$\frac{c_3}{y^2}$	$N_{0.5}$	$\frac{c_1}{y}$	$\frac{c_2}{y^2}$	$\frac{c_3}{y^3}$
0.1	15.7	14.85			15.5	14.0		
0.5	2.97	2.97			2.80	2.80		
1.0	1.40	1.48	1.40		1.33	1.40		
2.0	0.58	0.74	0.53	0.81	0.51	0.70	0.52	
3.0	0.31	0.39	0.30	0.35	0.22	0.47	0.23	
4.0	0.20	0.37	0.20	0.20	0.18	0.35	0.13	
6.0	0.090			0.090	0.071		0.058	0.036
8.0	0.050			0.051	0.018		0.032	0.015
10.0	0.028			0.032	0.078			0.0078
12.0	0.018			0.022	0.037			0.0045
14.0	0.012			0.016	0.0018			0.0028

* $N_{0.25}$ and $N_{0.5}$ are the values of $N(0,y)$, calculated from formulas (5a) and (5b), for $k = 0.25$ and $k = 0.05$.

3. Calculation of the total flux and the possibility of control. From Table 1 we see that, for different values of the coefficient k, the change in surface brightness can be satisfactorily represented by a power function of the distance R from the photometric center. For example, for $k = 0.5$

$$N(R) \sim R^{-1} \quad \text{for } 0 < R < 2 \cdot 10^9 \text{ cm},$$

$$N(R) \sim R^{-2} \quad \text{for } 2 \cdot 10^9 \leqslant R \leqslant 6 \cdot 10^9,$$

$$N(R) \sim R^{-2} \quad \text{for } 6 \cdot 10^9 < R \leqslant 14 \cdot 10^9.$$

For $k = 0.25$

$$N(R) \sim R^{-1} \quad \text{for } 0 < R < 2 \cdot 10^9,$$

$$N(R) \sim R^{-2} \quad \text{for } 2 \cdot 10^9 \leqslant R \leqslant 14 \cdot 10^9.$$

In the latter case we see that the law $R^{-1.4}$ for the interval $10^9 \leqslant R \leqslant 4 \cdot 10^9$ gives a better result than R^{-1} and R^{-2}.

Since the isophotes are circular, it is easy to find the total amount of energy emitted by the part of the coma bounded by a circular isophote of radius R. For this it is necessary to calculate the integral

$$I = \int_0^{2\pi} d\varphi \int_0^R N(r)\, r\, dr = 2\pi \int_0^R N(r)\, r\, dr. \tag{6}$$

Table 2 gives the values of integral (6), in arbitrary units, for different laws of surface-brightness distribution. The value $R_1 = 1$ for $2 \cdot 10^9$ cm. I_1 corresponds to the case in which the law R^{-1} is preserved out to a distance of $14 \cdot 10^9$ cm; I_2 gives the value of the integral if, for $0 \leq R \leq 2 \cdot 10^9$, the law R^{-1} holds, while for larger values $N \sim R^{-2}$. I_3 is obtained if $N \sim R^{-1}$ out to a distance of $2 \cdot 10^9$, $N \sim R^{-2}$ for $2 \cdot 10^9 \leq R \leq 6 \cdot 10^9$, and at greater distances $N \sim R^{-3}$.

Table 2

R/R_1	0.0	0.5	1.0	2.0	3.0	4.0	5.0	6.0	7.0
I_1	0.0	0.5	1.0	2.0	3.0	4.0	5.0	6.0	7.0
I_2	0.0	0.5	1.0	1.7	2.1	2.39	2.61	2.79	2.95
I_3	0.0	0.5	1.0	1.7	2.1	2.35	2.50	2.60	2.67

It is clear from Table 2 that measurement of the total flux through diaphragms isolating regions with different values of R, in the manner used by O' Dell (2), can confirm and, consequently, control measurements through light filters. At large R, I_2 and I_3 differ appreciably from I_1, which was calculated for the law $1/R$.

For a deeper study of these phenomena, observations through filters and diaphragms must be carried out with the highest possible accuracy. It would also be of interest to investigate, in parallel, the brightness distribution in the cyanogen coma. If the radiation of CN and C_2 gives different distribution laws, we can speak with greater confidence of photodissociation as the cause of the violation of the $1/R$ law.

The failure of Haser’ s model to explain the distribution observed by F. Miller apparently resulted from neglect of the acceleration factor g, which is of substantial importance for C_2.

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Note: Figure translations are in progress. See original paper for figures.

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