L. A. VASIL' EV

1964

SovietRxiv

Abstract

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L. A. VASIL' EV

A SHADOW DIFFRACTION METHOD FOR DETERMINING THE PHASE DIFFERENCE AT A DISCONTINUITY OF A PLANE WAVE-FRONT

(Presented by Academician I. V. Obreimov, January 13, 1964)

In studying optical inhomogeneities, researchers often have to deal with plane inhomogeneities. Typical examples of such objects are phase plates for phase-contrast microscopes, air-density variations (arising in the flow around wedge-shaped models, wing profiles, turbine blades), etc.

From geometrical theory it follows that the plane phase jumps under consideration cannot be visualized by means of a shadow instrument. This conclusion contradicts the experimentally obtained data: plane jumps are clearly visible in shadow photographs. The decisive role here is played by the wave nature of light.

Since the visibility of plane phase jumps is determined mainly by diffraction phenomena, the determination of any parameters of these inhomogeneities (the true position of the jump or the additional increase in optical path length produced by the jump) from the character of the shadow image can be carried out only by measuring the parameters of the diffraction pattern obtained on the shadow photograph.

To calculate the shadow pattern, let us assume that in the object plane of the shadow instrument the wavefront undergoes a discontinuity at the point $x = x_0$ (Fig. 1). The path difference ΔL, in wavelengths, between the points of the wavefront corresponding to $x < x_0$ and $x > x_0$, is equal to δ. The diaphragm in the object plane has the form of an infinitely long slit parallel to the y-axis. The boundaries of the slit have coordinates $x = R$ and $x = -R$.

As the light source we take a luminous point. The diaphragm of the focal plane of the principal objective of the receiving part of the shadow instrument is a Foucault knife edge—a slit diaphragm with edge coordinates $\xi = \xi_0$ and $\xi = r$.

Application of the theory set forth in ([1–3]) gives in this case the following formula for the distribution of light intensity in the image plane of the shadow instrument:

$$I(x') = 2c^2 \left\{ (1-\cos\delta)\left(\operatorname{ci}\frac{k}{F}x'r - \operatorname{ci}\frac{k}{F}x'\xi_0\right)^2 + \right.$$

$$+(1-\cos\delta)\left(\operatorname{si}\frac{k}{F}x'r - \operatorname{si}\frac{k}{F}x'\xi_0\right)^2 + (1+\cos\delta)\binom{0}{-\pi}^2 +$$

$$\left. +2\binom{0}{\pi}\sin\delta\left(\operatorname{ci}\frac{k}{F}x'r - \operatorname{ci}\frac{k}{F}x'\xi_0\right)\right\}. \quad (1)$$

The upper entries of the columns in (1) correspond to values $\xi_0 > 0$, and the lower ones to $\xi_0 < 0$.

In deriving (1) it was assumed that the edges of the diaphragm in the object plane, located at $x = R$ and $x = -R$, are sufficiently far from the jump that their influence on the distribution of illumination at points lying close to the image of the discontinuity may be neglected. The origin of coordinates has been transferred to the point $x = x_0$.

Examination of (1) shows that the intensity distribution is symmetric with respect to the image of the discontinuity under study. At the place where, according to geometri–

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Fig. 2. Shadow image of a phase jump $\delta = \lambda/4$

···in geometrical optics the image of the discontinuity must be located, there is always an intensity maximum.

The presence of symmetry is essential for finding the true position of the discontinuity, which is necessary in the qualitative interpretation of shadow photographs (for example, when measuring the distance from a detached density jump to the boundary of a model). The symmetry always present in the diffraction pattern substantially distinguishes it from the diffraction pattern at an opaque boundary.

The intensity of the central maximum is determined from (1) by substituting $x' = 0$

$$I(0) = 4c^2\left[\sin\frac{\delta}{2}\ln\frac{r}{\xi_0} + \binom{\theta}{-\pi}\cos\frac{\delta}{2}\right]^2. \quad (2)$$

The simplicity of expression (2) makes it possible to propose two methods for determining the increment of the optical path length in the jump by measuring the intensity of the central diffraction maximum.

Fig. 1. Shape of the wave front in the object plane of a shadow instrument

Figure 1: Fig. 1. Shape of the wave front in the object plane of a shadow instrument

The first of them is based on comparing the blackening of the photographic material in the "zero" and "working" photographs at the places corresponding to the position of the jump.

The light intensity in the image plane in the absence of a density jump is equal to (for $\xi_0 < 0$ and in the absence of influence of the field boundaries)

$$I_n(0) = 4\pi^2 c^2. \tag{3}$$

Fig. 1. Shape of the wave front in the object plane of a shadow instrument

The ratio of the light intensity in the "zero" and "working" photographs is related to the difference in optical densities of the blackening of the photographic material by

$$\frac{D - D_0}{\gamma} = \lg \frac{I(0)}{I_n(0)}. \tag{4}$$

Combining (2), (3), and (4), we obtain

$$\sin\frac{\delta}{2} = \frac{A \ln\frac{r}{\xi_0} + \left(\frac{\theta}{\pi}\right)\sqrt{\left(\frac{\theta}{\pi}\right)^2 + \ln^2\frac{r}{\xi_0} - A^2}}{-\left(\frac{\theta}{\pi^2}\right) + \ln^2\frac{r}{\xi_0}}, \tag{5}$$

where

$$A = \pi \cdot 10^{(D-D_0)/2\gamma}. \tag{6}$$

If, in the working position, the knife completely covers the image of the slit, and during the taking of the comparison photograph it does not cover it, then (5) takes the form

$$\sin\frac{\delta}{2} = \frac{A}{\ln\frac{r}{\xi_0}}. \tag{7}$$

In the case where, in the working position, the knife completely covers the image of the slit, one may apply a second method for determining the additional path difference in the jump, based on comparing the intensity of the diffraction maximum from the jump with the intensity of the maximum from an opaque boundary. In doing this it must be taken into account that both diffraction patterns must be obtained with one and the same adjustment of the shadow instrument.

If the experimental conditions permit and the illumination of the image is uniform over the entire field of the instrument, then the maxima located on one and the same photograph may be compared. If this proves difficult, then an additional photograph may be taken in which the boundary of an opaque body is placed approximately at the location where the jump of the wave front should be.

The additional path difference introduced by the jump is determined from

$$\sin^2\frac{\delta}{2} = \frac{I}{4I_1}, \tag{8}$$

where I is the intensity of the maximum located at the discontinuity, and I_1 is the intensity of the maximum at the boundary of the opaque body.

Taking (4) into account, we obtain

$$\sin\frac{\delta}{2} = \frac{10^{(D-D_0)/2\gamma}}{2}, \tag{9}$$

where D and D_0 are the optical densities of blackening at the maxima located at the discontinuity and at the opaque boundary.

The sensitivity of the method, defined as

$$= \frac{d(D-D_0)}{d\delta}, \tag{10}$$

is equal to

$$= \gamma \lg e \cdot \operatorname{ctg}\frac{\delta}{2}. \tag{11}$$

It follows from (11) that for $\gamma = 2.3$, $d(D-D_0) = 0.02$, and $\delta = \pi/4$, the relative measurement error is 0.5%.

When considering the character of the illumination distribution of the diffraction pattern at points that do not coincide with the geometrical image of the phase step, the action of the edge of the diaphragm in the focal plane of the main objective of the receiving part of the shadow instrument, located at $\zeta = r$, may be neglected, and (1) takes the form

$$I(x') = 2c^2\left\{(1-\cos\delta)\,\mathrm{ci}^2\,\frac{k}{F}x'\zeta_0 + (1-\cos\delta)\left[\left(\frac{\pi}{2}, -\frac{\pi}{2}\right) - \mathrm{si}\,\frac{k}{F}x'\zeta_0\right]^2 + \right.$$

$$+\left(\frac{0}{-\pi}\right)^2(1+\cos\delta)-2\left(\frac{0}{-\pi}\right)\sin\delta\,\mathrm{ci}\,\frac{k}{F}x'\zeta_0\Bigg\}. \quad (12)$$

The first terms in the rows correspond to positive values of x', the second to negative ones.

Equating, in order to determine the positions of the extrema of the light intensity, the derivative of (12) with respect to x' to zero, we obtain

$$\mathrm{tg}\,\frac{\delta}{2}=\frac{\left(\frac{0}{-\pi}\right)}{\mathrm{ci}\,\frac{k}{F}x'\zeta_0-\left[\left(\frac{\pi}{2},-\frac{\pi}{2}\right)-\mathrm{si}\,\frac{k}{F}x'\zeta_0\right]\mathrm{tg}\,\frac{k}{F}x'\zeta_0}. \quad (13)$$

For $\zeta_0>0$, this equation is not satisfied over the entire region of variation of x'. This means that, with the slit completely closed, there is only one diffraction maximum of light intensity, coinciding in position with the location of the geometrical image of the plane discontinuity of the wave front.

If the image of the slit is not closed by the knife edge, then a series of maxima is present in the image. Their position relative to the discontinuity depends on the magnitude of the additional path difference in the step and on the relative position of the knife edge and the image of the illuminating slit.

For a more detailed analysis of the problem and an experimental study of the methods described, let us consider diffraction patterns obtained experimentally on an -451 instrument. A narrow slit (0.02 mm), illuminated by a K-10 projection lamp with a light filter having a transmission maximum in the region of 6000 Å, served as the light source. The objects were glass plates on which phase steps had been deposited by the evaporation method, the additional path differences in them being $\lambda/4$ and $\lambda/2$.

The obtained series of diffraction patterns for $\zeta_0<0$ and $\delta=\lambda/4$ are shown in Fig. 2, see insert, p. 821.

Study of the obtained series of patterns shows, first of all, complete qualitative agreement between the practically obtained results and the theoretical conclusions. In each series of patterns for $\zeta_0<0$ (completely open slit), the diffraction pattern is a system of fringes symmetric with respect to the place where the geometrical image of the discontinuity should be located. At the center of the pattern, as was assumed, a maximum of intensity is always located.

As the distance from the knife edge to the image of the light source is decreased, the distance between the diffraction maxima increases; they broaden, occupying an ever larger part of the field. The most complicated phenomena occur when the knife begins to cover the image of the slit. Here the inaccuracies in the manufacture of the plates have a strong effect. Diffraction from the edges of the plates also has a strong influence. Therefore, in this zone (-0.04 mm $<\xi_0<$ 0.02 mm) the pattern is difficult to explain theoretically (especially if one takes into account the complexity of the wave fronts when real optical inhomogeneities are varied) and cannot be used for practical measurements.

When the knife completely covers the image of the slit, the diffraction pattern differs sharply from the one just considered. There is only one light maximum, located at the position of the image of the discontinuity. Its intensity decreases as the absolute value of ξ_0 increases.

To check the applicability of the proposed quantitative methods, photometry of the maxima was carried out with the image of the illuminating slit covered. The additional optical path difference was determined by comparing the optical density of the darkening of the photographic material in the diffraction maxima from a plane discontinuity and from an opaque boundary.

As a result of the measurement, values were obtained that were in complete agreement with the certified data, which had been obtained by the method of multiple-beam interferometry. In the range of increments of optical path length introduced by the plate of $\lambda/4$, the accuracy of the measurements was $3 \cdot 10^{-3}\lambda$. This is quite sufficient for most investigations. When measuring plates with an additional path difference equal to one half of the wavelength of light, the measurement error increases to $4 \cdot 10^{-2}\lambda$. In this region, large measurements of the phase of the wave correspond to small changes in the intensity of light in the diffraction maximum.

An attempt to determine the additional path difference in the discontinuity from the position of the diffraction maxima was not successful, since the distance between the maxima depends much more strongly on the position of the knife than on the quantity being measured. Therefore, a comparatively small error in measuring the position of the extrema leads to a large error in determining δ.

One should dwell in particular on one characteristic feature of diffraction methods: patterns of discontinuities that differ from one another in the magnitude of the change in optical path length by an integral number of wavelengths have one and the same distribution of light intensity.

Diffraction methods are suitable only for measuring discontinuities that introduce an additional path difference not greater than $\lambda/2$. But they can also be used to refine the value of the intensity of a strong density discontinuity, if a more approximate value, measured with an accuracy up to $\lambda/2$ by other methods, is known.

Received
2 I 1964

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Note: Figure translations are in progress. See original paper for figures.