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Abstract

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Renormalization of the Quasipotential Equation for a System of Two Particles

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In work ([1]) a quasipotential equation was obtained for the wave function of a system of two particles

$$(E^2 - p^2 - m^2)\,\psi(\mathbf{p}) = \int V(\mathbf{p}, \mathbf{p}'; E)\,\psi(\mathbf{p}')\,d\mathbf{p}', \tag{1}$$

where the quasipotential V is determined with the aid of the two-time Green function of two particles. Owing to its three-dimensional character, this equation has a number of advantages in comparison with the Bethe–Salpeter equation. In constructing the quasipotential V by perturbation theory, the problem of eliminating divergences arises. This problem could be regarded as solved if it were possible to show that the corresponding divergences reduce to divergences of the S-matrix, since the prescription for eliminating the latter is well known. This situation occurs in the case of the Bethe–Salpeter equation ([2]). On the contrary, in the Tamm–Dancoff method this cannot be achieved, which is one of the principal difficulties of the method.

In the present work the construction of the quasipotential V for a system of two particles of equal mass is considered. It will be shown that, in the case of renormalizable theories in which there are only triple point vertices (for example, electrodynamics), no divergences arise in the quasipotential V other than those characteristic of the S-matrix. These theories are distinguished by the fact that in them only the proper energy and vertex parts diverge, while diagrams describing scattering processes no longer have proper divergences.

Let us consider a system of two particles of equal mass (for example, an electron and a positron). Denote the four-momenta in the c.m. system of these particles in the initial state by $(E + \varepsilon_q, \mathbf{q})$ and $(E - \varepsilon_q, -\mathbf{q})$, and in the final state by $(E + \varepsilon_p, \mathbf{p})$ and $(E - \varepsilon_p, -\mathbf{p})$, where $2E$ is the total energy of the system.

Then the two-time Green function of two particles $\overline{G}$ is defined in terms of the four-dimensional Green function G as follows ([1]):

$$\overline{G}(\mathbf{p},\mathbf{q},E) = \int_{-\infty}^{\infty}\!\!\int d\varepsilon_p d\varepsilon_q\, G(\mathbf{p},\mathbf{q},\varepsilon_p,\varepsilon_q,E) \tag{2}$$

(the bar will henceforth be used to denote integration over energies).

The function $\overline{G}$ satisfies the equation ([1])

$$\overline{G}(\mathbf{p},\mathbf{q},E) = \overline{G}_0(\mathbf{p},\mathbf{q},E) - i\int \overline{G}_0(\mathbf{p},\mathbf{k}',E)\,V(\mathbf{k}',\mathbf{k},E)\,\overline{G}(\mathbf{k},\mathbf{q},E)\,d\mathbf{k}'\,d\mathbf{k}, \tag{3}$$

where

$$G_0(\mathbf{p},\mathbf{q},\varepsilon_p,\varepsilon_q,E) = S_1(\mathbf{p},E+\varepsilon_p)\,S_2(-\mathbf{p},E-\varepsilon_p)\,\delta(\mathbf{p}-\mathbf{q})\,\delta(\varepsilon_p-\varepsilon_q),$$

while S_1 and S_2 are the complete one-particle Green functions. In what follows we shall use the symbolic notation for equation (3)

$$\overline{G} = \overline{G}_0 - i\overline{G}_0 V\overline{G}. \tag{4}$$

In this case the quasipotential V is given by the expression

$$iV = \overline{G}^{-1} - \overline{G}_0^{-1}. \tag{5}$$

Instead of the Green function G, it is convenient formally to introduce the two-particle scattering amplitude off the mass shell

$$-iG_0TG_0 = G - G_0. \tag{6}$$

An analogous quantity can also be defined for the two-time Green function $\overline{G}$

$$-i\overline{G}_0\widetilde{T}\overline{G}_0 = \overline{G} - \overline{G}_0. \tag{7}$$

Comparing (6) and (7), we obtain the relation between $\widetilde{T}$ and T

$$\widetilde{T} = \overline{G}_0^{-1}\,\overline{G_0TG_0}\,\overline{G}_0^{-1} \tag{8}$$

(in what follows the sign $\sim$ is understood in the sense of this definition).

It follows from expression (8) that on the energy surface ($p^2 = q^2 = E^2 - m^2$) the function $\widetilde{T}$ coincides with the quantity T on the mass shell ($\varepsilon_p = \varepsilon_q = 0,\ p^2 = q^2 = E^2 - m^2$) and thus gives the physical scattering amplitude. Introducing the quantity diagram: two full vertex parts Γ connected by a wavy boson line through a propagator D, with an arrow labeled E

Figure 1: diagram: two full vertex parts Γ connected by a wavy boson line through a propagator D, with an arrow labeled E

$$F = -iG_0, \tag{9}$$

we obtain from (4) and (7) an equation for $\widetilde{T}$ in the form

$$\widetilde{T} = V + V\overline{F}\widetilde{T}. \tag{10}$$

Since equation (10) is equivalent to equation (4), it can also serve to define the quasipotential V, namely,

$$V = \widetilde{T}(1 + \overline{F}\widetilde{T})^{-1}, \tag{11}$$

where $\widetilde{T}$ is defined in (8).

For the further discussion it is convenient to represent T as the sum of two terms

$$T = M + L. \tag{12}$$

Here L is the set of weakly connected diagrams in which the particles of the initial state are connected with the particles of the final state by a single boson line. These diagrams have the following general form:

$$L = \Gamma D \Gamma, \tag{13}$$

where Γ is the full vertex part, and D is the full boson propagator. Expression (13) may be represented graphically:

Expanding (11) in perturbation theory, we obtain divergent expressions of the type $\widetilde{L}\overline{F}\widetilde{T}$. These divergences are not removed by the usual renormalization of the S-matrix. However, as we shall now show, in fact these terms are absent from expression (11). Let us represent the vertex part Γ in the form

$$\Gamma = \gamma(1 + FM), \tag{14}$$

where γ is the point vertex, E is defined in (9), and M in (12). Representation (14) can be depicted graphically:

Substituting (14) into (13), we obtain

diagram showing the vertex Γ represented as a point vertex γ plus a term with M

Figure 2: diagram showing the vertex Γ represented as a point vertex γ plus a term with M

$$L = (1 + MF)\,\gamma D\gamma\,(1 + FM). \tag{15}$$

Now we need to obtain the quantity $\widetilde{L} = \overline{F}^{-1}\,\overline{FLF}\,\overline{F}^{-1}$ according to (8). Since the function D in (15) depends only on the total energy E of the system, the relation

$$\overline{ADB} = \overline{A}D\overline{B} \tag{16}$$

holds for arbitrary functions A and B. Thus, using (15) and (16), we obtain

$$\widetilde{T} = \widetilde{M} + \widetilde{L} = \widetilde{M} + (1 + \widetilde{M}\overline{F})\,\gamma D\gamma\,(1 + \overline{F}\widetilde{M}) =$$

$$= \widetilde{M}\,[1 + \overline{F}\gamma D\gamma\,(1 + \overline{F}\widetilde{M})] + \gamma D\gamma\,(1 + \overline{F}\widetilde{M}). \tag{17}$$

It is convenient to represent the quantity $(1 + \overline{F}\widetilde{T})^{-1}$ in the following two forms:

$$(1 + \overline{F}\widetilde{T})^{-1} = (1 + \overline{F}\widetilde{M})^{-1}[1 + \overline{F}(1 + \widetilde{M}\overline{F})\gamma D\gamma]^{-1} =$$

$$= [1 + \overline{F}\gamma D\gamma(1 + \overline{F}\widetilde{M})]^{-1}(1 + \overline{F}\widetilde{M})^{-1}. \tag{18}$$

Now, substituting (17) and (18) into definition (11), we obtain

$$V = \widetilde{M}(1 + \overline{F}\widetilde{M})^{-1} + \gamma D\gamma\,[1 + \overline{F}(1 + \widetilde{M}\overline{F})\gamma D\gamma]^{-1}. \tag{19}$$

Using representation (14) for the vertex part and definition (8), we obtain

$$\gamma\overline{F}(1 + \widetilde{M}\overline{F})\gamma = \gamma F\Gamma. \tag{20}$$

Thus, taking (20) into account, expression (19) for the quasipotential V can be rewritten in the form

$$V = \widetilde{M}(1 + \overline{F}\widetilde{M})^{-1} + \gamma D(1 + \gamma F\Gamma D)^{-1}\gamma. \tag{21}$$

We now use the Dyson equation for the boson propagator:

$$D = \Delta + \Delta\gamma F\Gamma D, \tag{22}$$

where Δ is the Green' s function of the free boson. Then, obviously,

$$(1 + \gamma F\Gamma D)^{-1} = D^{-1}\Delta,$$

and, substituting this expression into (21), we finally obtain:

$$V = \widetilde{M}(1 + \overline{F}\widetilde{M})^{-1} + \gamma\Delta\gamma. \tag{23}$$

It follows from expression (23) that the quasipotential V splits into two terms, with only the diagrams M contributing to the first term, while from the diagrams L only the lowest-order term of perturbation theory remains (the second term), which obviously contains no divergences. In this respect the quasipotential V is analogous to the kernel of the Bethe–Salpeter equation—

Peter. The diagrams M contain the usual divergences of the S-matrix. Expanding the first term in (23) in perturbation theory, we obtain expressions of the type $\hat{M}\hat{F}\hat{M}$. In these expressions no additional divergences arise if one considers theories in which only the self-energy diagrams and the three-point vertex parts diverge (for example, electrodynamics). Thus, in this case the quasipotential V contains no divergences other than those characteristic of the S-matrix, and the renormalization of the quasipotential equation (1) is carried out completely analogously to the renormalization of the S-matrix. In the case of renormalizable theories in which the diagrams describing scattering processes diverge (for example, meson on meson), further investigation is required. In this case one can get rid of possible additional divergences if, in passing to the equation for partial waves, one discards the s- and p-waves [(3)]. In Ref. [(4)] it was shown that no new divergences appear in the kernel of the instantaneous Bethe—Salpeter equation in the ladder approximation. However, when the kernel contains terms describing the possibility of virtual annihilation of the particles of the system (for example, the diagrams L), then, as is clear from the above, an additional consideration is necessary.

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Note: Figure translations are in progress. See original paper for figures.