PHYSICS

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Abstract

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ANALYSIS OF QUASI-MAXWELLIAN EQUATIONS DESCRIBING COMPENSATING FIELDS

1. In this paper we study general quasilinear equations for an arbitrary field, first introduced for the case of gravitation in the work of one of us ([1]), in the form

$$\operatorname{rot} R = sv + \frac{\partial N}{\partial t}; \qquad \operatorname{div} N = \rho; \qquad L = \frac{4\pi G}{c^2} R;$$

$$\operatorname{rot} \Gamma = -\frac{\partial L}{\partial t}; \qquad \operatorname{div} L = 0; \qquad N = \frac{\Gamma}{4\pi G},$$

where the vectors R; Γ; L; N fully characterize the gravitational-inertial field; ρ is the density; v is the velocity of motion of the medium; G is Newton' s gravitational constant. Equations of this type are naturally derived from Noether' s theorem ([2–5]), generalized to the case of localized groups of transformations of reference systems.

2. In this case we have:

$$L_T = L(\psi, \nabla_\mu \psi) + L_0(A^a_\mu; A^a_{\mu,\nu}),$$

where L and L_0 correspond to the dynamical and field parts of the Lagrangian, and $\nabla_\mu = \partial_\mu - I_a A^a_\mu$, with

$$\delta A^a_\mu = \varepsilon^b f^a_{bc} A^c_\mu + \partial_\mu \varepsilon^a. \tag{1}$$

The form of the covariant derivative ∇_μ and the transformation law of the compensating field A^a_μ ([2–5]) can be obtained from the condition of covariance of the wave equation for particles of arbitrary spin 3 with respect to the local gauge group: $\delta\psi = \varepsilon^a(x) I_a \psi$, where

$$[I_a I_b] = f^c_{ab} I_c, \qquad \varepsilon^a = \varepsilon^a(x).$$

Let us formulate Noether' s theorem for local gauge transformations.

Varying L_T under the condition that

$$\frac{\partial L}{\partial \psi} - \nabla_\mu \frac{\partial L}{\partial \nabla_\mu \psi} = 0,$$

$$\frac{\partial L_0}{\partial A^a_\mu} - \partial_\nu \frac{\partial L_0}{\partial A^a_{\mu,\nu}} = J^\mu_a, \tag{2}$$

where

$$J^\mu_a = \frac{\partial L}{\partial \nabla_\mu \psi} I_a \psi, \qquad \frac{\partial L_0}{\partial A^a_\mu} + \frac{\partial L_0}{\partial A^b_{\nu,\mu}} f^b_{ac} A^c_\nu = 0,$$

and, moreover,

$$\nabla_\mu \frac{\partial L}{\partial \nabla_\mu \psi} = \partial_\mu \frac{\partial L}{\partial \nabla_\mu \psi} + A^a_\mu \frac{\partial L}{\partial \nabla_\mu \psi} I_a,$$

we obtain from $\delta L_T = 0$ the conservation law

$$\partial_\mu \left(J^\mu_a + \overset{0}{J}{}^\mu_a \right) = 0, \qquad \overset{0}{J}{}^\mu_a = \frac{\partial L_0}{\partial F^b_{\mu\nu}} f^b_{ac} A^c_\nu, \tag{3}$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - \frac{1}{2} f^a_{bc} \left(A^b_\mu A^c_\nu - A^b_\nu A^c_\mu \right).$$

The formula for the field strength A^a_μ, through which

$$L_0 = f^l_{am} f^m_{bl} F^a_{\mu\nu} F^{b;\mu\nu},$$

is expressed, follows from the generalized condition of gradient invariance $L_0[F^a_{\mu\nu}]$, since $\delta F^a_{\mu\nu} = \varepsilon^b f^a_{bc} F^c_{\mu\nu}$.

From the extremum condition (2) for A^a_μ there follows the quasi-Maxwell system of equations

$$f^l_{am} f^m_{bl} \partial_\nu F^{b;\mu\nu} = J^\mu_a + \overset{0}{J}{}^\mu_a. \tag{4}$$

In the particular case of an Abelian gauge group

$$\psi' = e^{i\alpha(x)}\psi, \qquad \text{where } f^a_{bc} = 0, \tag{4'}$$

the system reduces to Maxwell' s equations.

3. Analyzing the compensating field A^a_σ by the method given in (4), one can show that in A^a_σ longitudinal and transverse parts are distinguished:

$$A^a_\sigma = \overset{T}{A}{}^a_\sigma + \overset{L}{A}{}^a_\sigma,$$

so that $F^a_{\mu\nu}(\overset{L}{A}{}^a_\sigma) = 0$. In this case $F^a_{\mu\nu}$ for the complete A^a_σ has the form:

$$F^a_{\mu\nu} = \omega^{a\tau}_\lambda R^{\cdot\cdot\cdot\lambda}_{\mu\nu;\tau}, \qquad \omega^{a\tau}_\lambda = \langle \Omega^\tau \mid I^a \mid \Omega_\lambda \rangle,$$

where Ω^τ, Ω_λ define a basis of the representation of the local group specified by the generators I_a.

It follows from this that, in the general case, the quasi-Maxwell equation (4) reduces to the form:

$$f^l_{am} f^m_{bl} \partial_\nu (\omega^{b\tau}_\lambda R^{\mu\nu\cdot\lambda}_{\cdot\cdot\tau}) = J^\mu_a + \overset{0}{J}{}^\mu_a. \tag{5}$$

4. To analyze the physical meaning of the quasi-Maxwell equations, let us separate the spatial and temporal components of the fields. To this end we write:

$$F^a_{\mu\nu} = -\xi_p H^a_p - i\eta_p E^a_p,$$

where ξ_p, η_p correspond to spatial and temporal infinitesimal rotations. Then

$$H^a = \operatorname{rot} A^a - f^a_{bc}[A^b \vee A^c],$$

$$E^a = -\operatorname{grad}\varphi^a - \frac{1}{c}\frac{\partial}{\partial t}A^a - \frac{1}{2} f^a_{bc}(\varphi^b A^c - A^b \varphi^c). \tag{6}$$

In this case the transformations of A^a_σ preserving (4) have the form

$$\varphi^{a'} = \varphi^a - \frac{1}{c}\frac{\partial \varepsilon^a}{\partial t} + \varepsilon^b f^a_{bc}\varphi^c,$$

$$A^{a'} = A^a + \operatorname{grad}\varepsilon^a + \varepsilon^b f^a_{bc} A^c,$$

where $\{\varphi^a; A^a\} = A^a_\sigma$, with φ^a, A^a_σ being generalizations of the scalar and vector potentials, respectively.

Substituting (6) into (4), we obtain the three-dimensional notation for the field equations:

$$\operatorname{rot} E^a + \frac{1}{c}\frac{\partial H^a}{\partial t} = f^a_{bc}\{[A^c \vee \operatorname{grad}\varphi^b] - \varphi^b \operatorname{rot} A^c\}, \tag{7}$$

$$\operatorname{div} H^a = f^a_{bc}\{A^b \cdot H^c - A^c \cdot H^b\} + 4 f^a_{cb} f^c_{b''b} A^b \cdot [A^{b'} \vee A^{b''}],$$

$$\operatorname{div} E^a = s^a + \overset{0}{s}{}^a, \qquad \operatorname{rot} H^a - \frac{1}{c}\frac{\partial E^a}{\partial t} = G^a + \overset{0}{G}{}^a,$$

where G^a_0, G^a_0, S^a and S^a_0 are the spatial and temporal components of the currents J^μ_a and $J^\mu_{a\,0}$. In the particular case of the local Lorentz group (3), equations (7) take the form of equations for the gravitation tensor in orthogonal components $R_{\mu\nu}(i,k) = \Omega^\tau(i)\Omega_\lambda(k)R_{\mu\nu\tau}{}^\lambda$; for this it is sufficient to substitute into (7) the structure of the Lorentz group (3):

$$f^{ik}_{lm;i'k'} = g^k_{k'}\mu^{lm}_{ii'} + g^i_{i'}\mu^{lm}_{kk'},$$

where

$$\mu^{lm}_{ii'} = \frac{1}{2}\left(g^l_i g^m_{i'} - g^l_{i'} g^m_i\right).$$

Then

$$H^{ik} = \operatorname{rot} A^{ik} - 8\,[A^{ip} \vee A^{pk}], \tag{8}$$

$$E^{ik} = -\operatorname{grad}\varphi^{ik} - \frac{1}{c}\frac{\partial A^{ik}}{\partial t} - 4\,[\varphi^{ip}A^{pk} - A^{ip}\varphi^{pk}].$$

Thus we have arrived at the "second pair" of quasi-Maxwell equations for the case of a gravitational field.

The first pair has the form:

$$\operatorname{rot} H^{ik} - \frac{1}{c}\frac{\partial E^{ik}}{\partial t} = J^{ik} + J^{ik}_0,$$

$$\operatorname{div} E^{ik} = \rho^{ik} + \rho^{ik}_0.$$

The equations obtained, close to those published in ([1]), are of particular interest, since their quasi-Maxwell character is preserved also in a strong gravitational field, while the nonlinearity is completely contained in the current J_0.

It is also interesting to note that in the case of an arbitrary field A^a_μ, (4) reduces to (5), i.e. to an equation for $R_{\mu\nu}(i,k)$ close to (8).

Thus we arrive at a unified description of fields based on quasilinear equations of type (4).

In conclusion, let us note that the nonlinear term $J^{\mu}_{a\,0}$, entering into the extended conservation law (3), can probably be used in describing gravitational waves, as a term corresponding to an essentially strong gravitational field.

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Note: Figure translations are in progress. See original paper for figures.