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CONTINUUM MECHANICS

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ON THE MOTION OF IMMISCIBLE LIQUIDS IN A FRACTURED-POROUS MEDIUM

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A general scheme for the motion of a homogeneous liquid in media with double porosity was proposed in work (^{1}). In this note the motion of immiscible liquids, for example oil and water, in a hydrophilic fractured-porous medium is considered.

1. Let water begin to be injected, at time $t = 0$, with flow rate $q(t)$, into an oil-saturated formation consisting of a porous medium intersected by a branched system of fractures oriented in various ways. Water, penetrating into the formation through the fractures, is imbibed into the rock blocks under the action of capillary forces.

We shall regard the fracture system as highly developed. Then, using the concepts of continuum mechanics, we assume that each elementary volume of the formation dV contains a sufficiently large number of blocks dN. Let, at time t, the water that has penetrated into the formation through the fractures have encompassed N rock blocks contained in the volume V. Taking into account that the volume of the fractures is usually two or more orders of magnitude smaller than the volume of the pore space of the blocks, one may neglect the volume of the fractures themselves in comparison with the capacity of the blocks and assume that the entire flow rate of the water entering the formation, $q(t)$, is spent on the imbibition of the blocks. We assume the dependence of the rate of water entering the rock blocks in an elementary volume of the formation on time, $\varphi(t)$, to be known from experiment. Let

$$F(x_1, x_2, x_3, \vartheta) = 0$$

(x_i are coordinates) be the unknown surface bounding the formation volume $V(\vartheta)$ encompassed by water by the time $\vartheta \leq t$.

Figure 1

Figure 1: Figure 1

Fig. 2. Schematic of flooding of a fractured-porous rectilinear reservoir with formation of a "stabilized" zone. 1 —fractures; 2 —completely flooded blocks; 3 —blocks undergoing flooding; 4 —unflooded blocks

Figure 2: Fig. 2. Schematic of flooding of a fractured-porous rectilinear reservoir with formation of a "stabilized" zone. 1 —fractures; 2 —completely flooded blocks; 3 —blocks undergoing flooding; 4 —unflooded blocks

Fig. 1. Waterflooding of a model of a fractured-porous medium at successive moments of time $1, 2, 3, 4$ (motion of water from right to left)

Then, considering the balance of liquid imbibed into the rock blocks per unit time, we obtain, for determining the unknown function $\vartheta(x_1, x_2, x_3)$, the integral equation

$$\int\limits_V \varphi\left[t - \vartheta(x_1, x_2, x_3)\right] dV = q(t). \tag{1}$$

The motion of oil and water in fractures is described by a system of differential equations for the filtration of immiscible fluids (^{2}), which, taking capillary imbibition into account, assume the form indicated by V. M. Ryzhik:

$$\operatorname{div} \mathbf{u}_1 + m\frac{\partial \rho}{\partial t} + \varphi\left[t - \vartheta(x_1, x_2, x_3)\right] = 0; \tag{2}$$

$$\operatorname{div} \mathbf{u}_2 - m\frac{\partial \rho}{\partial t} - \varphi\left[t - \vartheta(x_1, x_2, x_3)\right] = 0; \tag{3}$$

$$\mathbf{u}_1 = -\frac{k}{\mu_1} f_1(\rho) \operatorname{grad} p, \qquad \mathbf{u}_2 = -\frac{k}{\mu_2} f_2(\rho) \operatorname{grad} p, \tag{4}$$

where k, m are the permeability and porosity in the fracture system; $\mathbf{u}_1, \mathbf{u}_2$ are the velocities of water and oil in the fractures; $f_1(\rho), f_2(\rho)$ are the relative permeabilities for water and oil in the fracture system; μ_1, μ_2 are the viscosities of water and oil; ρ is the water saturation in the fracture system; p is the fluid pressure.

Fig. 2. Schematic of flooding of a fractured-porous rectilinear reservoir with formation of a "stabilized" zone. 1 —fractures; 2 —completely flooded blocks; 3 —blocks undergoing flooding; 4 —unflooded blocks

The system of equations (1), (2), (3), (4) completely describes the process of motion of immiscible fluids in a fractured-porous medium.

2. Experimental studies of capillary imbibition by water of porous blocks initially saturated with hydrocarbon fluids, carried out by Mattax and Kyte (^{3}) and by D. Sh. Vezirov and A. A. Kocheshkov (^{4}), showed that the imbibition function is approximated sufficiently well for practical purposes by the dependence

$$\varphi(t)=\frac{A}{2}\,m_B\rho_k\,\frac{s^2\sigma\cos\theta\sqrt{k/m_B}}{\mu_2}\left(t\frac{s^2\sigma\cos\theta\sqrt{k/m_B}}{\mu_2}\right)^{-1/2},\quad t\leqslant t_k, \tag{5}$$

where m_B is the porosity of the blocks; ρ_k is the saturation of the blocks with water at the time t_k; s is the averaged specific surface area of the blocks; σ is the surface tension; θ is the contact angle; A is a constant coefficient.

At the time t_k, all the oil that can be displaced from the block by capillary imbibition enters the fractures and, thus, the capillary displacement of oil from the block ceases.

To clarify the mechanism of oil displacement by water from a fractured-porous medium, experiments were carried out on a model consisting of three blocks connected in series. Figure 1 presents photographs of the successive position of the displacement front in experiments on the indicated model. The dark regions correspond to water, the light regions to oil. The experiments showed that water, breaking through along the fracture of the first block and being imbibed into the regions of the blocks adjoining the fracture, then enters the second block, where water imbibition also occurs, and then the third block. In this process the velocity of water motion along the fracture decreases.

After a longer time has elapsed, the first block in Fig. 1 proves to be practically completely impregnated. Impregnation continues in the second and third blocks, i.e., the impregnation zone moves.

3. In the particular case of displacement of oil by water from a rectilinear fissured-porous bed (Fig. 2), the problem reduces to the solution of one integral and one differential equation:

$$\int_0^{\xi(\tau=T)}\varphi[T-\tau(\xi)]\,d\xi=\bar{q}(T); \tag{6}$$

$$\bar{q}(T)\Phi'(\rho)\frac{\partial\rho}{\partial\xi}+m\frac{\sigma\cos\theta\sqrt{k/m_B}s^2}{\mu_2}\frac{\partial\rho}{\partial T}+\varphi[T-\tau(\xi)]=0,$$

$$T=t\frac{\sigma\cos\theta\sqrt{k/m_B}s^2}{\mu_2},\qquad\Phi(\rho)=\frac{f_1(\rho)}{f_1(\rho)+\mu_0f_2(\rho)}, \tag{7}$$

Fig. 3. Dependence of ξ_* and ξ_*^1 on T (length of the stabilized zone $\lambda = 12$)

Figure 3: Fig. 3. Dependence of ξ_* and ξ_*^1 on T (length of the stabilized zone $\lambda = 12$)

$$\varphi(T) = \frac{A}{2} m_B \rho_k \frac{\sigma \cos\theta \sqrt{k/m_B} s^2}{\mu_2} T^{-1/2}, \qquad \xi = \frac{x}{l_*}, \qquad \mu_0 = \frac{\mu_1}{\mu_2},$$

where $\bar{q}(T)$ is the water flow rate, referred to a unit bed thickness h, bed width b, and the averaged block size l_*.

Fig. 3. Dependence of ξ_* and ξ_*^1 on T (length of the stabilized zone $\lambda = 12$)

Impregnation of the blocks in each element of a fissured-porous medium continues for a practically limited time. On the basis of experimental data we shall assume that impregnation in each element of the medium occurs only during the time T_*. Then we have the condition

$$T - \tau(\xi) \leq T_*. \tag{8}$$

Consider the case $\bar{q} = \text{const}$. The solution of equation (6) under the initial condition $\xi = 0$ at $T = 0$ has the form

$$\tau = a\xi^2, \qquad a = \left(\frac{\pi A}{4\bar{q}} \frac{s^2 \sigma \cos\theta \sqrt{k/m_B}\, m_B \rho_k}{\mu_2} \right)^2. \tag{9}$$

It is valid for $T \leq T_*$. For $T \geq T_*$, near $\xi = 0$ a rear front with coordinate ξ_*^1 is formed, behind which impregnation of the blocks is practically absent. This front will move following the forward front.

From physical considerations it follows that, for $\bar{q} = \text{const}$, after some time both fronts will move with the same velocity, forming a “stabilized” zone that moves uniformly.

The size of the stabilized zone λ is determined by the formula

$$\lambda = \xi_* - \xi_*^1 = \frac{T_*}{a_k}, \qquad \xi_* = \xi(\tau = T), \tag{10}$$

$$\xi_*^1 = \xi(\tau = T - T_*), \qquad a_k = \frac{\sigma \cos\theta \sqrt{k/m_B} s^2 m_B \rho_k}{\bar{q} \mu_2}.$$

Let us take real values of the initial parameters. Let $\sigma = 35$ dyn/cm; $\cos\theta = 0.6$; $k = 2$ md; $\mu_2 = 2.7$ cP; $l_* = 100$ cm; $\rho_k = 0.5$; bed thickness $h = 10$ m; bed width $b = 100$ m; $m_B = 0.07$; $A = 0.199$; $T_* = 25.3$.

The results of calculations of the dependences of ξ_* and ξ_*^1 on T for $\bar{q} = 1.9$ m^3/day, which corresponds to movement of the front of oil displacement by water at a velocity of 20 m/year, are given in Fig. 3.

The distribution of saturation and pressure in the fractures is determined from the solution of equation (7). Neglecting the second term in this equation in view of the extreme smallness of $m \sim 10^{-2} \div 10^{-3}$ and $\dfrac{\sigma \cos\theta \sqrt{k/m_B s^2}}{\mu_2} \sim 10^{-4}$, and integrating, we obtain, assuming that for $\xi = 0$, $\Phi(\rho) = 1$, the relation for determining ρ

$$\Phi(\rho) = 1 - \frac{1}{q(T)} \int_0^{\xi} \varphi[T - \tau(\xi)]\, d\xi. \tag{11}$$

The distribution of the liquid pressure is obtained directly from the law of motion (4).

The approach set forth above is also applicable to problems concerning the displacement of oil by water from heterogeneous strata.

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