Reports of the Academy of Sciences of the USSR

1964

SovietRxiv

Abstract

Full Text

Reports of the Academy of Sciences of the USSR

1964. Volume 156, No. 2

HYDROMECHANICS

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THE PROCESS OF ESTABLISHING A STEADY-STATE REGIME OF MELTING OF A VISCOUS GLASSLIKE MATERIAL

(Presented by Academician G. I. Petrov on December 24, 1963)

It has already been proved theoretically and experimentally that the time required for the establishment of a steady-state regime of material melting in a hypersonic gas flow can substantially exceed the time required to reach the destruction temperature at the surface ([1,2]). This applies especially to viscous glasslike materials of the quartz-glass type, in whose melting process evaporation of the melt film plays an important role. Nevertheless, a quantitative determination of the effect of nonstationarity has not yet been carried out, owing to difficulties caused by the necessity of taking into account the nonlinear relations between heat transfer and the melting of a viscous glasslike material, as well as evaporation and radiation from the surface of the film.

Of greatest practical interest is the case of material destruction in a gas flow whose parameters do not depend on time, since precisely this case occurs in the experimental study of the melting process ([1]), and since it is precisely in this case that neglecting the time required for establishment may lead to the complete unsuitability of the results obtained for estimating the effectiveness of the given material.

The formulation and method of solution of the problem of the unsteady regime of material melting in the neighborhood of the critical point in a high-temperature gas flow were set forth in work ([3]). A special feature of a glasslike material is the exponential dependence of the melt viscosity on temperature and, consequently, the absence of a sharp boundary separating the liquid and solid phases. The thickness and flow velocity of the molten film are determined, in addition to viscosity, by the magnitude of the shearing stresses: surface friction and pressure.

The time of establishment of a constant melting rate t_c may be defined as the value of time after which the difference between the given solution and the corresponding calculation by the steady-state program (marked by the subscript c) does not exceed a prescribed quantity ε (%):

$$\left|\frac{V_\infty(t) - (V_\infty)_c}{(V_\infty)_c}\right| < \frac{\varepsilon}{100},$$

where V_∞ is the mass removal rate.

The determination of the establishment time of the constant melting rate t_c was carried out under the broadest assumptions regarding the values of the thermophysical parameters of the material, and also with arbitrary variation of the individual characteristics of the incident gas flow. The stagnation pressure was varied from 0.1 to 10 bar, and the stagnation temperature from 3000 to 6000°K. The values found for t_c, as a function of the parameter $\lambda_0/C(V_\infty)_c^2$, are plotted in Fig. 1 ($\varepsilon = 10\%$); moreover, they form a "band," whose width reaches 15% in t_c.

It is interesting to note that the sign and magnitude of the deviation of individual points from the middle of the "band" are determined not only by the ratio between the values of the thermophysical properties, but also by the accuracy ε with which the nonstationary solution approaches the established one. As a rule, with increasing ε (decreasing t_c) the width of the band also increases. All this indicates that, even in the absence of evaporation and radiation from the surface of the melt film, the criterion $\lambda_0/C(V_\infty)_c^2$, apparently, is not completely

reflects the nature of the establishment. Nevertheless, taking into account the sufficiently arbitrary and wide range of variation of the various parameters of the problem (the number of variants exceeded 70), it may be considered that the tendency of the dependence of t_c on $\lambda\rho/C(V_\infty)_c^2$, shown in Fig. 1, is of a general character and can be extended to other cases of destruction of melting and evaporating materials.

From the theoretical point of view, it is most interesting to analyze the behavior of the obtained dependence for sufficiently large values of the independent variable.

Although, for considerations of machine time, calculations were not carried out in the range of small heat fluxes $(q_w)_{\mathrm{st}} < 1500$ kW/m^2, in Fig. 1 a branching of the curve into two segments has already become clearly apparent. The first of them (rectilinear) corresponds to the case when the decrease in the convective heat flux is not associated with a decrease in the stagnation pressure, while the second corresponds to a gradual transition from melting to pure sublimation.

As is known, the convective heat flux from a hot gas to the surface of a body can be represented in the form

$$q_w = (\alpha/c_p)(J_{00} - J_w),$$

Fig. 1. Calculation of the time required to establish a constant melting rate for one quartz-glass specimen. Different points—different calculation variants; dashed lines—calculation by formula (1)

Figure 1: Fig. 1. Calculation of the time required to establish a constant melting rate for one quartz-glass specimen. Different points—different calculation variants; dashed lines—calculation by formula (1)

where the heat-transfer coefficient (α/c_p) depends comparatively weakly on the stagnation temperature of the flow T_{00} and is proportional to the square root of the stagnation pressure P_{00}, while the enthalpy difference in the boundary layer $(J_{00} - J_w)$ is, to a first approximation, determined by the stagnation temperature.

Fig. 1. Calculation of the time required to establish a constant melting rate for one quartz-glass specimen. Different points—different calculation variants; dashed lines—calculation by formula (1).

On the other hand, the equilibrium rate of evaporation of the melt film can, according to the analogy between the equations of conservation of energy and concentration, be written in the form (3)

$$V_w = \frac{(\alpha/c_p)P_v}{\overline{M}(P_{00} - P_v)},$$

where P_v is the partial vapor pressure, and $\overline{M}$ is the ratio of the molecular weight of the gases in the oncoming flow to the molecular weight of the vapors.

It is now easy to see that if the decrease in heat flux is not associated with a decrease in stagnation pressure, then the share of evaporation in the overall process of mass removal cannot increase substantially, whereas the time of the onset of melting and, consequently, the time required to establish the stationary melting regime will tend to infinity.

A different picture is observed in the case when, at sufficiently large heat fluxes, the stagnation pressure gradually decreases. In this case, starting from a certain moment, the evaporation rate becomes so large that mass removal will occur practically only in the gaseous phase. This is also promoted by the fact that the shear stresses on ···

surface of the film—the pressure and friction gradients—also tend to zero as the braking pressure decreases.

Obviously, the branching point of the curve in Fig. 1 should, generally speaking, depend not only on the parameters of the incident flow, but also on the ratio between the viscosity and the partial pressure of saturated vapors. Thus, the subvariant marked in Fig. 1 by a triangle with the subscript corresponds to infinitely large viscosity and to the vapor pressure of quartz glass.

Fig. 2

Figure 2: Fig. 2

Fig. 2. Comparison of the character of establishment of the steady values of the surface temperature $\overline{T}_w$ and of the mass-entrainment velocity V_∞ for two calculation variants: solid lines—with allowance for the thickness of the melt film; dashed lines—without allowance for it.

Special calculations have shown that the process of establishing the steady regime is determined not by the character of the flow in the very thin melt film, but by thermodynamic equilibrium inside the solid body, whose outer surface moves with a velocity equal to the mass-entrainment velocity. In other words, the kinematic changes in the film occur much faster than the thermal changes in the solid body.

Figure 2 presents the results of comparing two variants, confirming the statement made. In one of them, the velocity profile in the film was replaced by the constant $V(t, y) \equiv V_\infty(t)$, while in the other it was calculated in accordance with the viscosity-temperature law for quartz glass.

Let us note two interesting facts: 1) both variants in Fig. 2 correspond well to each other; 2) the surface temperature is established much faster than the mass-entrainment velocity.

On this basis, a good approximation to reality should be the model of unsteady melting studied in Landau' s work (^{4}), according to which the melt film is blown off instantaneously ($V \equiv V_\infty(t)$), and the heat flux to the surface is constant, $\overline{q}_w = \text{const}$. It is only necessary, instead of the heat of melting L, to introduce that part of the heat effect of evaporation, ΓH , which is actually realized under the given flow conditions ($\Gamma = (V_w/V_\infty)_c$ is the ratio of the evaporation velocity to the mass-entrainment velocity).

In this formulation, the "time of onset of melting" may be represented in the form

$$t_m = \frac{\pi}{4}\frac{\lambda\rho C}{\overline{q}_w^2}(T_w - T_0)^2 = \frac{\pi}{4}\left(\rho\,\frac{\Gamma H}{\overline{q}_w}\,am\right)^2,$$

where T_w is the surface temperature; T_0 is the temperature of the unheated material; $a^2 = \lambda/\rho c$ is the thermal diffusivity coefficient; $m = C(T_w - T_0)/\Gamma H$. The established value of the mass-entrainment velocity is equal to

$$(V_\infty)_c = \overline{V}_\infty = \frac{\overline{q}_w}{\Gamma H \;\; + C(T_w - T_0)},$$

and the time of establishment of a constant melting velocity is

Fig. 3

Figure 3: Fig. 3

$$t_c = \frac{\lambda\rho}{C\bar{V}_\infty^2}\, f(m,\varepsilon), \tag{1}$$

or, in another notation:

$$t_c = \frac{\lambda\rho C}{\bar{q}_w^2}(T_w - T_0)^2 F(m,\varepsilon). \tag{2}$$

The form of the functions $f(m,\varepsilon)$ and $F(m,\varepsilon)$ for two values of the accuracy with which the steady melting regime is established, $\varepsilon = 10\%$ and $\varepsilon = 5\%$, is shown in Fig. 3, which summarizes the results of the numerical calculations of work (4). With the aid of formulas (1) and (2), it is easy to interpret the results presented in Fig. 1. It should only be kept in mind that $\bar{q}_w$ differs from the convective heat flux by the magnitude of the effect of vapor blowing and radiation from the film surface.

Fig. 3. Character of the variation of the functions $f(m,\varepsilon)$ and $F(m,\varepsilon)$ entering formulas (1) and (2),
$F(m = \infty, \varepsilon = 10\%) = 1.33; \quad F(m = \infty, \varepsilon = 5\%) = 1.88$

In the case of pure melting ($m = \infty$), the time of establishment is expressed by a linear function of the parameter $\lambda\rho/C\bar{V}_\infty^2$, and the slope of this dependence is determined by the accuracy ε with which it is desired to approach the steady-state value (decreasing ε from 10 to 1% leads to an increase of $f(\infty,\varepsilon)$ by more than an order of magnitude).

As the parameter $\Gamma = (V_w/V_\infty)_c$, or correspondingly the evaporation fraction, increases, the slope of the curve $t_c(\lambda\rho/C\bar{V}_\infty^2)$ begins to decrease and tends to a certain value $f(m_{\text{lim}},\varepsilon)$, determined by the surface-temperature magnitude

$$m_{\text{lim}} = C(T_w - T_0)/1 \cdot H_{\text{isp}}$$

(for quartz glass, m_{lim} is of the order of 0.3).

The presence of evaporation from the surface of the melt film at a given value of the heat flux $\bar{q}_w$ always increases the establishment time in comparison with the establishment time of the steady regime of pure melting, as follows from formula (2).

On the other hand, at a given rate of mass removal (for example, when $(V_\infty)_c$ has been found experimentally), the upper and lower bounds for the establishment time are readily determined from formula (1). For this it is sufficient to estimate t_c at $m = \infty$ and $m = m_{\text{lim}}$.

Thus, relations (1) and (2) make it possible, with a prescribed accuracy, to determine the time of establishment of a constant mass-removal rate for all cases of destruction of melting, evaporating, and sublimating materials. It is necessary, however, to take into account that the effective values of the thermal conductivity of many glassy materials under aerodynamic heating and melting turn out to be substantially higher than the tabulated values obtained at low temperatures (5).

Received
20 XII 1963

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