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MECHANICS

S. A. KHARLAMOV

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Abstract

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MECHANICS

S. A. KHARLAMOV

AN EXAMPLE OF HETEROPARAMETRIC EXCITATION OF A PENDULUM BY QUASIPERIODIC OSCILLATIONS OF THE SUSPENSION

(Presented by Academician A. Yu. Ishlinskii, 26 III 1964)

A mathematical pendulum with a vertical axis of rotation, under harmonic linear oscillations of the point of suspension, can rotate synchronously with these oscillations ([1,2]). In ([2]) such a mode of excitation of the pendulum' s rotation was called heteroparametric. Retaining this term for the definition of excitation of the pendulum' s rotation by linear oscillations of the point of suspension, we shall assume that the latter are quasiperiodic in character. In this case rotation of the pendulum with angular velocity equal to the beat frequency is possible.

Figure 1 shows a mathematical pendulum with a vertical axis of suspension; its parameters and generalized coordinates determining its position in the fixed reference system $\xi\eta\zeta$ are indicated. The equation of motion of the pendulum in the coordinate system xyz, which undergoes translational oscillations relative to the system $\xi\eta\zeta$ along the ξ-axis, is as follows ([1,2]):

$$ma^2\ddot{\theta} + B\dot{\theta} + ma\ddot{\xi}\sin\theta = 0, \tag{1}$$

where, in contrast to ([1,2]),

$$\ddot{\xi} = -\xi_0\omega^2 \sin\Omega t \sin\omega t. \tag{2}$$

Let us reduce equation (1) to the form

$$\ddot{\theta} + \lambda\dot{\theta} - \frac{\xi_0\omega^2}{a}\sin\Omega t \sin\omega t \sin\theta = 0, \tag{3}$$

where $\lambda = B/ma^2$, and then make the substitution $\theta = \Omega t + \psi$. Then (3) is transformed into an equation describing the motion of the pendulum in a reference system rotating uniformly relative to its axis,

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$$\ddot{\psi} + \lambda(\Omega + \dot{\psi}) - \frac{\xi_0 \omega^2}{a} \sin \Omega t \sin \omega t (\sin \Omega t \cos \psi + \cos \Omega t \sin \psi) = 0, \qquad (4)$$

which we shall solve by the averaging method ([1]), after first reducing it to standard form.

Let us introduce the dimensionless time $\tau = \omega t$, and set

$$\frac{\lambda}{\omega} = 2\alpha \frac{\xi_0}{a}, \qquad \frac{\Omega}{\omega} = n \frac{\xi_0}{a},$$

and choose as the small parameter $\varepsilon = \xi_0 / a$. In this case equation (4) is reduced to the equation

$$\psi'' + 2\varepsilon\alpha(\varepsilon n + \psi') - \frac{1}{2}\varepsilon \sin \tau [\cos \psi + \cos(\psi + 2\varepsilon n \tau)] = 0; \qquad (5)$$

here primes denote differentiation with respect to τ. By the change of variables

$$\psi = \Phi - \frac{1}{2}\varepsilon \sin \tau [\cos \Phi + \cos(\Phi + 2\varepsilon n \tau)],$$

$$\frac{d\psi}{d\tau} = \varepsilon U - \frac{1}{2}\varepsilon \cos \tau [\cos \Phi + \cos(\Phi + 2\varepsilon n \tau)]. \qquad (6)$$

Fig. 1

from (5) we obtain a system of two nonlinear first-order differential equations

$$\begin{aligned}
\frac{d\Phi}{d\tau} &= \varepsilon U - \varepsilon^2 \sin \tau \left\{ n \sin(\Phi + 2\varepsilon n \tau) + \right. \\
&\quad \left. + \frac{U}{2} [\sin \Phi + \sin(\Phi + 2\varepsilon n \tau)] \right\} + O(\varepsilon^3), \\
\frac{dU}{d\tau} &= -\varepsilon \left\{ 2\alpha(n + U) + n \cos \tau \sin(\Phi + 2\varepsilon n \tau) + \right. \\
&\quad + \frac{U}{2} \cos \tau [\sin \Phi + \sin(\Phi + 2\varepsilon n \tau)] - \\
&\quad - \alpha \cos \tau [\cos \Phi + \cos(\Phi + 2\varepsilon n \tau)] - \\
&\quad \left. - \frac{1}{4} \sin^2 \tau [\sin \Phi + \sin(\Phi + 2\varepsilon n \tau)][\cos \Phi + \cos(\Phi + 2\varepsilon n \tau)] \right\} + \\
&\quad + \varepsilon^2 \left\{ \frac{U}{4} \sin \tau \cos \tau [\sin \Phi + \sin(\Phi + 2\varepsilon n \tau)]^2 + \right. \\
&\quad + \frac{n}{2} \sin \tau \cos \tau [\sin \Phi + \sin(\Phi + 2\varepsilon n \tau)] \sin(\Phi + 2\varepsilon n \tau) - \\
&\quad \left. - \frac{1}{8} \sin^3 \tau [\cos \Phi + \cos(\Phi + 2\varepsilon n \tau)]^3 \right\} + O(\varepsilon^3).
\end{aligned} \qquad (7)$$

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In equations (7), terms up to and including order ε^2 have been retained, since it is desirable to study the motion of the pendulum in the second approximation, so that the time interval on which the averaged motion is studied exceeds the beat period by one order of magnitude. Retaining in (7) the terms containing ε and performing averaging with respect to τ, in accordance with (1) we find that the averaged system of the first approximation has the form

$$\frac{d\Phi}{d\tau} = \varepsilon U, \qquad \frac{dU}{d\tau} = -2\varepsilon\alpha(n+U) + \frac{1}{2}\varepsilon\cos\Phi\sin\Phi. \tag{8}$$

To construct the averaged equations of the second approximation, it is necessary to retain in (7) the terms with ε and ε^2 and to substitute into (7) the improved first approximation

$$\Phi = \widetilde{\Phi},$$

$$\begin{aligned} U = \widetilde{U} - \varepsilon \Bigg\{ & \frac{\widetilde{U}}{2}\sin\tau[\sin\widetilde{\Phi} + \sin(\widetilde{\Phi} + 2\varepsilon n\tau)] + \\ & + n\sin\tau\sin(\widetilde{\Phi} + 2\varepsilon n\tau) - \alpha\sin\tau[\cos\widetilde{\Phi} + \\ & + \cos(\widetilde{\Phi} + 2\varepsilon n\tau)] - \frac{1}{8}\sin\tau\cos\tau[\cos\widetilde{\Phi} + \\ & + \cos(\widetilde{\Phi} + 2\varepsilon n\tau)][\sin\widetilde{\Phi} + \sin(\widetilde{\Phi} + 2\varepsilon n\tau)] - \\ & - \frac{1}{8}\sin(2\widetilde{\Phi} + 2\varepsilon n\tau) - \frac{1}{16}\sin(2\widetilde{\Phi} + 4\varepsilon n\tau)\Bigg\}. \end{aligned}$$

After averaging the result of the substitution, which has the form

$$\frac{d\widetilde{\Phi}}{d\tau} = \varepsilon\widetilde{U} + \varepsilon^2 F_1(\widetilde{\Phi}, \widetilde{U}, \tau, 2\varepsilon n\tau) + O(\varepsilon^3),$$

$$\frac{d\widetilde{U}}{d\tau} = -2\varepsilon\alpha(n+\widetilde{U}) + \frac{1}{2}\varepsilon\cos\widetilde{\Phi}\sin\widetilde{\Phi} + \varepsilon^2 F_2(\widetilde{\Phi}, \widetilde{U}, \tau, 2\varepsilon n\tau) + O(\varepsilon^3),$$

with respect to τ, we find that, by virtue of the conditions

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T F_i(\widetilde{\Phi}, \widetilde{U}, \tau, 2\varepsilon n\tau)\,d\tau = 0, \qquad i = 1, 2,$$

the averaged system of equations of the second approximation coincides with system (8).

System (8) is equivalent to a single second-order equation

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Fig. 2

Figure 1: Fig. 2

$$\Phi'' + 2\varepsilon\alpha\Phi' - \frac{\varepsilon^2}{8}\cos\Phi\sin\Phi = -\varepsilon^2 M, \tag{9}$$

where $M = 2an$. The necessary condition for the possibility of exciting rotation of the pendulum follows from the existence of an equilibrium solution of equation (9),

$$\Phi' = 0, \qquad \Phi = \Phi^*,$$

where Φ^* is determined from

$$\frac{1}{8}\sin\Phi^*\cos\Phi^* = M. \tag{10}$$

From (10) we find that the necessary condition mentioned is expressed in the form of the inequality

$$B\Omega < \frac{mgaN}{16}\left(\frac{\xi_0}{a}\right),$$

where $N = \xi_0\omega^2/g$ will be called the vibrational overload.

Thus, "capture" of the pendulum is possible if the moment of viscous friction about its axis is 16 times smaller than the static moment of the pendulum multiplied by the vibrational overload and by the ratio of the amplitude of oscillations to the length of the pendulum. Sufficient conditions for "capture" are determined by the initial conditions.

Fig. 2

Figure 2 shows the phase trajectories of the dynamical system described by equations (8). It is easy to see that for certain initial conditions Φ_0, U_0 there exist trajectories tending to the equilibrium positions determined by condition (10). Since, for given Φ_0, U_0, with the aid of (6) the initial conditions for ψ, and consequently also for θ, are uniquely determined, by choosing the initial values θ_0 and $\dot{\theta}_0$ one can realize the regime of rotation of the pendulum under quasiperiodic oscillations of the suspension.

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Note: Figure translations are in progress. See original paper for figures.

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