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Figure 1

Figure 1: Figure 1

Abstract

Full Text

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STRUCTURE OF EXCITED STATES WITH $K\pi = 2+$ OF EVEN-EVEN DEFORMED NUCLEI

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Numerous experimental and theoretical works have been devoted to the study of collective states with $K\pi = 2+$ (i.e., with projection of the angular momentum on the symmetry axis of the nucleus equal to two, and with positive parity) in even-even nuclei. Thus, in ([1,2]), on the basis of the superfluid model of the nucleus with allowance for quadrupole-quadrupole interactions, the energies of states with $K\pi = 2+$ and the probabilities $B(E2)$ of electromagnetic transitions were calculated. The present work, which is a continuation of ([2]), is devoted to a calculation, with allowance for the blocking effect, of the energies of the two lowest states with $K\pi = 2+$ of even-even nuclei in the regions $150 < A < 190$ and $228 \leq A \leq 254$, using

Fig. 1. Energies of states with $K\pi = 2+$ for $\varkappa = kA^{-4/3}\hbar\omega_0$. Notation:
a—experimental data; *b*—neutron pole; *v*—proton pole;
g—$k = 9.5,\ \varkappa_{np} = \varkappa$; *d*—$k = 10,\ \varkappa_{np} = \varkappa$;
e—$k = 11,\ \varkappa_{np} = \varkappa$; *zh*—$k = 11.5,\ \varkappa_{np} = 0.7\varkappa$;
z—$k = 12,\ \varkappa_{np} = 0.7\varkappa$.

investigation of the properties of these states and to clarification of the relation between the collective and two-quasiparticle structure of the excited states.

In [2], on the basis of the variational principle within the framework of the method of approximate second quantization, a secular equation was obtained which determines the energies ω_i of excited states with $K\pi = 2+$. In the case when the constants of the quadrupole-quadrupole interaction are equal to one another, i.e. $\chi_n^{(2)} = \chi_p^{(2)} = \chi_{np}^{(2)} \equiv \chi$, the secular equation takes the form:

$$\frac{1}{\chi} = 2\sum_{\rho\rho'} \frac{f(\rho\rho')^2 U_{\rho\rho'}^2}{\varepsilon(\rho) + \varepsilon(\rho') - \omega_i^2/(\varepsilon(\rho) + \varepsilon(\rho'))} \equiv F(\omega_i), \tag{1}$$

where the summation over $\rho\rho'$ is carried out over the levels of the mean field

Figure 2

Figure 2: Figure 2

$E(\rho)$; $f(\rho\rho')$ is the matrix element of the quadrupole-moment operator (the wave functions and the scheme of single-particle levels of the Nilsson potential are used), $\varepsilon(\rho) = \sqrt{c^2 + \{E(\rho) - \lambda\}^2}$, $U_{\rho\rho'} = U_\rho V_{\rho'} + U_{\rho'} V_\rho$.

To improve the accuracy of the calculations, we shall take the blocking effect into account. It is very difficult to take the blocking effect into account consistently; therefore we shall do this in the following simplified way: the chemical potentials λ will be determined from the condition of conservation, on the average, of the number of protons and neutrons in the states with $K\pi = 2+$; the values of the poles $\varepsilon(\rho) + \varepsilon(\rho')$ in (1) will be replaced by the energies of two-quasiparticle states, calculated as in [3]. The energy values calculated for various χ, the first two poles of equation (1), and the corresponding experimental data are shown in Fig. 1. The second roots of (1) lie between the values of the first and second poles. Since in most cases the distances between these poles are small, the energies ω_2 are determined to a considerable extent by the position of the corresponding poles. It is seen from Fig. 1 that the values of the energies of the first states with $K\pi = 2+$, calculated for $\chi = 10A^{-4/3}\hbar\omega_0$ ($\chi_{np} = \chi$), are in fairly good agreement with the corresponding experimental data in both regions of strongly deformed nuclei, and allowance for the blocking effect has led to an improvement of this agreement. Agreement between theory and experiment improves if the calculations in the region $150 \leq A \leq 186$ are carried out for $\chi = 9.5A^{-4/3}\hbar\omega_0$, and in the region $228 \leq A \leq 254$ for $\chi = 11A^{-4/3}\hbar\omega_0$.

Fig. 2. Behavior of $F(\omega)$ for Pu^{240} and Cf^{250}.
a—intersections of the curves $F(\omega)$ with the straight lines $1/\chi$.

Let us discuss the features of the solutions of equation (1); for this purpose, Fig. 2 gives the values of $F(\omega)$ for Pu^{240} and Cf^{250}. The points of intersection of the straight line $1/\chi$ with the curve $F(\omega)$ are the roots of equation (1). As is known, the wave function of a collective state is a superposition of various kinds of two-quasiparticle states. If the state possesses distinctly pronounced collective properties, then the value of the root differs substantially from the value of the nearest poles, and $F(\omega)$ intersects the straight line $1/\chi$ at a small angle. If, however, the state is practically a two-quasiparticle one, then the value of the root almost coincides with the value of the pole, and $F(\omega)$ intersects $1/\chi$ at a right angle.

We shall use the normalization condition for the wave functions of one-phonon states to clarify the question of with what weights the two-quasiparticle states enter a given collective state. The studies carried out show that the overwhelming majority of the lowest states with $K\pi = 2+$ possess collective properties, and a large number of two-quasiparticle states make a noticeable contribution to them. Their structure is similar to the structure of the states of U^{234}, given in Table 1, in which the contribution of the most important two-quasiparticle states* to the first ω_1

Table 1

Contribution of two-quasiparticle states to collective states with $K\pi = 2+$ (in percent) for $\chi = 11A^{-4/3}\hbar\omega_0$, ($\chi_{np} = \chi$)

Configuration of two-quasiparticle states*	$f(\rho\rho')$	U^{234} ω_1	U^{234} ω_1^*	Pu^{238} ω_1	Pu^{238} ω_2	Pu^{240} ω_1	Pu^{240} ω_2	Cf^{250} ω_1	Cf^{250} ω_2
Neutron states									
633↓ −631↓	−0.82	40.3	51.0	63.7	4.3	0.5	13.4	0.1	10^{-4}
622↑ −631↓	−0.005	10^{-4}	10^{-4}	10^{-3}	95.3	94.9	10^{-3}	10^{-5}	10^{-8}
622↑ −620↑	−1.67	0.8	0.5	2.5	10^{-3}	1.0	17.3	26.7	0.07
624↑ −622↑	−1.42	0.2	0.2	0.6	10^{-3}	0.2	3.2	21.4	0.07
606↑ −604↑	−1.84	1.2	0.8	0.8	10^{-3}	0.1	1.5	1.2	10^{-3}
743↑ −761↑	−1.02	9.8	8.4	2.3	0.05	0.09	1.6	0.07	10^{-5}
734↑ −752↑	−0.89	2.1	1.5	1.9	0.01	0.2	3.9	0.8	10^{-4}
725↑ −743↑	−0.73	0.4	0.3	0.8	10^{-3}	0.1	2.3	1.2	10^{-3}
613↑ −611↑	−2.04	0.04	0.03	0.05	10^{-4}	0.01	0.2	1.8	10^{-3}
631↑ +631↓	−0.86	15.8	14.9	5.9	0.1	0.2	5.1	0.1	10^{-4}
622↓ +620↑	−1.71	0.1	0.07	0.2	10^{-4}	0.06	0.7	18.2	0.05
Proton states									
523↓ −541↓	−0.17	1.2	1.4	1.2	0.07	0.1	10.4	0.02	10^{-5}
523↓ −521↓	−0.94	0.6	0.4	1.2	10^{-3}	0.2	2.5	3.5	10^{-3}
512↑ −530↑	1.10	1.1	0.7	1.1	10^{-3}	0.1	2.1	1.8	10^{-3}

Configuration of two-quasiparticle states*	$f(\rho\rho')$	U^{234} ω_1	U^{234} ω_1^*	Pu^{238} ω_1	Pu^{238} ω_2	Pu^{240} ω_1	Pu^{240} ω_2	Cf^{250} ω_1	Cf^{250} ω_2
$514\downarrow -521\uparrow$	-0.098	10^{-3}	10^{-3}	10^{-3}	10^{-5}	10^{-3}	0.02	0.2	99.7
$642\uparrow -660\uparrow$	-0.96	2.5	1.9	1.0	10^{-3}	0.1	1.9	0.1	10^{-4}
$633\uparrow -651\uparrow$	-0.85	2.5	1.9	2.4	0.01	0.3	5.0	1.4	10^{-3}
$532\downarrow +530\uparrow$	0.74	2.9	2.4	0.7	10^{-3}	0.08	1.3	0.07	10^{-4}
$521\uparrow +530\uparrow$	0.60	2.4	1.9	3.6	0.2	0.4	10.0	1.2	10^{-3}
$521\uparrow +521\downarrow$	1.23	0.3	0.2	0.6	10^{-3}	0.07	1.1	9.5	0.02

* For $\chi_{np} = 0.7\chi$, $\chi = 12A^{-4/3}\hbar\omega_0$.

and second ω_2 states with $K\pi = 2+$. From Fig. 1 and Table 1 it is seen that the energy ω_1 and the structure of the states obtained in the case $\chi = 12A^{-4/3}\hbar\omega_0$ and $\chi_{np} = 0.7\chi$ are close to the case $\chi = 10A^{-4/3}\hbar\omega_0$ and $\chi_{np} = \chi$. Thus, a decrease of χ_{np} in comparison with χ_n and χ_p can be compensated by a certain increase of the latter.

The wave function of a one-phonon collective state transforms into the wave function of a two-quasiparticle state when the root of the secular equation ω comes very close to the pole. For the values of χ used, for the lowest states with $K\pi = 2+$ this occurs only when the matrix element $f(\rho\rho')$ corresponding to the first pole is very small. In this case the first or second state with $K\pi = 2+$ is two-quasiparticle. If the straight line $1/\chi$ intersects $F(\omega)$ first at a right angle and then at an acute angle, then the first state will be two-quasiparticle and the second collective. This case is realized in Yb^{172}, where the contribution of the neutron state $512\uparrow -521\downarrow$, calculated taking into account the effec-

* By $Nn_z\Lambda\uparrow$ we denote the state $K\pi[Nn_z\Lambda]$ of the Nilsson potential with $K = \Lambda + \Sigma$, and by $Nn_z\Lambda\downarrow$, with $K = \Lambda - \Sigma$.

the blocking, in the state with $K\pi = 2+$ and energy 1.468 MeV, is 99.6%. The calculations carried out confirm the correctness of the interpretation of this state, given in (3) on the basis of an analysis of the β-decay of Tm^{172}.

If $1/\chi$ intersects $F(\omega)$ first at an acute angle and then at an obtuse one, then the first state is collective and the second is two-quasiparticle. This case is realized in Cf^{250}, as is seen from Fig. 2 and Table 1. In U^{238} and Pu^{240} the situation is more complicated. A small change in χ, or a shift of the pole of the neutron state $622\uparrow - 631\downarrow$, leads to a change in the order of the collective and quasiparticle states. Thus, according to calculations without allowance for the blocking effect [(2)], the first states in U^{238} and Pu^{240} are collective, and the calculated probability $B(E2)$ of the electromagnetic transition agrees with the experimental data in [(4)]. In the present calculations, for $\chi = 11A^{-4/3}\hbar\omega_0$, the first state is two-quasiparticle and the second is collective; however, for $\chi = 13A^{-4/3}\hbar\omega_0$, the first state is collective and the second two-quasiparticle. Experimental data on Coulomb excitation of U^{238} [(4)] and on β-decay to Pu^{240} [(5)] indicate that the states observed in them with $K\pi = 2+$ are collective. To confirm the correctness of the assumptions made about the relation between collective and quasiparticle states, the structure of the first two states with $K\pi = 2+$ in Yb^{172}, U^{238}, Pu^{240}, and Cf^{250} should be studied experimentally.

Thus, within the framework of the superfluid nuclear model, a unified description has been obtained of two-quasiparticle and collective nonrotational excited states of deformed even-even nuclei. It has been shown that the mean field of the nucleus determines which of the lowest states with $K\pi = 2+$ are collective and which are two-quasiparticle.

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Note: Figure translations are in progress. See original paper for figures.