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Abstract

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PHYSICS

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THE BINDING ENERGY OF HYPERTRITIUM IN THE CASE OF A NONLOCAL INTERACTION

(Presented by Academician N. N. Bogolyubov, 9 III 1964)

The question of studying hypernuclei is important both from the point of view of elucidating a number of properties of atomic nuclei and for the study of elementary acts of interaction between strongly interacting particles—hyperons and nucleons. The production of nuclei of the type ${}_{\Lambda\Lambda}X$ made it possible to use hypernuclei also for studying the Λ—Λ interaction ([1]). Therefore, recently special attention has been devoted to the study of hypernuclei.

In the present work hypertritium is studied under the assumption of a nonlocal factorizing interaction between any pair of particles, with the use of Gell-Mann's hypothesis of global symmetry ([2]). The principle of global symmetry assumes that the $(\Sigma\Lambda\pi)$- and $(\Sigma\Sigma\pi)$-interactions are closely connected with the $(NN\pi)$-interaction, which makes it possible to relate the nucleon-nucleon potential to the pion component of the hyperon-nucleon potential. The hypothesis of global symmetry is an approximate one, since, for example, reactions are observed which would be forbidden if global symmetry were satisfied exactly. Therefore, in studying hypernuclei the hypothesis of global symmetry is regarded only as a convenient model for explaining experimental facts. We note that with its help one can obtain surprisingly good qualitative agreement with the results of observations ([3]).

Applying the variational method, Dalitz and Downs found the binding energies of light hypernuclei, including ${}_{\Lambda}\mathrm{H}^3$. They used a spin-dependent central Gaussian potential and found that the Λ—N potential is characterized by greater attraction in the singlet state than in the triplet state. In addition, they explained the fact of the nonexistence of the hyperdeuteron.

When considering the Λ—N interaction we must take into account the possibility of the transition $\Lambda N \rightleftarrows \Sigma N$, and therefore, in addition to the $\Lambda N \rightleftarrows \Lambda N$ and $\Sigma N \rightleftarrows \Sigma N$ potentials, we must also have a $\Lambda N \rightleftarrows \Sigma N$ potential. It is clear

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from this that the complete scattering problem must be described by coupled Schrödinger equations, in which the possibility of such transitions is taken into account.

As is known, it is impossible to solve the three-body problem for local potentials. On the other hand, in the case of a nonlocal factorizing two-particle potential, for the problem of three nonidentical particles one obtains a system of one-dimensional integral equations, which can easily be solved with the aid of an electronic computing machine. Such a potential for hypernuclei is considered in work ([4]), where, with the aid of the hypothesis of global symmetry, the parameters of the hyperon-nucleon interaction potential are established and the fact of the nonexistence of the hyperdeuteron is proved. In calculating the binding energy of ${}_{\Lambda}\mathrm{H}^3$, the authors of the cited work apply the variational method for local potentials with parameters chosen for the case of nonlocal interaction, and also do not take into account the possibility of $\Lambda N \rightleftarrows \Sigma N$ transitions.

In the present work the total binding energy of ${}_{\Lambda}\mathrm{H}^3$ is found by solving a system of coupled integral equations with allowance for $\Lambda N \rightleftarrows \Sigma N$ transitions.

It is known that hypertritium is in the state $T = 0$ and $I = \frac{1}{2}$, with the nucleons in ${}_{\Lambda}\mathrm{H}^3$ characterized by an ${}^3S$-state, while the $N - \Lambda$ and $N - \Sigma$ interactions occur in an ${}^1S$-state, so that it is sufficient for us to consider potentials for the state $l = 0$. It can easily be shown that the model of global symmetry in the case of a factorizable potential gives the following expressions for the interaction potentials ([4,5]):

$$V_{N\Lambda} = -\frac{3}{4}\left(\frac{\lambda_s}{2\mu_{NN}}\right) g_s(p) g_s(p'), \qquad V_{N\Sigma} = \frac{1}{3} V_{N\Lambda}, \qquad V_{\Lambda\Sigma} = \frac{1}{\sqrt{3}} V_{N\Lambda}, \quad (1)$$

$$V_{NN}^{s,t} = -\frac{\sqrt{3}}{4}\left(\frac{\lambda_{s,t}}{2\mu_{NN}}\right) g_{s,t}(p) g_{s,t}(p').$$

Here $\lambda_{s,t}$ is the depth of the $N - N$ interaction in the singlet and triplet states, respectively; μ_{NN} is the reduced mass of the $N - N$ system; all potentials entering (1) depend only on nucleon-nucleon parameters. As the $g_{s,t}(p)$-function we shall below take the Yukawa potential

$$g_{s,t}(p) = (\beta_{s,t}^2 + p^2)^{-1} \quad (2)$$

with parameters determined from the $N - N$ interaction at low energies:

$$\beta_t = 1.4488 f^{-1}, \qquad \beta_s = 1.158 f^{-1}, \qquad \lambda_t = 0.4143 f^{-3}, \qquad \lambda_s = 0.01467 f^{-3}. \quad (3)$$

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Let us first write down the coupled Schrödinger equations for hypertritium, taking into account the $\Lambda \rightleftarrows \Sigma$ transitions, in the case of a local interaction. In the obvious notation these equations have the form

$$[T_1 + T_2 + T_\Lambda - \varepsilon^{(\Lambda)}]\Psi_\Lambda(\mathbf{r}_1\mathbf{r}_2\mathbf{r}_\Lambda) = V_{N_1\Lambda}(\mathbf{r}_1 - \mathbf{r}_\Lambda)\Psi_\Lambda + V_{N_2\Lambda}(\mathbf{r}_2 - \mathbf{r}_\Lambda)\Psi_\Lambda +$$
$$+V_{N_1N_2}(\mathbf{r}_1 - \mathbf{r}_2)\Psi_\Lambda + V_{\Lambda\Sigma}(\mathbf{r}_\Lambda - \mathbf{r}_1)\Psi_\Sigma + V_{\Lambda\Sigma}(\mathbf{r}_\Lambda - \mathbf{r}_2)\Psi_\Sigma, \tag{4}$$

$$[T_1 + T_2 + T_\Sigma - \varepsilon^{(\Sigma)}]\Psi_\Sigma(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_\Sigma) = V_{N_1\Sigma}(\mathbf{r}_1 - \mathbf{r}_\Sigma)\Psi_\Sigma + V_{N_2\Sigma}(\mathbf{r}_2 - \mathbf{r}_\Sigma)\Psi_\Sigma +$$
$$+V_{N_1N_2}(\mathbf{r}_1 - \mathbf{r}_2)\Psi_\Sigma + V_{\Sigma\Lambda}(\mathbf{r}_\Sigma - \mathbf{r}_1)\Psi_\Lambda + V_{\Sigma\Lambda}(\mathbf{r}_\Sigma - \mathbf{r}_2)\Psi_\Lambda, \tag{5}$$

where the binding energy $\varepsilon^{(\Lambda)}$ differs from $\varepsilon^{(\Sigma)}$ by the mass difference $\delta = c^2(M_\Sigma - M_\Lambda)$.

One can pass to the equations for a nonlocal factorizable interaction in the known way ([7,8]). Taking formulas (1) into account, we finally obtain a system of integral equations in the momentum representation

$$\left[1 - \frac{3}{4}\lambda_s N_{23}^\Lambda(p_1)\right]\Phi_\Lambda^s(\mathbf{p}_1) - \frac{\sqrt{3}}{4}\lambda_s N_{23}^\Lambda(p_1)\Phi_\Sigma^s(\mathbf{p}_1) =$$
$$= \lambda_t \int \Gamma_\Lambda^{-1}\left(\mathbf{x} + \frac{\mathbf{p}_1}{a_\Lambda}, \mathbf{p}_1\right) g_s\left(\mathbf{x} + \frac{\mathbf{p}_1}{a_\Lambda}\right) g_t\left(\mathbf{p}_1 + \frac{1}{2}\mathbf{x}\right)\Phi_\Lambda^t(\mathbf{x})\,d\mathbf{x} +$$
$$+\frac{3}{4}\lambda_s \int \Gamma_\Lambda^{-1}\left(\mathbf{x} + \frac{\mathbf{p}_1}{a_\Lambda^0}, \mathbf{p}_1\right) g_s\left(\mathbf{x} + \frac{\mathbf{p}_1}{a_\Lambda^0}\right) g_s\left(\mathbf{p}_1 + \frac{\mathbf{x}}{a_\Lambda^0}\right) \times$$
$$\times \left[\Phi_\Lambda^s(\mathbf{x}) + \frac{1}{\sqrt{3}}\Phi_\Sigma(\mathbf{x})\right] d\mathbf{x}, \tag{6}$$

$$\left[1 - \frac{1}{4}\lambda_s N_{31}^\Sigma(p_2)\right]\Phi_\Sigma^s(\mathbf{p}_2) - \frac{\sqrt{3}}{4}\lambda_s N_{31}^\Sigma(p_2)\Phi_\Lambda^s(\mathbf{p}_2) =$$
$$= \lambda_s \int \Pi_\Sigma^{-1}\left(\mathbf{x} + \frac{\mathbf{p}_2}{a_\Sigma}, \mathbf{p}_2\right) g_s\left(\mathbf{x} + \frac{\mathbf{p}_2}{a_\Sigma}\right) g_s\left(\mathbf{p}_2 + \frac{1}{2}\mathbf{x}\right)\Phi_\Sigma^s(\mathbf{x})\,d\mathbf{x} +$$
$$+\frac{\sqrt{3}}{4}\lambda_s \int \Pi_\Sigma^{-1}\left(\mathbf{x} + \frac{\mathbf{p}_2}{a_\Sigma^0}, \mathbf{p}_2\right) g_s\left(\mathbf{x} + \frac{\mathbf{p}_2}{a_\Sigma^0}\right) g_s\left(\mathbf{p}_2 + \frac{\mathbf{x}}{a_\Sigma^0}\right) \times$$
$$\times \left[\Phi_\Lambda^s(\mathbf{x}) + \frac{1}{\sqrt{3}}\Phi_\Sigma^s(\mathbf{x})\right] d\mathbf{x},$$

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$$[1-\lambda_t N_{12}^{\Lambda}(p_3)]\Phi_{\Lambda}^{t}(\mathbf{p}_3)=$$

$$=\frac{3}{2}\lambda_s\int K_{\Lambda}^{-1}\left(\mathbf{x}+\frac{1}{2}\mathbf{p}_3,\mathbf{p}_3\right)g_t\left(\mathbf{x}+\frac{1}{2}\mathbf{p}_3\right)g\left(\mathbf{p}_3+\frac{\mathbf{x}}{a_{\Lambda}}\right)\times$$

$$\times\left[\Phi_{\Lambda}^{s}(\mathbf{x})+\frac{1}{\sqrt{3}}\Phi_{\Sigma}^{s}(\mathbf{x})\right]d\mathbf{x},$$

where, for example, the function $\Phi_{\Lambda}^{s}(\mathbf{p}_1)$ is defined as follows:

$$\Phi_{\Lambda}^{s}(\mathbf{p}_1)=\int g_s(\mathbf{k}_{23})\Psi_{\Lambda}(\mathbf{k}_{23},\mathbf{p}_1)\,d\mathbf{k}_{23}; \tag{7}$$

$\mathbf{k}_{23}$ and $\mathbf{p}_1$ are the momenta corresponding to the Jacobi coordinates, and by $a_{\Lambda}a_{\Lambda}^{0}$, a_{Σ}, a_{Σ}^{0} we denote the quantities

$$a_{\Lambda,\Sigma}=\frac{M+M_{\Lambda,\Sigma}}{M_{\Lambda,\Sigma}},\qquad a_{\Lambda,\Sigma}^{0}=\frac{M+M_{\Lambda,\Sigma}}{M}. \tag{8}$$

The functions $N_{ik}(p)$ are determined by the formulas:

$$N_{23}^{\Lambda,\Sigma}(p)=\int\Gamma_{\Lambda,\Sigma}^{-1}(\mathbf{x},\mathbf{p})\,g_s^2(\mathbf{x})\,d\mathbf{x}, \tag{9}$$

$$N_{12}^{\Lambda}(p)=\int K_{\Lambda}^{-1}(\mathbf{x},\mathbf{p})\,g_s^2(\mathbf{x})\,d\mathbf{x}, \tag{10}$$

$$N_{31}^{\Sigma}(p)=\int\Pi_{\Sigma}^{-1}(\mathbf{x},\mathbf{p})\,g_s^2(\mathbf{x})\,d\mathbf{x}, \tag{11}$$

where

$$K_{\Lambda}(\mathbf{x},\mathbf{p})=\gamma^2+\mathbf{x}^2+\frac{2M+M_{\Lambda}}{4M_{\Lambda}}\,p^2, \tag{12}$$

$$\Gamma_{\Lambda,\Sigma}(\mathbf{x},\mathbf{p})=\gamma^2+\frac{a_{\Lambda,\Sigma}}{2}\mathbf{x}^2+\frac{2M+M_{\Lambda,\Sigma}}{2(M+M_{\Lambda,\Sigma})}\,p^2, \tag{13}$$

$$\Pi_{\Sigma}(\mathbf{x},\mathbf{p})=\gamma^2+\gamma_0^2+\frac{a_{\Sigma}}{2}\mathbf{x}^2+\frac{2M+M_{\Sigma}}{2(M+M_{\Sigma})}\,p^2, \tag{14}$$

γ and γ_0 are determined by the formulas $\gamma^2=M\varepsilon(\Lambda)$, $\gamma_0^2=M\delta$.

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After integration over the angles, the system of integral equations (6) gives a system of three one-dimensional integral equations, which is easy to solve with the aid of computing machines. But we can simplify this system in advance by taking into account a special feature of the ${}_{\Lambda}H^3$ system. The binding energy of ${}_{\Lambda}H^3$ differs only slightly from the binding energy of the deuteron; this means that ${}_{\Lambda}H^3$ is a very loose system and at least one particle is always found comparatively far from the others. A large distance corresponds to a small momentum; in this case, for solving system (6) one may apply the approximation considered by Mitra [9] for solving the problem of ${}_1H^3$. This approximation consists in neglecting the product $(\mathbf{xp})$ in the integral equations, and for our problem it will be much better than for the case of ${}_1H^3$.

In the approximation just noted, the eigenvalues of the energy are determined from the condition that an eighth-order determinant vanish. The numerical values of the roots of the determinant were found with the aid of a computing machine. The smallest root of this determinant, $\alpha = \gamma/\beta_s^0 = 0.228$, corresponds to a total binding energy $\varepsilon({}_{\Lambda}H^3) = 2.904$ MeV, which is in satisfactory agreement with the experimental value of the binding energy $\varepsilon \simeq 2.3$ MeV.

Let us note that in our calculations one can pass to the limit in which the possibility of $\Lambda \rightleftarrows \Sigma$ transitions is excluded. In this case the energy is found from the condition that a third-order determinant vanish. The smallest root

this determinant corresponds to the binding energy $\varepsilon_{\Lambda}H^3 = 4.6$ MeV. This result shows that taking the $\Lambda \rightleftarrows \Sigma$ transitions into account plays an important role.

Thus, applying the model of global symmetry and taking the interaction between particles in the form of a nonlocal factorizing Yamaguchi potential, with allowance for the possibility of $\Lambda \rightleftarrows \Sigma$ transitions, one can explain the observed value of the total binding energy of hypertritium.

In conclusion, we consider it a pleasant duty to express our gratitude to V. G. Solov'ev for his constant interest in the work and for discussions, to V. I. Ogievetskii, who kindly informed us on questions in the field of strong interactions, and to A. V. Rakitskii for programming the problem.

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Note: Figure translations are in progress. See original paper for figures.

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