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Abstract

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CYBERNETICS AND CONTROL THEORY

Kh. A. MADATYAN

ON THE SYNTHESIS OF CIRCUITS CORRECTING THE OPENING OF CONTACTS

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The work is devoted to questions of reliability of control systems (^{1}). Usually the objects of consideration are contact circuits. Suppose that in a circuit, at any moment of time, due to the action of a fault source, some faults arise. A circuit is called self-correcting with respect to the given fault source if the faulty circuit realizes the same function.

Table 1

$x_1\ x_2\ \ldots\ x_k$	$0\ \ldots\ \sigma_{k+1}\ \ldots\ 1$	x_{k+1}
$0\ \ 0\ \ldots\ 0$	$0\ \ldots\ \sigma_{k+2}\ \ldots\ 1$	x_{k+2}
$\ldots\ldots$	$\ldots\ldots$	
$\ldots\ldots$	$0\ \ldots\ \sigma_n\ \ldots\ 1$	x_n
$\sigma_1\ \sigma_2\ \ldots\ \sigma_k$		$\}s$
$\ldots\ldots$		$\}s$
$1\ \ 1\ \ldots\ 1$	$\boxed{} \leftarrow f(\sigma_1, \ldots, \sigma_n)$	$\}s'$

The concept of a self-correcting circuit was introduced in (^{2}). In that work the problem of asymptotic estimates for the Shannon function of self-correcting circuits was solved, when the fault source produces only short circuits of contacts. The following asymptotic estimates were obtained:

$$1.\quad L_3^1(n) \sim \frac{2^n}{n}. \qquad 2.\quad L_3^m(n) \lesssim \left\{\left[\frac{m}{2}\right] + 1\right\} \frac{2^n}{n}.$$

The case when the fault source produces only openings of contacts remained open.

In the present note a similar problem is solved for this case, and analogous asymptotic estimates are obtained.

Let $L(\Sigma)$ be the number of contacts in the circuit. Denote by $L_p^m(f) = \inf L(\Sigma)$, where the lower bound is taken over all self-correcting circuits for an opening in m contacts that realize f. Let $L_p^m(n) = \max L_p^m(f)$ (the maximum is taken over all functions f depending on the arguments $x_1, \dots, x_n$).

Theorem 1. $L_p^1(n) \sim 2^n/n$.

Proof. We shall indicate a method which makes it possible, for each function $f(x_1, \dots, x_n)$, to construct a self-correcting circuit for one opening, having asymptotically $2^n/n$ contacts. For this we take a circuit, constructed by the method of O. B. Lupanov (^{3}), realizing the function f, and show how it must be transformed in order to obtain the required self-correcting circuit.

Divide the table (Table 1) specifying the function f into strips, each of which (except, perhaps, the last) contains exactly s rows, while the last contains s' ($s' \leq s$) rows. Let p be the number of strips ($p \leq 2^k/s + 1$). Within each strip we divide the columns into groups so that into one group

include all identical columns. Let $f_{ij}(x_1, \dots, x_n)$ be the function that coincides with f on the j-th group of the i-th strip and is equal to 0 on the remaining sets. Obviously,

$$f_{ij}(x_1, \dots, x_n) = f_{ij}^{(1)}(x_1, \dots, x_k) \;\&\; f_{ij}^{(2)}(x_{k+1}, \dots, x_n).$$

Let

$$\bigvee_j f_{ij}^{(1)} \cdot f_{ij}^{(2)} = f_i, \qquad \bigvee_{i=1}^{p} f_i(x_1, \dots, x_n) = f(x_1, \dots, x_n).$$

Take r equal to a power of two and less than $n-k$. Then the r-dimensional cube with axes $x_{k+1}, \dots, x_{k+r}$ can be partitioned into $2^r/r$ nonintersecting spheres of radius 1 (in the sense of the Hamming metric (4)) with centers $(\beta_{k+1}^h, \dots, \beta_{k+r}^h)$, where $h = 1, \dots, 2^r/r$.

The circuit $\mathfrak{A}'$, which realizes the function f (without the requirement of self-correction), as is known (^{3}), is constructed by connecting in parallel circuits $\mathfrak{A}_i'$ that realize f_i. Let us describe the structure of the circuits $\mathfrak{A}_i'$. To this end, construct a system of such $[1, q]$-multipoles $M_1', \dots, M_6'$ that each subsequent one is obtained by completing the preceding one and $M_6' = \mathfrak{A}_i'$ (Fig. 1).

M_1' is a $[1, 2^r]$-multipole representing a contact tree in the variables $x_{k+1}, \dots, x_{k+r}$; M_2' is a $[1, 2^r/r]$-multipole obtained from M_1' by joining the r outputs corresponding to the points of one and the same sphere. M_2' realizes the characteristic functions of the spheres

$$\varphi_h(x_{k+1}, \dots, x_{k+r}) \qquad (1 \leq h \leq 2^r/r).$$

Fig. 1

Fig. 1

Figure 1: Fig. 1

M_3' is a $[1, 2^{n-k}/r]$-multipole obtained from M_2' by connecting to each of the outputs contact trees in the variables $x_{k+r+1}, \dots, x_n$. M_3' realizes functions of the form

$$\varphi_h(x_{k+1}, \dots, x_{k+r}) x_{k+r+1}^{\sigma_{k+r+1}} \cdots x_n^{\sigma_n}.$$

M_4' is a $[1, 2^{n-k}]$-multipole obtained from M_3' by connecting to the outputs corresponding to the sphere with center $(\beta_{k+1}^h, \dots, \beta_{k+r}^h)$ the $[1, r]$-multipole shown in Fig. 2a $(h = 1, 2, \dots, 2^r/r)$. M_4' realizes all conjunctions of the form

$$x_{k+1}^{\sigma_{k+1}} \cdots x_n^{\sigma_n}.$$

M_5' is a multipole obtained from M_4' by joining (within each sphere) certain outputs so that M_5' realizes functions of the form

$$f_{ijh}^{(2)}(x_{k+1}, \dots, x_n) = f_{ij}^{(2)}(x_{k+1}, \dots, x_n) \;\&\; \varphi_h(x_{k+1}, \dots, x_{k+r})$$
$$(j = 1, 2, \dots; \quad 1 \le h \le 2^r/r).$$

M_6' is a $[1, 1]$-multipole obtained from M_5' by connecting to its outputs π-circuits and subsequently joining all outputs into one. Namely, to the output corresponding to the function $f_{ijh}^{(2)}$ there is connected the π-circuit corresponding to the perfect disjunctive normal form of the function $f_{ij}^{(1)}(x_1, \dots, x_n)$. The multipole M_6' realizes the function $f_i(x_1, \dots, x_n)$.

If in the given construction we take

$$r = 2^{[\frac{1}{2}\log_2 n]}, \qquad k = [2\log_2 n], \qquad s = [n - 2\sqrt{n}],$$

then we obtain the asymptotic inequality

$$L(n) \lesssim \frac{2^n}{n}.$$

The circuit $\mathfrak{A}'$ constructed by us has one property that will be used in the next part of the proof. Suppose the circuit $\mathfrak{A}'$ is decomposed into the direct sum of three subcircuits $\mathfrak{A}_1'$, $\mathfrak{A}_2'$, and $\mathfrak{A}_3'$, where $\mathfrak{A}_1'$ consists of all multipoles M_3', $\mathfrak{A}_2'$ consists of all multipoles $M_5' \setminus M_3'$, and $\mathfrak{A}_3'$ consists of all multipoles $M_6' \setminus M_5'$. Let $L(\mathfrak{A}_1')$, $L(\mathfrak{A}_2')$, and $L(\mathfrak{A}_3')$ be the number of contacts in the corresponding-

in the corresponding subcircuits. It is easy to see that, for the above values of the parameters r, k, and s, the relations

$$L(\mathfrak{A}_1') = o(L(\mathfrak{A}_2')), \qquad L(\mathfrak{A}_3') = o(L(\mathfrak{A}_2')). \tag{1}$$

Fig. 2

Figure 2: Fig. 2

Fig. 3

Figure 3: Fig. 3

hold. The latter means that $L(\mathfrak{A}_2') \sim 2^n/n$, i.e., that the subcircuit $\mathfrak{A}_2'$ contains almost all the contacts of the circuit $\mathfrak{A}'$.

We now proceed to describe those changes in the constructed circuit which make it possible to obtain from it a self-correcting circuit satisfying the condition of the theorem. To this end, in the subcircuits $\mathfrak{A}_1'$ and $\mathfrak{A}_3'$, each contact x^σ should be duplicated in parallel, as indicated in Fig. 2b. We denote the resulting subcircuits by $\mathfrak{A}_1$ and $\mathfrak{A}_3$.

Fig. 2

For the subcircuit $\mathfrak{A}_2'$ we shall give a special construction. Recall that in synthesizing the multipoles M_5' we performed, in a certain way, unions of the outputs of the multipoles M_4'; moreover, unions of this sort were carried out only within individual spheres of each multipole. We also note that in the $[1, r]$-pole (Fig. 3a), which was connected to the outputs of M_3', all contacts are controlled by different relays. Consider one of the unions performed for the outputs of the sphere with center $(\beta_{k+1}^h, \dots, \beta_{k+r}^h)$.

Number all those outputs of the multipole M_4' which participated in the union under consideration and which are controlled by one and the same relay x_{k+1}. Assign these numbers to those outputs of the multipole M_3' from which the contact $X_{k+1}^{\bar\beta_{k+1}^h}$ emerged, and then delete this contact. Now connect, by contacts $X_{k+1}^{\bar\beta_{k+1}^h}$, consecutively output M_3' with number 1 to output 2, output 2 to output 3, and so on, to the end. Connect the first and the last outputs with the corresponding π-circuit (Fig. 3a). We carry out this construction for all distinct relays in the given union, and then for all unions. Denote the subcircuit thus obtained by $\mathfrak{A}_2$.

Fig. 3

Thus, we have constructed a circuit $\mathfrak{A} = \mathfrak{A}_1 \cup \mathfrak{A}_2 \cup \mathfrak{A}_3$. It is easy to see that it realizes the same function $f(x_1, \dots, x_n)$. The fact that this circuit is self-correcting follows from the fact that every chain of the circuit $\mathfrak{A}$ having nonzero conductivity has been duplicated.

Let us estimate the complexity $L(\mathfrak{A})$ of the circuit $\mathfrak{A}$. From (1) and the relations

$$L(\mathfrak{A}_1) = 2L(\mathfrak{A}_1'), \qquad L(\mathfrak{A}_3) = 2L(\mathfrak{A}_3'), \qquad L(\mathfrak{A}_2) \leq L(\mathfrak{A}_2') + \sqrt{n}\, 2^{n-2\sqrt{n}}$$

it follows that $L(\mathfrak{A}) \lesssim 2^n/n$. Thus, $L_p^1 \le L(\mathfrak{A}) \lesssim 2^n/n$. The lower estimate for $L_p^1(n)$ follows from (5). The theorem is proved.

By a slight complication of this method of synthesis it is not difficult to obtain the following result:

Theorem 2.

$$L_p^m(n) \lesssim \left(\frac{m+1}{2}\right)\frac{2^n}{n}.$$

For this it is necessary:

1) In the circuits $\mathfrak{A}_1'$ and $\mathfrak{A}_3'$, duplicate each contact x^σ $(m+1)$ times.

2) If m is odd, then each contact in Fig. 3a must be duplicated $(m+1)/2$ times.

3) If m is even, then each contact in Fig. 3a must be duplicated $m/2$ times and, in addition, output number 1 must be connected with number 4, output number 3 with number 6, ⋯, i with $i+3$, etc., $q+1$ with 2 (Fig. 3b). Then we obtain

$$L(\mathfrak{A}_1) = (m+1)L(\mathfrak{A}_1'), \qquad L(\mathfrak{A}_3) = (m+1)L(\mathfrak{A}_3'),$$

$$L(\mathfrak{A}_2) \le \left(\frac{m+1}{2}\right) L(\mathfrak{A}_2') + \frac{m+1}{2}\sqrt{n2^{n-2}\sqrt[n]{n}}. \tag{2}$$

From (1) and (2) it follows that

$$L_p^m(n) \lesssim \left(\frac{m+1}{2}\right)\frac{2^n}{n}.$$

Theorem 2 gives, for almost all functions, self-correcting circuits that are twice as simple as those obtained by the trivial method.

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Moscow State University
named after M. V. Lomonosov

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References

[1] S. V. Yablonskii, *Problems of Cybernetics*, vol. 2, 1959.
[2] Yu. G. Potapov, S. V. Yablonskii, DAN, 134, No. 3 (1960).
[3] O. B. Lupanov, DAN, 119, No. 1 (1958).
[4] R. W. Hamming, Bell Syst. Techn. J., 29, No. 1 (1950).
[5] C. E. Shannon, Bell Syst. Techn. J., 28, No. 1 (1949).

Note: Figure translations are in progress. See original paper for figures.

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