PHYSICS

Academician of the Academy of Sciences of the Uzbek SSR É. I. ADIROVICH

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Abstract

Full Text

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ON THE THEORY OF PHOTOCONDUCTIVITY IN THIN SEMICONDUCTOR LAYERS

1. Consider a thin semiconductor layer (film, plate) $0 \leq x \leq l$, illuminated from one or from both sides. For $L_{\text{Deb}} \ll l \leq 2 \div 3 L_{\text{diff}}$, the electronic processes at high illumination intensities proceed under quasineutral conditions and are described by the equations of bipolar diffusion

$$D\frac{d^2p}{dx^2} = A_1(np - n_0p_0) + A_2(np_\ell - n_0p_{\ell 0}) - G;$$

$$0 = A_2(np_\ell - n_0p_{\ell 0}) - A_3(pn_\ell - p_0n_{\ell 0}); \qquad (1)$$

$$p_\ell + n_\ell = N_\ell; \qquad p \approx n.$$

Here $D = 2D_{pD}n/(D_p + D_n)$, and the quadratic terms correspond to recombination transitions through local centers (A_2, A_3) and directly from band to band (A_1).

If the volume photogeneration G is homogeneous or does not occur at all, then the general solution of the problem can be written in elliptic functions. Excluding the concentrations of electrons and holes at recombination centers (n_ℓ, p_ℓ) and passing to dimensionless units

$$\xi = \frac{\sqrt[4]{1 + 4b/a^2}}{\sqrt{2\sqrt{3}}} \frac{x}{\sqrt{D\tau_\infty}}; \qquad \eta = \frac{2p/a + 1}{2\sqrt{3}\,(1 + 4b/a^2)} \qquad (2)$$

we arrive at the equation

$$2\frac{d^2\eta}{d\xi^2} = 12\eta^2 - 1. \qquad (3)$$

Here

$$a = \frac{1}{A_1\tau_\infty}; \qquad \tau_\infty \sim \frac{1}{A_2N_\ell} + \frac{1}{A_3N_\ell}; \qquad b = \frac{G}{A_1} + a\,\frac{p_0n_{\ell 0} + n_0p_{\ell 0}}{N_\ell} + n_i^2. \qquad (4)$$

Fig. 1 diagram

Figure 1: Fig. 1 diagram

Equation (3) has a universal character and contains neither the parameters of the semiconductor nor the characteristics of photogeneration. It shows that all these quantities enter the solution only through the units of measurement and through the boundary conditions.

Integrating (3), we obtain

$$\left(\frac{d\eta}{d\xi}\right)^2 = 4\eta^3 - \eta - g_3; \tag{5}$$

$$\mp\xi = \int_{\eta_0}^{\eta} \frac{d\eta}{\sqrt{4\eta^3 - \eta - g_3}}. \tag{6}$$

2. Denote by $\varphi(\eta, g_3)$ the right-hand side of equation (5). We note that the extrema of the functions $\varphi(\eta, g_3)$ are located at the points $\eta' = \frac{1}{2\sqrt{3}}$ and $\eta'' = -\frac{1}{2\sqrt{3}}$, and the inflection point at $\eta''' = 0$. Figure 1 shows the family of curves $\varphi(\eta, g_3)$, superposed by shifting the coordinate axes. For each value of the integration constant g_3, one should choose its own position of the coordinate system, namely such that the origin is shifted along the ordinate axis by g_3 relative to point B.

The dependence on g_3 of the roots of the function $\varphi(\eta, g_3)$ is shown graphically in Fig. 2. The regions (∞, α), (α, β), and $(\beta, -\infty)$ are the real branches $e_1(g_3)$, $e_2(g_3)$, and $e_3(g_3)$. For $|g_3| > 1/3\sqrt{3}$ the function $\varphi(\eta, g_3)$ has only one real root. Since the roots of $\varphi(\eta, g_3)$ are determined from the conditions $g_3 = \varphi(\eta, 0)$, the graph $e(g_3)$ in Fig. 2b is obtained from the curve $\varphi(\eta, 0)$ of Fig. 1 by mirror reflection and rotation of the axes through $\pi/2$. Separately (Fig. 2a) the dependence $e(g_3)$ is shown over a wider interval of variation of g_3 (from -10 to $+10$). For $g_3 > 10$, $e_1 \approx \sqrt[3]{g_3/4}$; for $g_3 < -10$, $e_3 = -\sqrt[3]{g_3/4}$.

Fig. 1. Curves $\varphi(\eta, g_3)$ for
$g_3 < -1/3\sqrt{3}$ (1), $g_3 = -1/3\sqrt{3}$ (2),
$\varphi(\eta^*, 0) < g_3 < \varphi(\eta^*, 0)$ (3), $g_3 = 0$ (4),
$0 < g_3 < 1/3\sqrt{3}$ (5), $g_3 = 1/3\sqrt{3}$ (6),
$g_3 > 1/3\sqrt{3}$ (7)

The regions of possible values of η are determined by the conditions of reality of η and $d\eta/d\xi$, and moreover one must also have $p > 0$, i.e. $\eta > \eta^* = 1/2\sqrt{3(1 + 4b/a^2)}$. For a given g_3, the possible values of η correspond to portions of the curve $\varphi(\eta, g_3)$ lying to the right of the straight line $\eta = \eta^*$ and above the abscissa axis.

Fig. 2 diagram

Figure 2: Fig. 2 diagram

For any g_3, the solution permits all values of η lying in the interval $\eta^{**}(g_3) \leq \eta < \infty$, where $\eta^{**}(g_3) = \eta^*$ for $g_3 < -1/3\sqrt{3}$, and $\eta^{**}(g_3) = e_1(g_3)$ for $g_3 \geq -1/3\sqrt{3}$. Direct inversion of the elliptic integral (6) leads to a solution in the form of a Weierstrass function ([1,2]).

$$\eta = \wp(C \mp \xi;\ 1;\ g_3), \tag{7}$$

where C is a new constant of integration, introduced instead of η_0. The second constant of integration enters in the form of the invariant g_3. Analysis of expressions (5) and (6) shows that in the semiconductor layer $0 \leq \xi \leq l^*$, where

$$l^* = \sqrt[4]{1 + 4bl/a^2}\, l/\sqrt{2\sqrt{3}D\tau},$$

η either does not take values corresponding to the roots of $\varphi(\eta, g_3)$ at all, or takes only one such value, and moreover not more than once. For $e_1 \neq e_2 \neq e_3$, all roots of $\varphi(\eta, g_3)$ correspond to extrema of $\eta(\xi)$. Therefore $d\eta/d\xi$ changes sign in the interval $0 \leq \xi \leq l^*$ if and only if within this interval $\eta = e_1$ or $\eta = e_2$ (since $e_3 < 0$). In the solution (7) both signs must be taken into account only if within the interval $0 \leq \xi \leq l^*$ η takes the value e_1. If $\eta \neq e_1$, then in solution (7) only the plus sign before ξ may be retained, since, without loss of generality, the direction of bipolar diffusion may always be taken as the positive direction of ox. If, however, $\eta = e_1$ within the interval $(0, l^*)$, then the sign of $d\eta/d\xi$ in the semi-

Fig. 2. Dependence of the roots of $\varphi(\eta, g_3)$ on g_3

semiconducting layer varies, and $\eta(\xi)$ is written in the form of a piecewise function

$$\eta = \begin{cases} \wp(C' + \xi;\ 1;\ g_3), & \text{for } 0 \leqslant \xi \leqslant \xi', \\ \wp(C'' - \xi;\ 1;\ g_3), & \text{for } \xi' \leqslant \xi \leqslant l^*, \end{cases} \tag{8}$$

where

$$C'' - C' = 2\xi', \qquad C'' + C' = \frac{2}{\sqrt{e_1 - e_3}} F\left(\sqrt{\frac{e_2 - e_3}{e_1 - e_3}}\right);$$

$$e_1 + e_2 + e_3 = 0; \qquad e_1e_2 + e_2e_3 + e_3e_1 = -\frac{1}{4}; \qquad e_1e_2e_3 = \frac{g_3}{4}. \tag{9}$$

3. For $\varphi(\eta',0) < g_3 < \varphi(\eta^*,0)$, where $\varphi(\eta^*,0) = (1+6b/a^2)/[3(1+4b/a^2)^{3/2}]$; $\varphi(\eta',0) = -\frac{1}{3}\sqrt{3}$, the problem has, in addition to (7), one more solution $\tilde{\eta}(\xi)$, with the possible values of $\tilde{\eta}$ restricted by the conditions $\eta^* \leqslant \tilde{\eta} \leqslant e_2$. If $\tilde{\eta} \neq e_2$ inside the interval $0 \leqslant \xi \leqslant l^*$, then from (6) we obtain[1]

$$\tilde{\eta} = e_3 + (e_2 - e_3)\,\mathrm{sn}^2\left[\sqrt{e_1 - e_3}\,(C + \xi)\right], \tag{10}$$

where $k^2 = (e_2 - e_3)/(e_1 - e_3)$. If, however, $\tilde{\eta} = e_2$ inside $(0, l^*)$, then the solution is written in the form of a piecewise function

$$\tilde{\eta} = \begin{cases} e_3 + (e_2 - e_3)\,\mathrm{sn}^2\left[\sqrt{e_1 - e_3}\,(C' + \xi)\right], & \text{for } 0 \leqslant \xi \leqslant \xi', \\ e_3 + (e_2 - e_3)\,\mathrm{sn}^2\left[\sqrt{e_1 - e_3}\,(C'' - \xi)\right], & \text{for } \xi' \leqslant \xi \leqslant l^*, \end{cases} \tag{11}$$

where C' and C'' are connected by the same relations (9) as in solution (8), and $\tilde{\eta}(\xi') = e_2$.

4. The solutions (7), (8), (10), and (11) degenerate for $g_3 = -\frac{1}{3}\sqrt{3}$, when $e_2 = e_1$, and for $g_3 = \frac{1}{3}\sqrt{3}$, when $e_2 = e_3$. Since the solutions (10) and (11) have physical meaning only for $g_3 < 0$, their degenerate form should be considered only for $g_3 = -\frac{1}{3}\sqrt{3}$.

In the case $g_3 = \frac{1}{3}\sqrt{3}$, the roots of $\varphi(\eta, g_3)$ are $e_2 = e_3 = -\frac{1}{2}\sqrt{3}$, $e_1 = 1/\sqrt{3}$, and $k^2 = (e_2 - e_3)/(e_1 - e_3) = 0$. From the formulas $\mathrm{sn}(u\sqrt{e_1 - e_3}) = \sqrt{e_1 - e_3}\,[\wp(u) - e_3]^{-1/2}$ and $\mathrm{sn}(u, 0) = \sin u$, it follows that, in the case of multiple roots $e_2 = e_3$, the degenerate form of solution (8) has the form

$$\eta = -\frac{1}{2\sqrt{3}} + \frac{\sqrt{3}}{2}\csc^2\left[\frac{\sqrt[4]{3}}{\sqrt{2}}(C \pm \xi)\right], \tag{12}$$

where the plus sign before ξ should be written in the interval $0 \leqslant \xi \leqslant \xi'$, and the minus sign in the interval $\xi' \leqslant \xi \leqslant l^*$ (respectively, C is equal to C' and C''). The conditions (9), relating the constants in the piecewise solution, take the form

$$C'' - C' = 2\xi'; \qquad C'' + C' = \frac{\sqrt{2}\pi}{\sqrt[4]{3}}. \tag{13}$$

If $\eta \neq e_1$ in the interval $(0, l^*)$, then in (12) only the plus sign before ξ is retained.

In the second case, leading to a degenerate solution, $g_3 = -\frac{1}{3}\sqrt{3}$, the roots of $\varphi(\eta, g_3)$ are $e_2 = e_1 = \frac{1}{2}\sqrt{3}$, $e_3 = -1/\sqrt{3}$, and $k^2 = (e_2 - e_3)/(e_1 - e_3) = 1$. The solutions (7) and (10) then become the expressions[2]

$$\eta = -\frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{2}\,\mathrm{cth}^2\left[\frac{\sqrt[4]{3}}{\sqrt{2}}(C+\xi)\right],$$
$$\tilde{\eta} = -\frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{2}\,\mathrm{th}^2\left[\frac{\sqrt[4]{3}}{\sqrt{2}}(C+\xi)\right]. \tag{14}$$

Both solutions tend asymptotically to $e_1 = \frac{1}{2}\sqrt{3}$, and for all finite values of ξ, $\eta > e_1$, while $\tilde{\eta} < e_1$. Consequently, everywhere in the semi-

semiconducting layer $d\eta/d\xi \neq 0$ and $d\tilde{\eta}/d\xi \neq 0$, and the direction of the bipolar-diffusion current does not change.

5. The expressions obtained above exhaust all possible solutions of the problem posed. The total photoconductivity of the semiconductor layer is expressed through

$$\int_0^{l^*} \eta(\xi)\, d\xi$$

and, according to the formulas

$$\int \mathrm{sn}^2\, u\, du =$$

$$= \frac{1}{k^2}[u - E(\mathrm{am}\, u, k)]$$

and

$$\int \wp(u)\, du = -\zeta(u) \tag{3}$$

can be represented in the form of well-known tabulated functions. The difficulties associated with finding the constants and choosing the solution corresponding to the given boundary conditions are due mainly to the determination of g_3. The following sequence of operations may be proposed: 1) by specifying g_3, we find η_0 and η_l with the aid of the boundary conditions $\Phi_0(\eta,\, d\eta/d\xi) = 0$ and $\Phi_l(\eta,\, d\eta/d\xi) = 0$; 2) knowing η_0 and η_l, we find C and l^* from the solution $\eta(\xi)$; 3) by virtue of the uniqueness of solution (3), the true values of C and g_3 are determined from the condition that the calculated and actual values of l be equal. These operations can be tabulated or nomographed.

Fig. 3. Distribution of nonequilibrium carriers when bulk photogeneration predominates (a) and when surface photogeneration predominates at both boundaries of the semiconductor layer (b).

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Figure 3: Fig. 3. Distribution of nonequilibrium carriers when bulk photogeneration predominates (a) and when surface photogeneration predominates at both boundaries of the semiconductor layer (b)

In most cases the choice of solution can be made on the basis of physical considerations. Thus, for example, under illumination in the long-wavelength region of the spectrum, when only bulk and practically homogeneous photogeneration occurs, the distribution of the concentrations of nonequilibrium charge carriers is described by formulas (11) and corresponds to currents caused by surface recombination at the boundaries of the layer $x = 0$ and $x = l$. When a semiconductor is illuminated from two sides by short-wavelength light of approximately equal intensity, the solution has the form (8) and expresses the process of flow of two opposing currents of bipolar diffusion from the surfaces into the interior of the semiconductor layer (Fig. 3).

Physical-Technical Institute
Academy of Sciences of the Uzbek SSR

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Note: Figure translations are in progress. See original paper for figures.