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Abstract

Full Text

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ON HIGH-FREQUENCY PULSATIONS DURING THE COMBUSTION OF SOLID FUEL

According, for example, to ([1]), the frequencies of pulsations in the cylindrical cavity of a fuel element (grain) coincide with the natural frequencies of acoustic oscillations in a closed cylindrical volume

$$\nu_{mnk} = \frac{c_4}{2}\sqrt{\left(\frac{\alpha_{mn}}{R}\right)^2 + \left(\frac{k}{L}\right)^2}. \tag{1}$$

Here c_4 is the speed of sound in the combustion products, R and L are the radius and length of the cavity, k is the wave number for longitudinal oscillations, α_{mn} is a root of the Bessel equation, and m and n are the tangential and radial wave numbers.

In pulsating combustion, elastic oscillations of the fuel element are observed; their frequency is matched to the frequency of the acoustic pulsations ([2]). In an elastic medium ([3]), expansion waves (longitudinal waves) and shear waves (transverse waves) propagate. The velocity of the latter is

$$c_1 = \sqrt{G/\rho}, \tag{2}$$

where G is the shear modulus and ρ is the density of the solid. It may be supposed that, in pulsating combustion, surface elastic oscillations are essential. Expansion and shear waves, propagating over the surface of the solid, merge into a single wave whose amplitude decreases exponentially with distance from the surface into the depth of the elastic medium, while the velocity c_2 is somewhat smaller than c_1 and depends on Poisson' s ratio ν. For $\nu = 0.25$, $c_2 = 0.9194\,c_1$; for $\nu = 0.50$, $c_2 = 0.9553\,c_1$. In what follows, for simplicity it is assumed that $c_2 = c_1$.

The period of oscillations in a standing elastic surface wave (an eigenvalue problem) will be determined by the same velocity. The frequency of the standing waves situated on the inner surface of the cylindrical cavity, by analogy with (1), should be equal to

$$\nu_{NK} = c_2\sqrt{\left(\frac{N}{2\pi R}\right)^2 + \left(\frac{K}{4L}\right)^2}, \tag{3}$$

where N is the tangential and K the longitudinal wave number.

From comparison of (1) with (3) it is seen that the conditions for coincidence of the frequencies of acoustic oscillations of the gas with the frequencies of elastic standing waves on the inner surface of the fuel element are not stringent. If longitudinal oscillations are neglected, the criterion for frequency matching has the form

$$\frac{\pi c_4 \alpha_{mn}}{c_2 N} = 1. \tag{4}$$

The frequencies (1) and (3) depend in the same way on the radius of the cavity. Matching between them, once it has arisen, will not be disturbed as the fuel burns out and the radius of the cavity increases.

The energy for sustaining and amplifying high-frequency vibrations, as has been pointed out in the literature ([4]), is drawn from the combustion zone, but the mechanism by which combustion energy is converted into oscillation energy requires clarification. In a thin layer of solid fuel heated by heat conduction and, probably, by radiation, melting and gasification of the material occur. For us it is not essential whether a chemical reaction takes place in it or not. The gas flowing out of the surface layer burns (or completes burning) in the flame front,

separated from the solid surface by a "dark zone." It is assumed below that combustion in the gas phase is similar to normal gas combustion, and that the width of the "dark zone" obeys the same laws as the width of the front of a normal flame. The flame velocity in the gas phase can be expressed in terms of the burning rate of the solid propellant as

$$u = u_\tau(\rho/\rho_g - 1) \simeq u_\tau \rho/\rho_g, \tag{5}$$

where ρ_g is the density of the gas flowing into the flame.

If, for some reason—not essentially important which—the flame velocity suddenly increases, then two shock waves will arise in the combustion zone: one travels through the unburned gas, the other through the combustion products. For us, the significant wave is the one traveling through the unburned gas toward the solid surface. The relation between the pressure jump in such a shock wave, if it is weak, and the increase in flame velocity is given in ([5])

$$\frac{p' - p}{p} = \Delta p = \frac{c_3}{c_3 + c_4} qM\Delta u; \qquad \Delta u = \frac{u' - u}{u}, \tag{6}$$

where $q = Q/E = \gamma(\gamma-1)Q/c_3^2$ is the ratio of the heat effect to the initial internal energy of the unburned gas, $M = u/c_3$ is the ratio of the flame velocity in the steady regime to the speed of sound in the unburned gas, u' is the increased flame velocity, and p' is the increased pressure. Thus, the search for the causes of periodic pressure pulses that drive oscillations of the gas and the solid surface reduces, according to equation (6), to the search for the cause of a periodic increase in the flame velocity in the gas phase. This cause may be the entrainment into the flame zone of small droplets and particles—a suspension —from the surface of the solid phase, occurring as a result of the development of elastic oscillations of the solid surface. This mechanism of increasing the flame velocity resembles the mechanism for increasing the burning rate of solid explosives during the transition from slow combustion to detonation, proposed by K. K. Andreev ([6]). The difference between them consists only in the fact that in K. K. Andreev' s case the solid and liquid phases enter the gas-combustion zone as a result of an increase in the reaction rate in the solid substance, whereas in the case considered here the liquid phase and small particles are torn from the surface as a result of its vibration.

The suspension is torn from the surface in that phase of the oscillation when the surface, moving toward the gas, passes through the equilibrium position. Consequently, if it is assumed that the burning rate increases without delay, precisely at the moment when the suspension enters the flame (this is plausible, since the size of the droplets is always smaller than the width of the reaction zone in the solid phase), then in order to amplify the surface oscillations it is necessary that the suspension traverse the dark zone during the time from 1/4 to 3/4 (or from 5/4 to 7/4, or from $l+1/4$ to $l+3/4$, where l is zero or a positive integer) of the period of oscillation of the solid surface T. We shall take this time to be equal to $(l+1/2)T$.

Taking the droplet velocity in the dark zone to be equal to the sum of the gas velocity w and the maximum velocity of the solid surface v, we obtain the necessary (but not sufficient) condition for amplification of the oscillations in the form

$$\frac{\lambda}{w+v} \simeq (l+1/2)T \simeq (l+1/2)\frac{2\pi R}{Nc_2}. \tag{7}$$

Let us recall that $|w| = u$. Condition (7), like all subsequent results, applies to the case in which there are no longitudinal oscillations. Generalization to the case of longitudinal and combined oscillations is elementary. It is not carried out here for fear of obscuring the essence of the proposed method.

The width of the dark zone and the maximum velocity of the solid surface, from dimensional considerations, are equal to

$$\lambda = \frac{\varkappa}{u} = \frac{\varkappa\rho_g}{u_\tau\rho}; \qquad v = A\frac{c_2 N}{R}. \tag{8}$$

Here χ is the thermal diffusivity of the gas flowing into the gas-combustion zone, and A is the amplitude of the surface oscillations. Taking (5) and (8) into account, (7) is transformed into the condition

$$(l+1/2)\frac{2\pi R u_T^2 \rho^2}{\chi N c_2 p_r^2}(1+v/u)=1, \tag{9}$$

which assumes the limiting value

$$(l+1/2)A\frac{2\pi u_T \rho}{\chi p_r}=1, \tag{10}$$

if $v \gg u$, and

$$(l+1/2)\frac{2\pi R u_T^2 \rho^2}{\chi N c_2 p_r^2}=1, \tag{11}$$

when $v \ll u$.

The criterion for the onset of oscillations is equation (11), which applies to the case when the oscillations are weak. The criterion for the increase in the amplitude of strong oscillations, (10), leads to a paradoxical result: in order for strong oscillations to increase in amplitude, it is necessary that, simultaneously, either the burning rate of the solid propellant decrease or the gas density in the dark zone increase. Otherwise the system goes out of resonance. Criterion (11) can be combined with (4):

$$\frac{R u_T^2 \rho^2}{\chi \lambda_{mn} c_4 p_r^2}=\text{const}. \tag{12}$$

Condition (12) states: if there are no oscillations, then, in order that they should not arise when any of the parameters is changed, it is necessary simultaneously to change the other parameters, keeping criterion (12) constant. Or, conversely: if, when any of the parameters entering criterion (12) is changed, oscillations arise, then to eliminate them the other parameters must be changed so that criterion (12) remains what it was when there were no oscillations.

As was already said, criterion (12) is a necessary but insufficient condition for the occurrence of oscillations. Shock waves arising when the suspension enters the combustion zone may coincide in phase with the oscillations of the solid surface, but their intensity may prove insufficient to compensate the energy losses of the gas oscillations and of the solid surface. Then the initial disturbances will decay, or weak pulsations of constant amplitude will be established, causing no harm.

A second criterion for the amplification of oscillations can be obtained by following the method developed earlier ((5), p. 198). To do this, in addition to the dependence of the shock-wave intensity on the increase in flame velocity (6), it is necessary to find the dependence of the increase in burning rate on the shock-wave intensity (feedback). Because of surface irregularities and the different sizes of the droplets, their velocities are not identical, and the suspension enters the front of the gas flame over a certain time interval $\Delta\tau$. The mean increase in the velocity of the gas flame can be estimated as

$$\overline{u'} - u = \frac{\lambda_l \rho}{\Delta\tau\, p_r} \quad \text{or} \quad \Delta u = \frac{\chi_T}{u_T^2 \Delta\tau}. \tag{13}$$

Here λ_l is the thickness of the liquid phase, and χ_T is the thermal diffusivity of the solid propellant. The time interval $\Delta\tau$ is proportional to the thickness of the dark zone; from dimensional considerations it is equal to

$$\Delta\tau \simeq \frac{\lambda}{v} = \frac{\chi p_r}{u_T \rho v}. \tag{14}$$

Upon reflection of a gaseous shock wave from a solid surface, the pressure at the wall (taking, for example, a diatomic gas) changes from 1, in very

weak wave, to 8 in a very strong one. Taking the wave to be very weak, we set the amplification factor equal to 1. From the condition of equality of pressure at the gas-solid boundary, one can estimate the velocity of motion of the solid surface

$$v = \frac{p}{\rho c_2}\,\Delta p. \tag{15}$$

Approximately equal to this same quantity (neglecting damping) is the velocity of the surface after 1/2 period of oscillation, i.e., the velocity entering into (14).

From (13), (14), and (15) one obtains the dependence of the dimensionless increment of the flame velocity on the dimensionless pressure drop in the shock wave

$$\Delta u = \frac{\varkappa_t c_3^2}{\gamma \varkappa u_t c_2}\,\Delta p. \tag{16}$$

The criterion for amplification of oscillations, according to ([5]), p. 229, is

$$\left.\frac{d\Delta p}{d\Delta u}\right|_{(6)} > \left.\frac{d\Delta p}{d\Delta u}\right|_{(16)}. \tag{17}$$

Differentiating (6) and (16), we obtain the criterion for amplification of high-frequency oscillations during the combustion of solid fuel

$$\frac{\varkappa_t c_3^2 \rho}{\gamma \varkappa (c_3 + c_4) c_2 \rho_g} q > 1. \qquad (18)$$

For the occurrence and amplification of high-frequency oscillations in the cavity of a fuel element (longitudinal modes were not considered), the criteria (12) and (18) must be satisfied simultaneously. If the proposed physical theory is correct, they provide certain possibilities for controlling oscillations. More radical means of combating high-frequency pulsations, according to the theory set forth, may be suppression of oscillations in the solid body by structural means or a sharp change in the width of the combustion zone (dark zone), which is probably achieved by additions of aluminum to the solid fuel ([4]).

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Note: Figure translations are in progress. See original paper for figures.