# Reports of the Academy of Sciences of the USSR

1964

SovietRxiv

Fig. 1

Figure 1: Fig. 1

## Abstract

## Full Text

# Reports of the Academy of Sciences of the USSR

1964. Volume 154, No. 2

## ASTRONOMY

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# COLLAPSE OF A STAR WITH COUNTER-PRESSURE TAKEN INTO ACCOUNT

*(Presented by Academician Ya. B. Zel’ dovich on 2 IX 1963)*

At the present time all information on nonstationary solutions of the equations of general relativity for spherically symmetric stars is restricted to the case of dust ($^1$). For the real equation of state of a cold Fermi gas, a set of static solutions for spherical stars is known, and their masses have a maximum $m_{\max} = 0.73\, m_{\odot}$ ($^2$).

Below are given the results of numerical integration of Einstein’ s equations for the case of a cold Fermi gas, when the mass of the star exceeds the value $m_{\max}$.*

**Fig. 1.** Profiles $\zeta(R)$: 1—specified in the numerical calculation; 2—equilibrium profile for $m_{\max}$.

We shall use the comoving coordinate system and the units of ($^2$): $c = 1,\ k = 1,\ m_1^4 c^5 / 8\pi\hbar^3 = 1$; here $m_1$ is the neutron mass.

The interval is written in the form

$$ds^2 = e^{\sigma} d\tau^2 - e^{\omega} dR^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2).$$

The system of equations, given in ($^3$), was written in divergence form; namely, the function

$$m = \frac{1}{2} r \left[1 + e^{-\sigma}\dot{r}^2 - e^{-\omega}{r'}^2\right]$$

was introduced, having the meaning of the mass of matter bounded by the radius $R$ (a dot denotes differentiation with respect to $\tau$, a prime with respect to $R$).

Fig. 2 and Fig. 3

Figure 2: Fig. 2 and Fig. 3

The mass $m$ obeys the equations $m' = 4\pi\varepsilon r^2 r'$ and $\dot{m} = -4\pi p r^2 \dot{r}$, where $\varepsilon$ is the energy density, $p$ the pressure.

The latter relation, by virtue of the boundary condition: at the boundary of the star $p = 0$, ensures conservation of the mass of the star in the course of the calculation.

The equation of state has the form:

$$\varepsilon = \frac{1}{\pi}\left[\zeta(2\zeta^2+1)\sqrt{\zeta^2+1} - \operatorname{Arsh}\zeta\right], \quad p = \frac{1}{\pi}\left[\zeta\left(\frac{2}{3}\zeta^2 - 1\right)\sqrt{\zeta^2+1} + \operatorname{Arsh}\zeta\right],$$

$$\rho = m_1 n = \frac{8}{3\pi}\zeta^3.$$

At the time $\tau = 0$ the initial conditions were specified: $r = R$, $\dot{r} = 0$ and the profile of the quantity $\zeta_0(R)$. In Fig. 1 the curve $\zeta_0(R)$ is shown, as well as the profile $\zeta(R)$ for an equilibrium configuration with $m = m_{\max}$ (in the adopted units $m_{\max} = 0.07665$). With the adopted ($\zeta_0$) density profile $m = 0.1031$, and the rest mass $M = 0.1112$, where $M$ is the total number of particles multiplied by the particle rest mass:

$$M = 4\pi \int_0^{R_0} \rho e^{\omega/2} r^2 \, dR.$$

* The calculation was carried out on a high-speed computer according to a difference scheme compiled by V. L. Zaguskin.

The results of the calculation are shown in Figs. 2 and 3.

Figure 2 gives the general picture of the motion in the $(\tau, R)$ plane. The curves $r = \text{const}$ plotted on this graph show that the motion has the character of an unrestrained fall toward the center, with each particle passing inside its own gravitational radius (the totality of these events gives the line $r = 2m$) and with an unlimited increase of the density at every point.

Naturally, the numerical calculation could be continued only up to the point at which the density became infinite. This first occurred at the center at the instant

Fig. 2. Graph of the motion: 1—lines $r = \text{const}$; 2—line of passages through gravitational radii; 3—light ray, the last to leave the star; 4—line of fall to the center

Fig. 3. Profiles $\xi(R)$ for different $\tau$: 1–$\tau = 0$; 2–1.42; 3–1.67; 4–1.906; 5–1.96; 6–$\xi(R)$ on the trajectory of the light ray last leaving the star

$\tau = 1.96$. The dashed line denotes extrapolation to later times. Extrapolation is reliably carried out in the variables $\tau, r$. From above, the entire region of the solution is bounded by the curve of fall to the center $r = 0$. The same graph also shows the trajectory of a light ray leaving the star at the moment when it passes inside its own gravitational radius $r_{\text{gr}} = 0.2062$. This trajectory bounds the region of states inside the star that are, in principle, accessible to observation by an external observer.

Figure 3 shows the curves $\xi(R)$ at different instants of time. Also shown there is the dependence $\xi(R)$ along the trajectory of the light ray last leaving the star. It is noteworthy that these density values differ little from the initial ones and, consequently, from the equilibrium ones as well. This circumstance may be important in estimating the loss of mass by the star through neutrino radiation during collapse$^{(4)}$.

Received
2 IX 1963

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*Note: Figure translations are in progress. See original paper for figures.*