GENERATION AND ACCELERATION OF NEUTRINOS IN A TURBULENT PLASMA

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Abstract

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GENERATION AND ACCELERATION OF NEUTRINOS IN A TURBULENT PLASMA

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1. In the presence of a weak $(e\nu)(e\nu)$ interaction, neutrinos can be generated not only in elementary collision events ([1–4]), but also by macroscopic motions of matter, for example by plasma turbulence.* We shall show that in a turbulent plasma, in the presence of a weak $(e\nu)(e\nu)$ interaction, acceleration of neutrinos must also occur, i.e., a process in which the number of neutrinos (as well as the number of antineutrinos) is conserved while their mean energy increases. The successes achieved in constructing a theory of plasma turbulence ([5–7]) make it possible to consider both effects for a weakly turbulent plasma. Weak turbulence can be described in the language of plasmons (high-frequency—Langmuir—and low-frequency—hydrodynamic) ([5]), with the principal role being played by induced processes involving plasmons ([8,9]). According to ([10,11]), plasmons with $k > \omega$ can generate neutrino-antineutrino pairs, while plasmons with $k < \omega$ can absorb and emit neutrinos ([12]). The induced processes of Cherenkov radiation and absorption of plasmons by neutrinos lead to acceleration of neutrinos. It should be noted that in the region $k > \omega$ there are various mechanisms of plasma turbulization, beginning with beam instability ([6]) and ending, for example, with processes of decay and coalescence of transverse waves into plasma waves ([13]). For the question of neutrino generation it is necessary to know the possible turbulence at $k < \omega$. We shall show that the nonlinear interaction of plasma waves leads to a transfer of the energy of plasma oscillations from the region $k > \omega$ into the region $k < \omega$.

2. To analyze the acceleration effect, we use the following equation for the change of the neutrino distribution function φ_p:

$$\frac{\partial \varphi_p}{\partial t} = -\int \frac{d\mathbf{k}\, N_k^l}{(2\pi)^3} \left\{ w_p(\mathbf{k}) \left(\varphi_p - \varphi_{p-\mathbf{k}}\right) + w_{p+\mathbf{k}}(\mathbf{k}) \left(\varphi_p - \varphi_{p+\mathbf{k}}\right) \right\}, \qquad (1)$$

where $w_p(\mathbf{k})$ is the probability of emission of a plasmon of momentum $\mathbf{k}$ by a neutrino of momentum $\mathbf{p}$; N_k^l is the number of plasmons; in (1) only induced processes are taken into account. For $k \ll p$, (1) yields a diffusion equation analogous to the quasilinear one ([6]). In astrophysical applications, both low-energy neutrinos and high-density plasma are of interest, when k is of the order of p

$$\left(k \simeq \frac{\omega_0}{v_\Phi} \sim \sqrt{n} \quad \text{for Langmuir oscillations, } \omega_0 \text{ is the Langmuir frequency} \right).$$

From (1), in the general case, we find the change of the mean neutrino energy

$$\langle \varepsilon \rangle = \int d\mathbf{p}\, (2\pi)^{-3}\, \varepsilon \varphi_p, \qquad \varepsilon = |\mathbf{p}|,$$

$$\frac{d}{dt}\langle \varepsilon \rangle = \langle H \rangle, \qquad H = \int \frac{\omega_k N_k^l\, d\mathbf{k}}{(2\pi)^3} \left\{ w_{p+\mathbf{k}}(\mathbf{k}) - w_p(\mathbf{k}) \right\}. \tag{2}$$

To find the energy lost in 1 sec by a unit volume of turbulent plasma in the generation of neutrino-antineutrino pairs, we use–

* The question of the generation of neutrinos by turbulence was posed by Ya. B. Zel' dovich at a seminar of the P. N. Lebedev Physical Institute of the Academy of Sciences of the USSR.

expression

$$Q^l = \int u_{\mathbf{k}}\, \frac{N_{\mathbf{k}}^l}{(2\pi)^3}\, \omega_{\mathbf{k}}\, d\mathbf{k}, \tag{3}$$

where $u_{\mathbf{k}}$ is the probability of pair production by a plasmon. For $N_{\mathbf{k}}^l$ corresponding to thermal equilibrium, (3) coincides with (17) of Ref. (11).

3. To find the probabilities $w_p(\mathbf{k})$ and $u_{\mathbf{k}}$, it is expedient to use perturbation theory (11). Using the expression for the quantized fields of plasmons (14), we obtain, by means of the standard rules (15),

$$u_{\mathbf{k}} = \frac{g^2 \omega_{\mathbf{k}}^4 (1 - k^2/\omega_{\mathbf{k}}^2)^2}{6\pi^2 e^2\, \partial\varepsilon^l/\partial\omega}. \tag{4}$$

This expression differs from (11) by the factor $(1 - k^2/\omega^2)^{-1}$.* Similarly to the calculation (12), using L_1 (see (12)), we obtain the probability of Cherenkov emission of a neutrino by a plasma wave

$$w_p(\mathbf{k}) = \frac{4g^2 \omega_{\mathbf{k}}^2 (k^2/\omega_{\mathbf{k}}^2 - 1)}{e^2\, \partial\varepsilon^l/\partial\omega}\, \frac{|\mathbf{p}|}{||\mathbf{p}| - \omega_{\mathbf{k}}|} (1 - x^2)\, \delta(|\mathbf{p} - \mathbf{k}| - |\mathbf{p}| + \omega_{\mathbf{k}}); \tag{5}$$

$$x = \frac{\omega}{k} + \frac{k^2 - \omega^2}{2k|\mathbf{p}|}. \tag{6}$$

Here g is the weak-interaction constant; x is the cosine of the angle between the emitted plasmon and the neutrino,

$$\frac{\partial \varepsilon^l}{\partial \omega} = \frac{\partial}{\partial \omega} \varepsilon^l(\omega, \mathbf{k})\Big|_{\omega=\omega_{\mathbf{k}}},$$

and ε^l is the longitudinal dielectric permittivity of the plasma.

4. Let us consider the acceleration of neutrinos by isotropic turbulence $N_{\mathbf{k}} = N_{|\mathbf{k}|}$. From (5), (6), (2) we obtain

$$H = \int \frac{N^l_{|\mathbf{k}|}\, d\mathbf{k}}{2\pi^3} \, \frac{g^2}{e^2} \, \frac{\omega^2_{|\mathbf{k}|}(1 - \omega^2_{|\mathbf{k}|}/|\mathbf{k}|^2)^2}{|\mathbf{p}|\, \partial\varepsilon^l/\partial\omega}. \tag{7}$$

Let us note, first, that the acceleration effect tends to zero for waves whose phase velocity tends to the speed of light, $\omega_{\mathbf{k}} \to k$. This reflects the fact that absorption and emission of electromagnetic waves in vacuum are forbidden. Second, high-frequency turbulence (Langmuir plasmons) accelerates neutrinos more efficiently than hydrodynamic turbulence. For Langmuir oscillations at $v_\Phi = \omega_{\mathbf{k}}/|\mathbf{k}| \ll 1$, we obtain the estimate

$$\frac{\varepsilon^2}{\hbar^2\omega_0^2} \simeq \frac{2g^2}{e^2} \, W^l \omega_0 \left\langle \frac{1}{v_\Phi} \right\rangle t \simeq 2.5 \cdot 10^{-21} \left\langle \frac{c}{v_\Phi} \right\rangle (\omega_0 t) \, \frac{W^l}{m^2_{ec}} \left(\frac{\hbar}{m_{ec}} \right)^3, \tag{8}$$

where

$$W^l = \int \frac{\omega_0 N_{|\mathbf{k}|}\, d\mathbf{k}}{(2\pi)^3}$$

is the energy of Langmuir oscillations in cm^{-3}. Neutrino acceleration can be effective in explosive processes, when intense turbulence develops, for example, in supernova explosions and in so-called superstars. Thus, for W^l of the order of $\frac{1}{10} n m^2_{ec}$, $v_\Phi \sim c$, $n \sim 10^{25}$, over $t \sim 10^6$ sec the energy of accelerated neutrinos is ~ 1 eV, which appreciably exceeds the possible Fermi energy 10^{-2} eV of the degenerate gas of the neutrino background.

5. Let us show that, owing to nonlinear effects in the interaction of plasma waves, an effective transfer of waves into the region of phase velocities greater than the speed of light, $k < \omega$, is possible. To describe the nonlinear effects we use the equation

$$\frac{\partial N^i_{\mathbf{k}}}{\partial t} = N^l_{\mathbf{k}} \int N^l_{\mathbf{k}'} \frac{d\mathbf{k}'}{(2\pi)^3} \, w^{ll}_p(\mathbf{k}, \mathbf{k}') \, \frac{3}{2\omega_0} (\mathbf{k}'^2 - \mathbf{k}^2) \, f_p \, d\mathbf{p}; \tag{9}$$

f_p is the distribution function of the plasma particles, $w_p^{ll}(\mathbf{k},\mathbf{k}')$ is the scattering probability, found taking into account the nonlinearities of the plasma for relativistic

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* See the note in the proof correction [12].

plasma waves in (14). Analysis of (9) gives the characteristic growth time τ of plasma waves with $k < \omega$, arising because of interaction with nonrelativistic waves*,

$$\tau^{-1} = \frac{3v_{Te}\omega_0}{2nm_e(2\pi)^{1/2}} \int |\mathbf{k}'|\, d\mathbf{k}'\, N^l_{k'}\chi^2(1-\chi^2), \qquad \chi = \frac{(\mathbf{k}\mathbf{k}')}{|\mathbf{k}|\,|\mathbf{k}'|}, \tag{10}$$

with the same accuracy for isotropically distributed plasma waves

$$\frac{1}{\tau\omega_0} = \left\langle \frac{v_{Te}}{v_\Phi} \right\rangle \frac{W^l}{nmc^2}\,\frac{2\sqrt{2\pi}}{5}. \tag{11}$$

One may speak of turbulence at $k < \omega$ in the case where the characteristic time (11) is much shorter than the time of Coulomb collisions,

$$\frac{\tau_{\text{Coul}}}{\tau} \simeq \left\langle \frac{v_{Te}}{v_\Phi} \right\rangle r_D^3\,\frac{W^l}{mc^2}\,\frac{3(2\pi)^3}{5\pi\ln 4\pi n r_D^3}, \tag{12}$$

where $r_D = v_{Te}/\omega_0$ is the Debye radius of the plasma electrons.

If W^l is of order $< nmv_{Te}^2$, and v_Φ is of order v_{Te}, then (12) gives

$$\frac{nr_D^3}{\ln 4\pi n r_D^3} \gg \frac{c^2}{v_{Te}^2}\,\frac{5}{24\pi^2}. \tag{13}$$

Note that for $v_{Te} \gtrsim \frac{1}{10}c$ ($T \sim 10^8 - 10^9\,{}^\circ$K) condition (13) is automatically satisfied if the number of particles in the Debye sphere is, as usual, large.

6. In a turbulent plasma there arises a substantially larger generation of neutrino-antineutrino pairs than in an equilibrium plasma. From (3), (4) we obtain, in order of magnitude,

$$Q^l \simeq \frac{g^2\omega_0^5}{12\pi^2 e^2}\,W^l_{k<\omega}, \tag{14}$$

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* This result is obtained in the approximation

$$\left(\frac{m_e}{m_i} v_{Te}\right)^{1/2} \ll \frac{v_{Te}}{v_\Phi} \ll 1.$$

In this approximation, result (10) can be obtained from equations (2,1) [(17)] and (10) [(18)], derived for nonrelativistic plasma waves: $\omega_0/|\mathbf{k}| \ll 1$, $\omega_0/|\mathbf{k}'| \ll 1$. Result (10) of the present work shows that such equations may also be used in the case when one of the waves is relativistic, $\omega_0/|\mathbf{k}| \sim 1$. Such a situation is connected with mutual compensation of additional terms in the probability w_p^{ll}, describing scattering by plasma electrons. We emphasize that (10) corresponds to small pumping compared with that which occurs when the written inequalities are not fulfilled (thus, for

$$\left(\frac{m_e}{m_i} v_{Te}\right)^{1/2} \gg v_{Te}/v_\Phi \gg (m_e/m_i)^{1/2}$$

the pumping increases by $\sim (m_e v_\Phi^2/m_i v_{Te}^3)^2$ times, and for $(v_{Te}/v_\Phi) \lesssim (m_{eT} i/m_{iT} e)^{1/2}$, when scattering by ions plays the main role–by $v_{Te}^{-1} v_{Ti}^{-1}$ times). Taking v_Φ to be of order $v_{Te} \gg m_e/m_i$, we shall restrict ourselves to (10). Waves of large amplitudes of hydrodynamic turbulence, excited, for example, by gravitational instability, generate plasma waves with v_Φ of order v_{Te}. Beams of charged particles [(19)] and directed radiation fluxes accompanying explosions can generate waves with $v_\Phi \sim 1$. Further, in a high-temperature plasma with $v_{Te} \gg \frac{1}{3}\sqrt{m_e/m_i}$, decays of plasma waves into hydrodynamic waves (plasma sound), which increase v_Φ, and coalescence processes, which decrease v_Φ, are possible. Analysis shows that decay processes dominate in the case of small-scale hydrodynamic turbulence with characteristic sizes

$$\lambda \ll 3 v_{Te} \left(\frac{m_i}{m_e}\right)^{1/2} \frac{1}{\omega_0}.$$

In the case

$$\lambda \gg \frac{3 v_{Te}}{\omega_0} \sqrt{m_e/m_i},$$

the pumping (11) dominates over coalescence effects if W^l/W^s is sufficiently large (W^s is the energy of the hydrodynamic turbulence). For $v_{Te} \ll \frac{1}{3}\sqrt{m_e/m_i}$, the decrease of v_Φ due to coalescence effects is usually negligibly small.

where $W_{k<\omega}^l$ is the energy of plasma waves $k < \omega$. If $W_{k<\omega}^l$ is taken to be of order $\frac{1}{10} n m v_{Te}^2$, then (14) exceeds the equilibrium value by a factor of $2 \cdot 10^{20} \frac{1}{\sqrt{n}}$, which even for $\rho = \sum_a n_a m_a \sim 10^6$ g/cm^3, $n \sim 10^{30}$ cm^{-3}, amounts to a factor

$\sim 10^5$. We emphasize that the result depends substantially on the adopted value of the energy of turbulent motion.

It should also be noted that high-frequency turbulence (corresponding to plasma waves) creates many more neutrino pairs than hydrodynamic turbulence of the same energy.

In the work of Adams, Ruderman, and Wu [11] it was shown that radiation of neutrino-antineutrino pairs by plasmons in equilibrium stellar plasma can dominate over other types of neutrino radiation [1–4], and all the more over light radiation, appreciably decreasing the time scales of stellar evolution. It is necessary, however, to take into account the presence of convection and turbulence in stars. According to (14), radiation of neutrino pairs by turbulence begins to exceed the mechanisms [1–4] at substantially lower densities and temperatures of stellar plasma. This may shorten the time scales of evolution. We shall not detail this picture, since the results depend substantially on the possible values of W^l. We note only that in all processes of an explosive character (supernova explosions, etc.), irrespective of the causes producing them, intense turbulence arises and, consequently, a large neutrino flux.

Finally, it should be noted that intense turbulence is possible at the initial stage of the evolution of the Universe [20].

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Note: Figure translations are in progress. See original paper for figures.