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Abstract

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CYBERNETICS AND CONTROL THEORY

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UNIVERSALITY PROBLEMS IN THE THEORY OF GROWING AUTOMATA

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In the present note broader classes of elements are considered than pulsating ones ([1]). It is assumed that they possess the ability to "reproduce,"and therefore we shall call them reproducing. From such elements one can construct networks that are growing automata. The concept of a growing automaton obtained in this way essentially embraces the Kolmogorov–Uspenskii algorithm ([2]) and Neumann networks ([3]) and, apparently, coincides with the conception of a growing automaton that has been repeatedly expressed by A. N. Kolmogorov. For reproducing elements, results are established analogous to the results of ([1]) for pulsating elements. In addition, the properties of operators realizable in networks constructed from reproducing elements are investigated. The definitions given in ([1]) are assumed to be known.

1°. A **reproducing element** (RE) A, like a pulsating element, has a finite number of states and a finite number of inputs. It is allowed that in networks over A some elements may be declared **input** elements (networks with **input elements**).

The rules of operation of an RE A with pulsation depth (p.d.) l and reproduction coefficient (r.c.) p: suppose that at time t some copy α of the element A belongs to some network $L(a)$ over A (possibly L is a network with input elements), and let χ be its l-neighborhood in this network; then at time $t+1$, depending only on χ (considered up to isomorphism): a1) α gives rise to a numbered sequence of copies of the element A ("direct descendants") $\alpha_1, \dots, \alpha_{\rho(\chi)}$ ($\rho(\chi) \le p$) in certain states*; a2) α disconnects part of its inputs from the elements to which these inputs were connected at time t, and connects some of them, as well as some of its inputs that were free at time t, to elements belonging to $\chi \cup (\alpha_1, \dots, \alpha_{\rho(\chi)})$; its remaining inputs α leaves free; a3) the direct descendant α_i ($i = 1, \dots, \rho(\chi)$) connects part of its inputs to α, and leaves the remaining ones free; a4) if α is not an input element, then it passes into a definite state (depending only on χ); but if α is an input element**, then its state at each moment is set from outside (in the general case independently of χ).

By $\nabla_A(L(a), T)$ we denote the network into which the network $L(a)$ passes after T moments. If the element $\alpha \in L(a)$, then the composition of the network $\nabla_A(L(a), T)$, besides α itself, will also include the set $U(\alpha, T)$, consisting of the direct descendants of the element α generated by it during the last T moments, of the direct descendants of these direct descendants, and so on. We shall say of the set $U(\alpha, T)$ that it consists of descendants of the element $\alpha \in L(a)$ generated in T moments.

* If at time $t+1$ α gives rise to α_i, then: a) α_i at time t does not belong to L, b) α_i is a direct descendant only of the element α.

** If α is an input element at time t, then it remains an input element at time $t+1$.

Example. GE A_1 has states $0, 1$, inputs R_1, R_2, R_3, g.p. $= 2$, c.r. $= 1$, and the following rules of operation: 1) if at time t in the neighborhood of an element α (α is an arbitrary copy of GE A_1) there is an element with coordinate $\alpha R_1 R_1$, having state 1 and not coinciding with α, then at time $t+1$: a) α generates one direct descendant (denote it by $\alpha(t+1)$) in state 0; b) α connects input R_1 to the element $\alpha R_1 R_1$, input R_2 to the descendant $\alpha(t+1)$, and leaves input R_3 free; c) $\alpha(t+1)$ connects input R_3 to α, and leaves the remaining inputs free; d) if

[Fig. 1 and Fig. 2 diagrams]

Fig. 1 *Fig. 2*

α is not an input element, then it also passes into state 1; 2) in all other cases the element α does not generate new elements and does not change its state or switching.

Thus, for example, if the network over A_1 at time t has the form of Fig. 1, then at time $t+1$ it will pass into the network of Fig. 2, then into the network of Fig. 3, and so on.

2°. In this subsection the concept of simulation, introduced in (1) for a pulsating element (Subsec. 2), will be extended to GE. In this and the following subsections we shall restrict ourselves to considering networks without input elements.

Let a GE A be given, having states $q_1, \dots, q_N$ and inputs $R_1, \dots, R_M$. Let, along with A, another GE A' and a block B over A' with M inputs $P_1, \dots, P_M$ and one output be given. Let some $(qA \to qB)$-correspondence ψ and some natural number T be given. Define the relation

$$A' \overset{\psi}{\underset{T}{\geq}} A$$

as follows: whatever the networks $L(a)$ over A and $V(b)$ over A' may be, if only $V(b)$ is a (φ, ψ)-replacement of $L(a)$ (where φ is some $(L \to B)$-

correspondence), then the network $\nabla_{A'}(V(b), T)$ is a (φ', ψ)-replacement of the network $\nabla_A(L(a), 1)$ such that: a) if an element $\alpha \in \nabla_A(L(a), 1)$ and $\alpha \in L(a)$, then $\varphi'\alpha$ and $\varphi\alpha$ are one and the same copy of the block B; b) if an element $\alpha_i \in \nabla_A(L(a), 1)$, but $\alpha_i \notin L(a)$, consequently α_i is a direct descendant of some $\alpha \in L(a)$, then $\varphi'\alpha_i$ is a copy of the block B, consisting only of descendants of the elements of the block $\varphi\alpha \subset V(b)$, generated in T instants.

[Fig. 3 diagram]

Fig. 3

We shall say that A' **simulates** A ($A' \geq A$), if for some ($qA \to qB$)-correspondence ψ and some T the relation $A' \underset{T}{\overset{\psi}{\geq}} A$ holds.

A GE A^0 will be called **universal** for the class $\mathfrak{M}$ of GE, if

$$\forall A[(A \in \mathfrak{M}) \to (A^0 \geq A)].$$

Theorem 1. *There exists a GE A^0, universal for the class of all GE, and A^0 has 3 states, 5 inputs, g.p. $= 3$, and c.r. $= 1$.*

3°. Until now we have studied elements whose operation was determined locally. We shall now consider a somewhat more general case, where the operation of each element will also depend on certain global properties of the network to which it belongs.

Let an element B have states $q_1, \dots, q_N$ and inputs $R_1, \dots, R_M$. We shall say

that the network $L(a)$ over B has the property S_i, if it contains at least one element in the state $q_i (i = 1, \dots, N)$. We shall call an element B a **generalized RE** (ORE) with g.p. $= l$ and c.p. $= p$, if its rules of functioning differ from the rules of functioning of an RE with g.p. $= l$ and c.p. $= p$ only in that actions a 1)–a 4) at the moment $t + 1$ are determined not only by χ, but also by which of the properties $S_i (i = 1, \dots, N)$ held for the network $L(a)$ at the moment t.

Theorem 2. *There exists an ORE B^0, universal* for the class of all OREs, *and B^0 has 3 states, 5 inputs, g.p. $= 3$, and c.p. $= 1$.*

4°. Let $\mathfrak{A}(a)$ be a network over an RE (or ORE) C, in state a, containing m input elements $\beta_1, \dots, \beta_m$. Suppose that in $\mathfrak{A}(a)$ there is also distinguished a sequence of elements $\gamma_1, \dots, \gamma_n$, which we shall declare to be **output elements**. Each input element will be interpreted as an input channel with alphabet Q (where Q is the set of states of the element C), and each output element as an output channel with the same Q. Now one may say that the network $\mathfrak{A}(a)$ has one "glued" input channel with alphabet Q^m and one "glued" output channel with alphabet Q^n (see (4), pp. 87–89). Suppose that at the moment $t = 0$ the network $\mathfrak{A}(a)$ is given and that, beginning with the moment $t = 1$, along the "glued" input channel there is fed a sequence of letters $\xi(1), \dots, \xi(t), \dots$ in the alphabet Q^m (i.e. at the moment t the states of the input elements are set in accordance with the letter $\xi(t)$). Let $\eta(1), \ldots, \eta(t), \ldots$ be the sequence of letters in the alphabet Q^n which, beginning with the moment $t = 1$, is generated on the "glued" output channel (i.e. at the moment t the states of the output elements $\gamma_1, \ldots, \gamma_n$ correspond to the letter $\eta(t)$). In this case we shall say that the network $\mathfrak{A}(a)$ transforms the input sequence $\xi(1), \ldots, \xi(t), \ldots$ into the output sequence $\eta(1), \ldots, \eta(t), \ldots$. Let $z(t) = \theta[x(t)]$ be an operator without anticipation with input alphabet X and output alphabet Z (see (4), p. 89). We shall say that $\mathfrak{A}(a)$ realizes the operator θ, if there exist a one-to-one mapping h of X into Q^m and a one-to-one mapping g of Z into Q^n such that the following condition is satisfied: if the operator θ transforms the input sequence $x(1), \ldots, x(t), \ldots$ into the output sequence $z(1), \ldots, z(t), \ldots$, then the network $\mathfrak{A}(a)$ transforms the input sequence $hx(1), \ldots, hx(t), \ldots$ into the output sequence $gz(1), \ldots, gz(t), \ldots$.

The question arises of the relation between REs and OREs. It is easy to see that not for every ORE B does there exist an RE A such that $A \geq B$. However, with operators, as the following theorem shows, the situation is different.

Theorem 3. *The class of operators realizable in networks constructed from REs coincides with the class of operators realizable in networks constructed from OREs.*

5°. For a given operator Ω and a fixed word $W = \xi_1 \xi_2 \ldots \xi_\tau$ (ξ_i belongs to the input alphabet of the operator Ω), define the **residual** operator Ω_W by the condition: let Ω transform the sequence $x'(1), \ldots, x'(\tau), x'(\tau+1), \ldots, x'(\tau+t), \ldots$, where $x'(i) = \xi_i$ for $i \leq \tau$, into the sequence $z'(1), \ldots, z'(\tau), z'(\tau+1), \ldots, z'(\tau+t), \ldots$; then Ω_W transforms the sequence $x(1), \ldots, x(t), \ldots$, where $x(t) = x'(\tau+t)$, into the sequence $z(1), \ldots, z(t), \ldots$, where $z(t) = z'(\tau+t)$.

Let $\mathcal{E}(X, Z)$ be some class of operators with input alphabet X and output alphabet Z. We shall say that the operator Ω is **quasi-universal** for the class $\mathcal{E}(X, Z)$, if for every $\theta \in \mathcal{E}$ there exist a word W_θ (the "code" of the operator θ) and a natural number k (the "scale") such that the operator $\Omega_{W_\theta}[x(t)]$ is a k-stretching of the operator $\theta[x(t)]$ (for the definition of k-**stretching**, see (4), p. 128). We shall say that the operator Ω is **universal** for the class $\mathcal{E}(X, Z)$,

* Modeling and a universal element for OREs are defined in exactly the same way as for REs.

** If γ_i is an output element at the moment t, then it remains an output element at the moment $t + 1$.

if there exists a number σ such that for every $\theta \in \mathcal{G}$ there exist a word W_θ and a number $k \leqslant \sigma$ such that the operator $\Omega_{W_\theta}[x(t)]$ is a k-extension of the operator $\theta[x(t)]$.

Let X and Z be alphabets consisting of at least two letters. Denote by $\mathcal{G}(X, Z)$ the class of all operators with input alphabet X and output alphabet Z that are realizable in networks constructed from RE (or, equivalently, according to Theorem 3, in networks constructed from ORE).

Theorem 4. *The class $\mathcal{G}(X, Z)$ contains an operator that is quasi-universal for this class, but does not contain an operator that is universal for this class.*

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Note: Figure translations are in progress. See original paper for figures.