Aerodynamics

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Abstract

Full Text

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A Wing of Finite Span in the Presence of a Moving Shock Wave

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1. We investigate unsteady spatial flows of a compressible fluid caused by the motion of a wing and by a shock wave incident upon it.

Consider the motion of a thin, slightly cambered wing at a small angle of attack with velocity $u > a$. Let the wing encounter a weak shock wave whose front moves with the speed of sound a. The shock-wave front is a plane inclined to the plane of motion of the wing at an angle ω.

The normal component of the velocity on both sides of the wing surface is given by

$$v_n = -u\beta + B = A_0, \tag{1}$$

where the function β—the angle of attack of the elements of the wing surface —and B are specified at each point of the wing surface. These functions are integrable functions of their arguments. The first term corresponds to the basic motion of the wing with constant velocity u, and the second to small additional unsteady motions in which the wing surface may be deformed.

Considering the disturbances of the medium to be small, we make generally accepted simplifying assumptions and consider the problem in a linearized formulation ([1,2]).

Let us take fixed coordinate axes $O_1x_1y_1z_1$, defining the space of motion of the wing, as indicated in Fig. 1 (the axis Oz_1 is directed perpendicular to the plane of the figure).

The velocity potential of the disturbed flow $\Phi(x_1, y_1, z_1, t)$ satisfies the three-dimensional wave equation. On the basis of the prescribed law (1), we have the flow-tangency condition

$$\Phi_{z_1} = A_0(x_1, y_1, t), \tag{2}$$

which must be satisfied on both sides of Σ, the projection of the wing onto the fixed plane $x_1O_1y_1$. The velocity potential Φ in the region disturbed by the

Fig. 1

Figure 1: Fig. 1

motion of the wing, where the influence of the shock wave is felt, will be sought in the form

$$\Phi(x_1, y_1, z_1, t) = \varphi_0(x_1, y_1, z_1, t) + \varphi_1(x_1, y_1, z_1, t). \tag{3}$$

The function φ_0 is the velocity potential in the moving shock wave. The function φ_0 and its derivatives are continuous functions of their arguments. The potential φ_1 satisfies the condition

$$\varphi_{1z_1}(x_1, y_1, 0, t) = A_0(x_1, y_1, t) - \varphi_{0z_1}(x_1, y_1, 0, t) \tag{4}$$

on that part of the projection Σ which, at the time t under consideration, has come to lie behind the front of the moving shock wave. The boundary of the region on which condition (4) is prescribed moves relative to the wing surface with constant velocity $a_1 - u$, where $|a_1| = a \sin\omega$. The derivative $\varphi_{1t} = 0$ on the projection Σ_1 of the vortex sheet onto the plane $x_1O_1y_1$. The function φ_1 vanishes on the Mach wave and on the plane $x_1O_1y_1$ outside the region $\Sigma + \Sigma_1$. In addition, at each instant of time the Chaplygin–Zhukovsky principle must be satisfied at the trailing edge of the wing.

2. To solve the problem we shall apply the method developed in papers ([3–5]). We take the solution of the wave equation in the form

$$\varphi_1(x_1, y_1, z_1, t) =$$

$$= -\frac{a}{2\pi} \iint\limits_{S_1(x_1,y_1,z_1,t)} \frac{\varphi_{1z_1}\left\{\xi_1, \eta_1, 0, t - \frac{1}{a}\sqrt{(x_1-\xi_1)^2 + (y_1-\eta_1)^2 + z_1^2}\right\}}{\sqrt{(1+a^2)(x_1-\xi_1)^2 + (1+a^2)(y_1-\eta_1)^2 + a^2 z_1^2}}\, dS_1, \tag{5}$$

where the integration extends over the branch of the hyperboloid

$$(x_1 - \xi_1)^2 + (y_1 - \eta_1)^2 + z_1^2 - a^2(t-\tau)^2 = 0, \quad \tau < t.$$

Along with the fixed system of coordinate axes, introduce a moving system $Oxyz$ (Fig. 1). The variables x, y, z are related to the variables x_1, y_1, z_1 by the relations $x = x_1 - ut,\ y = y_1,\ z = z_1$. The solution (5)

Fig. 1

Fig. 2

Figure 2: Fig. 2

Fig. 2

in the moving coordinate axes has the form

$$\varphi(x,y,z,t) =$$

$$= -\frac{(u^2-a^2)}{2\pi}\iint\limits_{S(x,y,z,t)} \frac{\varphi_z\left\{\xi,\eta,0,t+\frac{u(x-\xi)}{u^2-a^2}+\frac{a}{u^2-a^2}r\right\}}{\sqrt{(u^2-a^2)^2r^2+\left[a(x-\xi)ur\right]^2+a^2k^4(y-\eta)^2}}\,dS \quad (6)$$

$$\left(r=\sqrt{(x-\xi)^2-k^2(y-\eta)^2-k^2z^2},\quad k=\sqrt{u^2/a^2-1}\right),$$

where the region of integration S is the surface of the hyperboloid

$$(x-\xi)^2+(y-\eta)^2+z^2+2u(x-\xi)(t-\tau)+(u^2-a^2)(t-\tau)^2=0. \quad (7)$$

By φ we denote the required potential φ_1 in the new variables.

In formula (6) we pass from the surface integral to a double integral with a plane region of integration in the plane of the wing xOy:

$$\varphi(x,y,z,t) =$$

$$= -\frac{1}{2\pi}\iint\limits_{S_1^*}\frac{\varphi_z(\xi,\eta,0,\tau_1)}{r}\,d\xi d\eta - \frac{1}{2\pi}\iint\limits_{S_2^*}\frac{\varphi_z(\xi,\eta,0,\tau_2)}{r}\,d\xi d\eta, \quad (8)$$

where

$$\tau_1 = t+\frac{u(x-\xi)+ar}{u^2-a^2}, \qquad \tau_2 = t+\frac{u(x-\xi)-ar}{u^2-a^2}. \quad (9)$$

The regions S_1^* and S_2^* are the projection of the surface S onto the plane xOy. These regions are bounded by the hyperbola M (Fig. 3).

3. Let us turn to the space of the variables x, y, t [4,5] (Fig. 2). Consider the cylindrical surface Σ^*, whose generators are parallel to the axis Ot, and whose directrix is the contour of the wing $AOBD$, given by the equation $\eta = \psi(\xi)$. The surface Σ^* bounds the region V^*, where the values of the derivatives φ_z are known. The plane

Fig. 3

Figure 3: Fig. 3

$$\xi \sin\gamma + \eta \cos\gamma + (u \sin\gamma +$$

$+a \sin\omega)\tau = 0$ divides the region V^* into two parts: where $\varphi_z = A_0$, and where $\varphi_z = A_1$.

Putting the parameter $z = 0$ in (7), let us consider the family of cones with vertices on the line of intersection of the surface Σ^* with the plane W, and find the envelope surface of this family (for $\tau > t$). The surface Ω and the plane W divide the region V^* into parts: $V^* = V_1^* + V_2^* + V_3^* + V_4^*$. The regions V_1^* and V_4^* are situated outside the envelope Ω, respectively to the left and to the right of the plane W (Fig. 2). The regions V_2^* and V_3^* are situated inside the envelope, likewise respectively to the left and to the right of the plane W.

Fig. 3

The solution of the problem has a different analytic form depending on the part of the region V^* in which the vertex of the surface S (a hyperboloid, or, for $z = 0$, a cone) lies for the given set of variables x, y, z, t (Fig. 2).

For sets of variables x, y, z, and t for which the vertex of the surface S lies in the region V_1^*, the velocity potential can be calculated by formula (8), where the integration in both integrals is extended over the part of the plane xOy lying inside the hyperbola M (Fig. 3), and under the integral signs one sets $\varphi_z = A_0(\xi, \eta, \tau)$. The solution of the problem in the case of the region V_4^* differs from the case of the region V_1^* only in that under the integral signs one sets $\varphi_z = A_1(\xi, \eta, \tau)$.

In investigating the regions V_2^* and V_3^*, an essential role is played by the line of intersection L of the plane W with the surface S, which divides the surface S into parts with different values of the derivative φ_z. The projection L_W of the curve L onto the plane xOy, for $\gamma = \pi/2$, is determined by the equation

$$b_0^2(\xi - x_0)^2 - a_0^2(\eta - y)^2 = 1, \tag{10}$$

where

$$x_0 = -\frac{u + a \sin\omega}{a^2 - a^2 \sin\omega} \left[a^2 t + uat \sin\omega + ax \sin\omega\right],$$

$$a_0^2 = x_0^2 + (x + ut)^2 - a^2 t^2 + z^2, \qquad b_0^2 = \frac{a^2 - a^2 \sin^2\omega}{(u - a \sin\omega)^2}\, a_0^2.$$

Fig. 4

Figure 4: Fig. 4

For $\omega = \pi/2$ the curve L_W is a parabola, and for $\omega < \pi/2$ it is a hyperbola.

Let the vertex of the surface S lie in the region V_2^*. In this case the plane W intersects the surface S only along that part of it which corresponds to the values $\tau = \tau_1$.

According to formula (8), the solution of the problem in the case of the region V_2^* is obtained in the form

$$\varphi(x, y, z, t) = \varphi_k(x, y, z, t) + \frac{1}{2\pi} \iint_{\sigma_2} \frac{A_W(\xi, \eta, \tau_1)}{r} \, d\xi \, d\eta, \tag{11}$$

where $A_W = [\varphi_{0z}]_{z=0}$; the region of integration σ_2 is shown in Fig. 3, and the potential φ_k represents the solution of the problem in the case of the region V_1^*.

Let the vertex of the surface S be located in the region V_3^*. In this case the plane W intersects the surface S along both its branches, corresponding to the values $\tau = \tau_1$ and $\tau = \tau_2$. The curve L_W always has points of tangency K_1 and K_2 with the hyperbola M, or, for $z = 0$, with the Mach lines (Fig. 3). The solution of the problem in the case of the region V_3^* is obtained in the form

$$\varphi(x, y, z, t) = \varphi_k(x, y, z, t) + \frac{1}{2\pi} \iint_{\sigma_1' + \sigma_2'} \frac{A_W(\xi, \eta, \tau_1)}{r} + \frac{1}{2\pi} \iint_{\sigma_1'} \frac{A_W(\xi, \eta, \tau_2)}{r} \, d\xi \, d\eta, \tag{12}$$

where the regions σ_1' and σ_2' are shown in Fig. 3.

4. Let us turn to the time instant t. In the space $xy\tau$ draw the plane $\tau = t$ (Fig. 2). The projection of the line of intersection of the plane $\tau = t$ with the plane W onto the plane xOy will likewise be denoted by the letter W (Fig. 4). The projections of the line of intersection of the plane $\tau = t$ with the envelope surface onto the same plane will be denoted by Ω_1 and Ω_2. The equations of the curves Ω_1 and Ω_2 are found in parametric form:

$$(u+a\sin\omega)^2(x-\xi)^2+(u+a\sin\omega)^2[\psi(x^*)-\eta]^2-2u(u+a\sin\omega)(x^*-\xi)(x^*+ut+at\sin\omega)+(u^2-a^2)(x^*+ut+at\sin\omega)^2 \tag{13}$$

Fig. 4

$$(u+a\sin\omega)^2(x^*-\xi)+(u+a\sin\omega)^2[\psi(x^*)-\eta]\psi'(x^*)+(u^2-a^2)(x^*+ut+at\sin\omega)-u(u+at\sin\omega)(x-\xi)-u(u+a\sin\omega$$

where x^* is a parameter. The curve Ω_1 corresponds to the solution with the smaller value of the variable ξ, and the curve Ω_2 to the larger value of this variable.

The lines Ω_2, W, Ω divide the plane of the wing into the regions S_1, S_2, S_3, S_4 –regions with different analytic character of the solution of the problem (Fig. 4). The plane regions S_1, S_2, S_3, S_4 correspond respectively to the regions $V_1^*, V_2^*, V_3^*, V_4^*$ in the space $xy\tau$ at the time instant t.

If the shock wave moves in the direction of motion of the wing, i.e., the wing overtakes the shock wave, then everywhere in the formulas, instead of the expression $u + a\sin\omega$, one should put $u - a\sin\omega$.

The formulas above have been given for the case when the angle $\gamma = \pi/2$. When the angle $\gamma \neq \pi/2$, the solution of the problem is given in an analogous form, with the curve L_W, generally speaking, not located symmetrically with respect to the Mach lines.

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CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.