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Abstract

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Estimating the Degree of Stability of Nonlinear Pulse Systems

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Absolute stability of processes in nonlinear pulse automatic systems (NIPAS) is a necessary, but not sufficient, condition for their operability. In addition to stability of the process, it is often necessary to ensure an appropriate rate of its settling. In the present work an estimate of the rate of settling of processes in NIPAS is given on the basis of the concept of degree of stability.

Consider a NIPAS consisting of a nonlinear element NE and a linear pulse part (LPP) ([1]). As was shown in ([1], [2]), if the characteristic of the NE $\Phi(x)$ satisfies the conditions

$$\Phi(0) = 0, \qquad r < \frac{d\Phi(x)}{dx} < k + r, \tag{1}$$

then, for stability of the possible forced processes, it is sufficient that the inequality

$$\operatorname{Re} \frac{W^*(j\bar{\omega})}{1 + rW^*(j\bar{\omega})} + \frac{1}{k} > 0 \tag{2}$$

be satisfied, where $W^*(j\bar{\omega})$ is the frequency characteristic of the LPP, and that, for the given value of r, the linearized system obtained from the NIPAS by replacing the characteristic of the nonlinear function $\Phi(x)$ by the linear one rx be stable. The degree of stability δ_l of the linearized system is determined by the pole of its transfer function $W^*(q)/[1 + rW^*(q)]$ having the largest (in the algebraic sense) real part. Let us estimate the degree of stability δ of the NIPAS.

Denote the deviation of the process $x[n]$ in the NIPAS from the established process $x^0[n]$, caused by an arbitrary bounded external action, by

$$\xi[n] = x[n] - x^0[n]. \tag{3}$$

The equation with respect to the deviation may be represented in the form ([1], [2])

$$\xi[n] = f_{\mathrm{n}}[n] - \sum_{m=0}^{n} w_{\mathrm{e}}[n-m]\Phi_1(\xi[m], m), \tag{4}$$

where $f_{\mathrm{n}}[n]$ is the response of the LPP to instantaneous disturbances, $w_{\mathrm{e}}[n] = D^{-1}\{W^*(q)/[1+rW^*(q)]\}$ is the impulse characteristic of the equivalent or transformed LPP,* and

$$\Phi_1(\xi[n], n) = \Phi(x^0[n] + \xi[n]) - \Phi(x^0[n]) - r\xi[n]. \tag{5}$$

In view of (1), $\Phi_1(\xi[n], n)$ satisfies, for $n \geq n_0$, the condition

$$0 < \frac{\Phi_1(\xi[n], n)}{\xi[n]} < k. \tag{6}$$

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* Here $D^{-1}\{\ \}$ is the operation of inverse discrete Laplace transformation (3).

Repeating now the derivation of the absolute-stability condition given in (2), with somewhat different auxiliary functions, namely,

$$\varphi_N[n]e^{\delta n} = \begin{cases} \Phi_1(\xi[n], n)e^{\delta n}, & 0 \leq n \leq N, \\ 0, & 0 < n,\ n > N; \end{cases} \tag{7}$$

$$e^{\delta n}\psi_N[n] = \xi_N[n]e^{\delta n} - \frac{1}{k}\varphi_N[n]e^{\delta n}, \tag{8}$$

where $0 \leq \delta < \delta_l$, we arrive at the following conclusion.

In order that

$$\lim_{n\to\infty} \xi[n]e^{\delta n} = 0, \tag{9}$$

it is sufficient that the characteristic of the NE satisfy condition (1), that the linearized system have degree of stability $\delta_l > \delta$, and that the inequality

$$\mathrm{Re}\, \frac{W^*(-\delta + j\bar{\omega})}{1 + rW^*(-\delta + j\bar{\omega})} + \frac{1}{k} > 0, \qquad 0 \leq \bar{\omega} \leq \pi \tag{10}$$

hold.

But condition (9) corresponds to the fact that the NIAS has degree of stability not less than δ, i.e., the deviation from the forced process satisfies the condition

$$|\xi[n]| < Me^{-\delta n}. \tag{11}$$

Thus, the following theorem is valid:

Theorem. In order that an NIAS with an NE characteristic satisfying conditions (1), and an LP having, for the given r, degree of stability δ_l, have degree of stability not less than δ, it is sufficient that inequality (10) be satisfied.

Condition (10) can be given a simple geometric interpretation. In order that an NIAS with an NE characteristic satisfying conditions (1) have degree of stability not less than δ, it is sufficient that the shifted characteristic $W^*(-\delta + j\bar{\omega})$, satisfying the Nyquist frequency criterion, not intersect the circle with center on the real axis and with diameter coordinates $-1/r$ and $-1/kr$. For $r = 0$ this circle degenerates into the straight line $-1/k$. In the special case $\delta = 0$, we obtain the absolute-stability conditions for processes established earlier (1, 2, 4).

The established theorem and its geometric interpretation make it possible to investigate the degree of stability of an NIAS on the basis of the well-known concept from the theory of linear systems—the degree of stability—and with the aid of ordinary frequency methods.

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Note: Figure translations are in progress. See original paper for figures.

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