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Abstract

Full Text

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ON THE INERTIAL MECHANISM OF DEPOSITION OF A COARSE-DISPERSE AEROSOL ON THE EARTH'S VEGETATION COVER

(Presented by Academician E. K. Fedorov, 18 IV 1964)

Modern theory of the diffusion and deposition of a heavy admixture takes into account only gravitational deposition. However, within the vegetation cover, as well as above it, there is wind, whose velocity u may exceed the velocity w of gravitational deposition of particles by tens and hundreds of times; therefore, for coarse-disperse aerosols, not only gravitational but also inertial deposition should play an important role.

In field experiments on the deposition of coarse-disperse aerosols* to estimate the role of inertial deposition, 20 vane samplers were placed in the experimental field; on each of them a horizontal glass plate and a vertical microscope slide were fastened. It was assumed that droplets settle on the surface of the horizontal plate gravitationally, and that deposition on the windward side of the vertical glass occurs as a result of inertia. The glasses of the vane sampler were placed near the upper boundary of a comparatively sparse vegetation cover with a mean plant height $h = 30$ cm. From the results of microscopic examination of the glasses, for each aerosol fraction the quantity n_v/n_g was determined, averaged over 20 points, where n_v is the mean number of droplets of the given fraction that settled per unit area of the windward side of the vertical glass, and n_g is the same for the upper side of the horizontal glass. The values of n_v/n_g obtained for fractions with different mean droplet diameter d , at different wind velocities $u(h)$ at a height of 30 cm, are given in Table 1.

Table 1

$u(h),$ m/sec	$d,$ μ	13	25	42	58	75	92	108	133	167	217	293
1.8	0.84	2.8	4.8	4.4	4.5	4.3	4.1	3.6	2.5	2.1	2.2	
3.6	4.6	8.3	7.1	7.5	5.0	3.2	2.4	3.0	3.1	2.9	4.1	
1.6	2.6	5.2	6.1	5.6	5.5	4.4	3.2	2.4	3.5	1.2	0.76	

It is evident from the table that on the vertical glasses (which may be regarded as crude models of a plant leaf) the density of deposits of droplets of the same class was several times greater than on the horizontal ones ($n_v/n_g > 1$). This result is direct evidence that inertial deposition of aerosol on plants can predominate over gravitational deposition.

The equation of the material balance of an admixture for an element of the vegetation layer of length dx and height h is

$$\int_S C(x, z) u_n(z) dS + \int_S N(x, z) dS + \int_V P(x, z) dV = 0,$$

where C is the admixture concentration; N is the diffusion flux of the admixture;

$$P = -[\alpha(z)\beta(z)u(z) + \beta_g(z)w] C(x, z)$$

is the loss of admixture within the vegetation layer due to inertial and gravitational deposition; α is the coefficient of capture of particles by plants; β is the specific area of the projection of plants on a plane normal to u ; β_g is the specific area of the horizontal projection of plants.

* Conducted with the participation of I. F. Evdokimov, V. M. Krasilnikov, K. P. Mikulina, and Z. M. Yuzhnyi.

After integration with averaging over z and certain simplifications, we obtain the following boundary condition at the upper boundary of the vegetation cover $z = h$:

$$\partial c(x, h)/\partial z = aC(x, h), \quad (1)$$

where

$$a = \frac{h\alpha\beta u(h) + w(\beta_r - 1 + \xi)}{K(h)}; \quad (2)$$

$\xi = C(x, 0)/C(x, h)$, K is the coefficient of convective diffusion.

Thus, the problem of the propagation and deposition of a heavy impurity (for example, in the presence of a linear continuous source), taking into account not only gravitational but also inertial deposition, reduces to solving the diffusion equation

$$u(z) \frac{\partial C(x, z)}{\partial x} - w \frac{\partial C(x, z)}{\partial z} = \frac{\partial}{\partial z} \left[K(z) \frac{\partial C(x, z)}{\partial z} \right] \quad (3)$$

with boundary condition (1) at the underlying surface*, the value of a being determined by equation (2), the usual source condition

$$C(0, z) = \frac{G}{u(H)} \delta(z - H)$$

(δ is the delta-function symbol, H is the height of the source, G is its productivity), the usual condition at infinity $C \rightarrow 0$ as $\sqrt{x^2 + z^2} \rightarrow \infty$, and the power laws ($K(z) = kz$, $u(z) = u_0 z^q$).

Applying the Laplace transform in x , we obtain

$$z^2 \frac{d^2 \tilde{C}}{dz^2} + \left(1 + \frac{w}{k}\right) z \frac{d\tilde{C}}{dz} - \frac{u_0 p z^{1+q}}{k} \tilde{C} = -\frac{GH^{-q}}{k} z^{1+q} \delta(z - H), \quad (4)$$

where p is the transform parameter, and $\tilde{C}(p, z)$ is the image of the function $C(x, z)$.

The solution of equation (4), obtained by the method of variation of arbitrary constants and valid for the region of interest to us $z < H$, is

$$\tilde{C}(p, z) = AK_\nu(v_2 \sqrt{p}) I_\nu(v_1 \sqrt{p}) + AK_\nu(v_2 \sqrt{p}) \frac{[E\sqrt{p} I_{\nu+1}(v_3 \sqrt{p}) - I'_\nu(v_3 \sqrt{p})]}{[E\sqrt{p} K_{\nu+1}(v_3 \sqrt{p}) + K_\nu(v_3 \sqrt{p})]} K_\nu(v_1 \sqrt{p}), \quad (5)$$

where

$$A = \frac{2G}{(1+q)k} \left(\frac{H}{z}\right)^{w/2k}, \quad v_1 = \frac{2}{1+q} \sqrt{\frac{u_0}{k}} z^{(1+q)/2},$$

$$v_2 = \frac{2}{1+q} \sqrt{\frac{u_0}{k}} H^{(1+q)/2}, \quad v_3 = \frac{2}{1+q} \sqrt{\frac{u_0}{k}} h^{(1+q)/2},$$

$$E = \frac{\sqrt{u_0/k} h^{(q-1)/2}}{a}, \quad \nu = \frac{w}{k(1+q)};$$

I and K are the symbols of the Bessel functions of imaginary argument and the Macdonald functions.

The solution for the original function at $z = h$, obtained by the inverse Laplace transform, is

$$C(x, h) = C_0(x, h) + C_1(x, h), \quad (6)$$

where

$$C_0(x, h) = \frac{G(H/h)^{w/2k}}{(1+q)kx} \exp\left[-\frac{u_0}{k(1+q)^2} \frac{H^{1+q} + h^{1+q}}{x}\right] I_\nu\left[\frac{2u_0(Hh)^{(1+q)/2}}{k(1+q)^2 x}\right] \quad (7)$$

is Rounds' s solution ⁽¹⁾ for $z = h$ for the same problem, but without allowance for inertial deposition (instead of condition (1), the condition $K(z) \frac{\partial C(x, z)}{\partial z} \rightarrow 0$ as $z \rightarrow 0$ is adopted);

$$C_1(x, h) = \frac{A}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \frac{e^{px} K_\nu(v_2\sqrt{p}) K_\nu(v_1\sqrt{p}) [E\sqrt{p} I_{\nu+1}(v_3\sqrt{p}) - I_\nu(v_3\sqrt{p})]}{E\sqrt{p} K_{\nu+1}(v_3\sqrt{p}) + K_\nu(v_3\sqrt{p})} dp. \quad (8)$$

* Condition (1) in general form was proposed by A. S. Monin; we have specified it as applied to the inertial mechanism of deposition.

Choosing the contour of integration by the method of M. E. Berlyand ([2], p. 27), one can represent integral (8) in a form convenient for numerical integration with respect to the real variable ω :

$$C_1(x, h) = \int_0^\infty \frac{2A\{M(FH + QK) - L(FK - QH)\}e^{-\omega^2 x}}{\pi(L^2 + M^2)} d\omega, \quad (9)$$

where

$$F = \frac{\pi^2}{16}(A_1A_2 - B_1B_2); \quad Q = \frac{\pi^2}{16}(A_1B_2 + A_2B_1);$$

$$H = E\omega D_1 + C_2; \quad K = E\omega C_1 - D_2;$$

$$L = \frac{\pi}{4}(E\omega B_3 - A_1); \quad M = \frac{\pi}{4}(E\omega A_3 + B_2);$$

$$A_1 = \frac{J_\nu(v_2\omega) - J_{-\nu}(v_2\omega)}{\sin \pi\nu/2}; \quad B_1 = \frac{J_\nu(v_2\omega) + J_{-\nu}(v_2\omega)}{\cos \pi\nu/2};$$

$$A_2 = \frac{J_\nu(v_3\omega) - J_{-\nu}(v_3\omega)}{\sin \pi\nu/2}; \quad B_2 = \frac{J_\nu(v_3\omega) + J_{-\nu}(v_3\omega)}{\cos \pi\nu/2};$$

$$A_3 = \frac{J_{\nu+1}(v_3\omega) - J_{-(\nu+1)}(v_3\omega)}{\sin \pi\nu/2}; \quad B_3 = \frac{J_{\nu+1}(v_3\omega) + J_{-(\nu+1)}(v_3\omega)}{\cos \pi\nu/2};$$

$$C_1 = \cos \left[\frac{\pi}{2}(\nu + 1) \right] J_{\nu+1}(v_3\omega); \quad D_1 = \sin \left[\frac{\pi}{2}(\nu + 1) \right] J_{\nu+1}(v_3\omega);$$

$$C_2 = \cos\left(\frac{\pi\nu}{2}\right) J_\nu(v_3\omega); \quad D_2 = \sin\left(\frac{\pi\nu}{2}\right) J_\nu(v_3\omega);$$

J is the symbol for Bessel functions.

The density of impurity deposits on the ground beneath the plants is

$$g_o = \xi wc(x, h); \quad (10)$$

the density of impurity deposits on the plants (referred to a unit area of ground) is

$$g_p = [kh\alpha + w(1 - \xi)]C(x, h). \quad (11)$$

For practical calculations using these formulas, it is necessary to be able to determine a new dimensionless characteristic of the vegetation cover—the effective capture coefficient $\alpha\beta h$, which is a function of the Stokes criterion

$$\text{Stk} = \frac{\rho u d^2}{18\mu X},$$

where ρ is the particle density, μ is the viscosity of air, and X is the characteristic size of obstacles.

Direct experimental determination of this characteristic, for example by direct measurements of impurity deposits on plants, is difficult. We used an indirect method for determining $\alpha\beta h = f(\text{Stk})$ from the material balance of the impurity $G = G_o + G_p$ (G_o is the total amount of impurity deposited on the ground beneath the plants, G_p the same on the plants):

$$\alpha\beta h \simeq \frac{\xi w}{u(h)} \left[\xi \left(\frac{G}{G_o} - 1 \right) - \beta_r h \right] \simeq \frac{\xi w}{u(h)} \left(\frac{G}{G_o} - 1 \right). \quad (12)$$

The values of $\alpha\beta h$ (taken as constants in the region $\text{Stk} > 8$), found by this method from the results of field experiments, as well as the corresponding values of ξ , h , X , and the roughness parameter z_0 , are given in Table 2.

For these parameter values, selective comparisons were made between the experimental data and solution (6). The agreement proved to be much better than in calculations by Rounds' s formula. The good agreement between the calculated and experimental values of the density g_o on the ground gives grounds to believe that the densities of deposits on the plants g_p , found by formula (11), are close to the actual values.

Calculations show that taking inertial settling into account radically changes the picture of the process. The plant layer, absorbing the impurity, causes

Fig. 1

Figure 1: Fig. 1

its intense diffusion from the upper layers downward, i.e., as it were, washes the impurity out of the atmosphere. As a result, the entire concentration field changes accordingly.

The exact solution (6) given above is of fundamental importance, but for practical approximate calculations simpler formulas are desirable. For this purpose one may use any solution of the diffusion equation obtained without taking inertial settling into account (for example, Rounds' solution $C_0(x, z)$), but divide the impurity flux into the two parts of interest to us, using relations (10) and (11), from which it follows that

$$g_p \approx \frac{wC_0(x, 0)}{1 + w\xi/\alpha\beta hu(h)}, \quad (13)$$

$$g_o \approx \frac{\xi wC_0(x, 0)}{\alpha\beta hu(h)/w + \xi}. \quad (14)$$

Fig. 1

As an example, Fig. 1 gives the results of determining g_o from the exact formulas found (6), (7), and (9) (line 1), from Rounds' formula (7) (2), and from the approximate formula (14), with $C_0(x, 0)$ determined by Rounds' formula (7) (3). In the calculations the following parameter values were adopted: $G = 802$ mg/m, $H = 1.6$ m, $h = 0.2$ m, $k = 0.166$ m/sec, $w = 0.0208$ m/sec, $q = 0.281$, $u_0 = 4.12$ m^{0.719}/sec, $\xi = 1.0$, $\alpha\beta h = 0.486$, $u(h) = 2.62$ m/sec, $K(h) = 0.0332$ m²/sec. The corresponding experimental values of g_o are shown by points.

Table 2

	z_0 , cm	ξ	h , cm	$\alpha\beta h$	X , cm
Steppe, natural vegetation cover	0.52	1.0	5	0.0015	—
Steppe, natural vegetation cover	0.66	1.0	8	0.006	—

	z_0 , cm	ξ	h , cm	$\alpha\beta h$	X , cm
Steppe, natural vegeta- tion cover	1.8	1.0	20	0.14	0.20
Steppe, natural vegeta- tion cover	2.7	1.0	30	0.055	0.38
Wheat at the flower- ing stage, 50-80 plants per 1 m ²	—	0.38	60	0.20	0.30

It is seen from the graph that not only the exact solution, but also the approximate formula (14), is considerably closer to the experimental data than Rounds' solution; as already noted, this gives grounds for considering formula (13) reliable, since its direct experimental verification is difficult.

The possibility of calculating separately the impurity flux to the ground beneath plants (losses of chemical) and the impurity flux to the plants (the amount of chemical used beneficially) is especially important for agricultural applications.

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CITED LITERATURE

1. W. Rounds, *Trans. Am. Geophys. Union*, **36**, 395 (1955).
2. M. E. Berlyand, *Prediction and Regulation of the Thermal Regime of the Surface Layer of the Atmosphere*, L., 1956.

Note: Figure translations are in progress. See original paper for figures.

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