A. Ya. Sagomonyan

1964

SovietRxiv

Abstract

Full Text

A. Ya. Sagomonyan

On the Problem of the Interaction of Bodies at Very High Velocities

(Presented by Academician A. Yu. Ishlinskii, 23 XI 1963)

1. Introduction

A number of important practical problems pose for investigation the problem of the encounter of a body of small mass with a body of considerably larger mass at velocities on the order of 5–10 kilometers per second and higher. At the present time there exist numerous theoretical works devoted to this problem. Works devoted to a rigorous mathematical study are based on the model of the steady motion of an incompressible fluid ([1,2]). A model of the interaction of bodies at high velocities, taking into account the wave and nonstationary character of the phenomenon, is so complex that an analytic study of the problem is possible only under substantial simplifications. On the basis of observation and the study of experimental results on the interaction of bodies at velocities of 3-7 km/sec and of X-ray photographs of the interaction process at velocities up to 2 km/sec, in the present work, as a first step toward solving this problem, a scheme is proposed which apparently preserves the main features of the interaction phenomenon but makes it possible, for a number of interesting cases, to determine approximately the principal parameters of the interaction.

For convenience, the body of large mass before the beginning of the encounter will be regarded as immobile and will be called the *barrier*. The body of smaller mass, which encounters the barrier at high velocity, will be called the *projectile*. Let us consider the scheme of interaction of a projectile with a semi-infinite barrier. The velocity of the projectile is directed along the normal to the plane of the barrier. Studying X-ray photographs of the collision process and the results of experiments in collisions, one may arrive at the following conclusion.

If the velocity of encounter of the projectile with the barrier is greater than a certain definite velocity, then on penetration the projectile enters a plastic state, is strongly deformed, and begins to flow. The velocity at which this phenomenon begins depends on the shape of the projectile and on its strength properties. However, there exists such an encounter velocity above which, on penetration into the barrier, a projectile of any shape behaves as a plastic body. For ordinary metals this maximum velocity apparently has the order of 2 km/sec. At encounter velocities above the indicated threshold velocity, the time of the deformation process and the corresponding depth of penetration depend on the shape of the projectile. For example, other conditions being equal, a narrow cone on penetration preserves its initial shape longer than a cone with a large aperture angle. The deformation process is characterized by the fact that the projectile assumes a mushroom-like shape with a cap in the form of a spherical shell. As penetration proceeds, the stem of this "mushroom" is shortened as a result of the spreading of the projectile material. After the complete "consumption" of the mushroom stem, the material of the projectile in the form of the "cap" of this mushroom continues retarded penetration until complete stopping.

When the encounter velocity exceeds another certain critical velocity (for ordinary metals this value apparently ranges from 3 to 4 km/sec), the material of the projectile and the barrier behind the shock waves is in a state close to that of a liquid. In the process of penetration into the barrier, the liquid jet formed behind the shock wave of the projectile, under the action of the high counter-pressure, turns toward the free surface, forming an axisymmetric mushroom-like shape with boundary lines AB, A_1A_2, B_1B_2 in the meridional plane, shown in Fig. 1. In svo-

in relative motion in the jet, the fluid particles, turning toward the free surface, are ejected outward with high velocity through the annular section AA_1, BB_1, on which the pressure is equal to atmospheric pressure. This latter circumstance is confirmed by high-speed photography, which shows the scattering of a luminous jet from the free surface. The glow of the material, on the other hand, indicates that the material of the striker and of the target is strongly heated, and melting and evaporation may occur.

Fig. 1
Fig. 1

Fig. 2
Fig. 2

In what follows it is assumed that the impact velocity of the striker with the target exceeds the indicated critical velocity and that the phenomenon described here takes place.

The boundary surface AB, separating the materials of the striker and the target, is a contact surface on which the pressure and the normal component of velocity are continuous. The law of motion of this surface also determines the law of penetration of the striker into the target.

As the contact surface moves, the tail of the jet (the "stem of the mushroom") will shorten until it disappears completely. At this moment the interaction of the striker with the target ends. Under the action of the free surface, part of the target material behind the shock wave, being in a liquid state, will likewise be thrown outward. The volume of this part of the target, the volume formed as a result of the inertial motion caused by the remaining part of the target material behind the shock wave, and the volume determined by the contact surface at the moment when the mushroom stem disappears give the crater size in the interaction of the striker with a semi-infinite target.

2. **Supersonic interaction of a striker with a semi-infinite target.** Let a metallic striker, whose length considerably exceeds its transverse dimensions, meet, in the direction normal to the plane of a semi-infinite target, with a high velocity v_0. Suppose that, as a result of the interaction, the material of the striker instantaneously assumes a mushroom-like shape consisting of a spherical shell and a stem (Fig. 1). The surface of this shell, separating the materials of the striker and the target, is the contact surface. Denote the radius of the spherical contact surface by b, and the velocity of its translational motion by u. Let the shock wave formed in the target (line kk in Fig. 1) also be a spherical surface of radius a. It is assumed that both spheres have a common center. The target material between the shock wave and the contact surface, and the striker material behind the shock wave that has arisen in it, are regarded as fluids. Consider the case in which the velocity u is greater than the speed of sound in the target, and the velocity difference $v_0 - u$ is greater than the speed of sound in the striker. Under this assumption, the shock waves in the striker (line nn in Fig. 1) and in the target, relative to the contact surface, will be at a finite distance.

A certain small time is required for such a picture to become established. But in our approximate formulation it is assumed that it arises instantaneously. We begin the investigation of the problem by determining the force action of the target on the spherical contact surface. It is probable that, consid-

the pressure distribution in the vicinity of the forward point of the contact surface will be approximately the same as in the supersonic motion of a sphere of the same radius in an unbounded liquid. Obviously, what is decisive in the force action during penetration of the contact surface is the pressure distribution in the indicated part of this surface. Therefore we assume that, during supersonic penetration of the contact surface, the pressure distribution will be the same as in the supersonic motion of a sphere of the same radius in an unbounded liquid, with the formation of a spherical shock wave having a common center. The liquid between the shock wave and the sphere is assumed to be homogeneous and incompressible. Consequently, the density of the medium changes only at the shock wave, by the same amount at all its points. In calculating the force action we shall assume that, at those points of the contact surface where the pressure reaches the value of atmospheric pressure, the liquid separates from the shell and thereafter exerts no action on it. The proposed scheme for determining the force, in essence, excludes from consideration the complicated process of the influence of the free boundary on the flow pattern.

Thus, the problem of determining the force action on the contact surface has been reduced to the study of the motion of an incompressible liquid between two concentric spheres: the contact surface and the shock wave, moving with translational velocity u. In what follows the radius of the contact surface is assumed constant. In the meridional plane for this axisymmetric problem we obtain the following boundary conditions (Fig. 2).

On the contact surface

$$R = b; \qquad v_n = u\cos\theta. \tag{1}$$

On the shock wave, by means of the laws of conservation of mass and momentum, we shall have

$$R = a; \qquad v_n = \lambda u\cos\theta; \qquad p_1 - p_0 = \rho_0 u^2 \lambda \cos^2\theta, \tag{2}$$

where R, θ are polar coordinates, p_0 is atmospheric pressure, introduced for convenience, and λ is related to the densities ρ and ρ_0 behind and ahead of the shock front by the relation $\lambda = 1 - \rho_0/\rho$.

This problem is solved by the well-known classical methods of hydrodynamics ([3]). Under the assumption that the motion is potential, the solution is given in elementary form for any dependence of the velocity u on the time t. However, it can be shown that, during the small time of the interaction process, the large accelerations $\dot{u}(t)$ that arise have no appreciable influence on the magnitude of the velocity. Therefore, in what follows the velocity u is assumed constant. In that case the inverse problem will be stationary. The solution for the vortical motion of this inverse problem is given in ([4]).

As a result of the calculations carried out, the pressure p, the separation angle θ_0, and the drag force F on the motion of the contact surface are expressed by the formulas:

$$\frac{p - p_0}{\rho} = \frac{u^2}{2}\left(1 - \lambda^2 - f^2\sin^2\theta\right); \qquad F = \frac{\rho_0 \pi b^2 u^2 (1+\lambda)(1-\lambda^2)}{4f^2};$$

$$\sin\theta_0 = \sqrt{1-\lambda^2}/f. \tag{3}$$

The quantity f for potential and vortical motions, respectively, is expressed by the formulas

$$f = \frac{(3-\lambda)}{2}; \qquad f = \frac{1}{15(1-\lambda)}\left[\lambda(5-6\lambda)\left(\frac{a}{b}\right)^3 + 6\lambda^2\left(\frac{b}{a}\right)^2 + 5(3-4\lambda)\right]. \tag{4}$$

The ratio a/b, exactly for potential motion and with good approximation for vortical motion, is determined by the formula

$$a/b = ((3-\lambda)/2\lambda)^{1/3}. \tag{5}$$

Strictly, for vortical motion this ratio is determined from the equation

$$3\lambda^2(b/a)^4 + 5(3-4\lambda)(b/a)^2 - \lambda(5-6\lambda)(b/a)^{-1} = 0.$$

Let us denote by ρ_{01} and ρ_1 the values of the density of the projectile material before and after the discontinuity, and let $\lambda_1 = 1 - \rho_{01}/\rho_1$. From the condition of continuity of pressure at the contact surface we obtain

$$u = v_0 \Big/ \left(1 + \sqrt{\frac{\rho_0}{\rho_{01}} \frac{1+\lambda}{1+\lambda_1}}\right). \tag{6}$$

The quantities λ and λ_1 are determined with the aid of the Hugoniot adiabats on the shock fronts. By the momentum theorem, the force F_1, acting on the contact surface from the side of the projectile in the direction of the velocity u, without allowance for the loss of velocity due to the shock front, is expressed by the formula

$$F_1 = \rho_{01}(v_0 - u)^2(1 + \sin\theta_0)S, \tag{7}$$

where S is the cross-sectional area of the projectile before the beginning of the interaction. From the condition $F = F_1$, introducing the projectile diameter d ($S = \pi d^2/4$), we obtain

$$\frac{b}{d} = \sqrt{\frac{f + \sqrt{1-\lambda^2}}{(1+\lambda_1)(1-\lambda^2)}\, f}. \tag{8}$$

If l is the length of the projectile, then the distance L of the contact surface from the free boundary (at the point $\theta = 0$) at the moment the interaction ends will be equal to

$$L = \frac{l}{v_0 - u} u = l\sqrt{\frac{\rho_{01}}{\rho_0} \frac{1+\lambda_1}{1+\lambda}}. \tag{9}$$

If one introduces the distance h between the shock wave and the contact surface, $h/d = (b/d)(a/b-1)$, then the distance H of the shock wave in the obstacle (at the point $\theta = 0$) from the surface at the moment the interaction ends can be represented in the form

$$H = L + a - b = L + d(a/b - 1)b/d. \tag{10}$$

In Table 1, for the case $\lambda = \lambda_1$, the dimensionless quantities a/b, b/d, h/d are given for five values of λ. The calculations were made for potential and vortex flows. Steel was taken as the material of the projectile and the obstacle

Table 1

v_0, km/sec	λ	Potential flow a/b	Potential flow b/d	Potential flow h/d	Vortex flow a/b	Vortex flow b/d	Vortex flow h/d
10.7736	0.2	1.9129	1.7005	1.5524	1.9361	1.6829	1.5754
13.6544	0.3	1.6510	1.6214	1.05559	1.6793	1.5818	1.0745
21.1769	0.4	1.4800	1.5654	0.7358	1.5133	1.4934	0.7666
39.1845	0.5	1.3572	1.5333	0.5478	1.3914	1.4151	0.5538
88.2859	0.6	1.2599	1.5282	0.3972	1.2947	1.3450	0.3964

with the Hugoniot adiabat [5]

$$p_1 - p_0 = A(\rho/\rho_0)^n + B;$$

$A = 54 \cdot 10^2$ kg/cm^2; $B = 13 \cdot 10^4$ kg/cm^2, $\rho_0 = 795$ kg$\cdot$sec^2/m^4; $n = 8.13$. The calculations show a small discrepancy of the values for potential and vortex flows. If, at the moment the interaction of the projectile with the obstacle ends, as a result of ejection outward all the material of the obstacle behind the shock wave disappears, then the shock wave will damp practically instantaneously, and there will be no inertial motion. In this case the depth of the crater is determined by formula (10). If, however, part of the obstacle material behind the shock wave remains in the crater, then after the interaction ends, when the pressure in the crater is equal to atmospheric pressure, inertial motion of the obstacle will take place. Preliminary calculations have shown that in cases where a considerable part of the obstacle material behind the shock wave is not ejected outward, the effect of inertial motion can be substantial.

Moscow State University
named after M. V. Lomonosov

Received
21 XI 1963

Cited Literature

1. M. A. Lavrent' ev, *UMN*, **12**, issue 4, 41 (1957).

2. G. Birkhoff, G. Taylor, *Appl. Phys.*, **19**, 563 (1948).

3. G. Lamb, *Hydrodynamics*, Moscow, 1947.

4. M. Laikhtkhill, *Problems of Rocket Technology*, No. 6 (1957).

5. L. P. Orlenko, *Applied Mechanics and Technical Physics*, No. 2, 77 (1961).