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O. V. SOROKIN, Yu. P. SAMARIN, Corresponding Member of the Academy of Sciences of the USSR

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Abstract

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THEORY OF ELASTICITY

O. V. SOROKIN, Yu. P. SAMARIN, Corresponding Member of the Academy of Sciences of the USSR
I. A. ODING

ON THE CALCULATION OF CREEP OF BEAMS IN BENDING

1°. A general phenomenological approach to problems of high-temperature deformation of metals may be based on a four-element elastic-viscous model ([1,2]), obeying the system of equations

$$E_2\varepsilon_{\mathrm{y}} = \sigma, \qquad E_1\varepsilon_{\mathrm{v}} + \eta_1\dot{\varepsilon}_{\mathrm{v}} = \sigma, \qquad \eta_2\dot{\varepsilon}_{\mathrm{p}} = \sigma, \qquad \varepsilon_{\mathrm{y}} + \varepsilon_{\mathrm{v}} + \varepsilon_{\mathrm{p}} = \varepsilon, \quad (1)$$

where $\varepsilon_{\mathrm{y}}, \varepsilon_{\mathrm{v}}, \varepsilon_{\mathrm{p}}$ are, respectively, the magnitudes of the elastic, elastic-plastic, and plastic strains; E_1, E_2, η_1, η_2 are the elastic and viscous characteristics of the model; σ is the stress acting on the model; ε is the total strain of the model.

Here we assume that the quantities E_1, η_1, and η_2 entering (1) are not constant, but depend on σ (such dependences may be obtained from creep tests at constant stress). Hence follows the nonlinearity of the equation between σ and ε (with respect to σ) obtained from system (1) after elimination of $\varepsilon_{\mathrm{y}}, \varepsilon_{\mathrm{v}}$, and ε_{p}. Integration of such an equation presents great difficulties and, evidently, is of little use in comparison with the method proposed below for linearizing system (1), based on the fact that the dependences $E_1(\sigma), \eta_1(\sigma)$, and $\eta_2(\sigma)$ can never be determined exactly, and therefore they may be represented, for example, by piecewise-constant functions, which transforms system (1) into a linear one with discontinuous coefficients.

2°. From this point of view let us consider the creep of a beam in pure bending. We write the equilibrium equation of an element of the beam in the form

$$\int_{y_1}^{y_2} \sigma(y,t) y \, dS = M, \quad (2)$$

where y is the distance of an element of the cross-sectional area of the beam to the neutral axis; dS is an element of the cross-sectional area of the beam, perpendicular to the neutral axis; M is the bending moment acting on the beam element.

The strains along the neutral axis must satisfy the compatibility relation

$$\varepsilon(y,t) = yk(t), \tag{3}$$

where $k(t)$ is the curvature of the beam along the neutral axis.

Let us divide the whole process in time into intervals of length $\Delta t = h$, within each of which all characteristics of the model will be regarded as constant. Then for the i-th time interval ($t_i \leq t \leq t_i + h = t_{i+1}$), from (1) we obtain

$$\begin{aligned}\sigma(y,t) = {} & c_1^i(y)e^{(t-t_i)r_1^i(y)} + c_2^i(y)e^{(t-t_i)r_2^i(y)} + E_2\varepsilon(y,t) + \\ & + \frac{E_2}{r_1^i(y) - r_2^i(y)} \int_{t_i}^{t} \varepsilon(y,\tau) \left\{ r_1^i(y) \left[r_1^i(y) + \frac{E_1^i(y)}{\eta_1^i(y)} \right] e^{(t-\tau)r_1^i(y)} - \right. \\ & \left. - r_2^i(y) \left[r_2^i(y) + \frac{E_1^i(y)}{\eta_1(y)} \right] e^{(t-\tau)r_2^i(y)} \right\} d\tau,\end{aligned} \tag{4}$$

where

$$r_{1,2}^i(y) = -\frac{E_2}{2\eta_1^i(y)} \left[1 + \frac{\eta_1^i(y)}{\eta_2^i(y)} + \frac{E_1^i(y)}{E_2} \right] \pm \left\{ \left\{ \frac{E_2}{2\eta_1^i(y)} \left[1 + \frac{\eta_1^i(y)}{\eta_2^i(y)} + \frac{E_1^i(y)}{E_2} \right] \right\}^2 - \frac{E_1^i(y)E_2}{\eta_1^i(y)\eta_2^i(y)} \right\}^{1/2}. \tag{5}$$

For $t = t_i + h$, in view of the smallness of h, from (4) we have the approximate equality

$$\sigma(y, t_i + h) = \sigma(y, t_i) - a(y, t_i) + b(y, t_i)\varepsilon(y, t_i + h), \tag{6}$$

where

$$\begin{aligned} a(y,t_i) &= E_2\varepsilon(y,t_i) \left\{ 1 - \left[r_1^i(y) + r_2^i(y) + \frac{E_1^i(y)}{\eta_1^i(y)} \right] \frac{h}{2} \right\} \\ &\quad - \left[c_1^i(y) r_1^i(y) + c_2^i(y) r_2^i(y) \right] h, \\ b(y,t_i) &= E_2 \left\{ 1 + \left[r_1^i(y) + r_2^i(y) + \frac{E_1^i(y)}{\eta_1^i(y)} \right] \frac{h}{2} \right\}. \end{aligned} \tag{7}$$

Using (6), (3), and (2), we obtain:

$$k(t_i + h) = \int_{y_1}^{y_2} a(y, t_i) y \, dS \Big/ \int_{y_1}^{y_2} b(y, t_i) y^2 \, dS. \tag{8}$$

$c^i_{1,2}(y)$ is found with the aid of the continuity condition for the strains ε, ε_y, ε_p, and ε_v:

$$c^i_1(y) = \frac{B^i(y) - A^i(y) r^i_2(y)}{r^i_1(y) - r^i_2(y)}, \qquad c^i_2(y) = \frac{A^i(y) r^i_1(y) - B^i(y)}{r^i_1(y) - r^i_2(y)}, \tag{9}$$

where

$$A^i(y) = \sigma(y, t_i) - E_2 \varepsilon(y, t_i), \tag{10}$$

$$B^i(y) = \frac{E_2 E^i_1(y)}{\eta^i_1(y)} \varepsilon_\text{v}(y, t_i) + E_2 \left[\frac{1}{\eta^i_1(y)} + \frac{1}{\eta^i_2(y)} \right] \left[E_2 \varepsilon(y, t_i) - \sigma(y, t_i) \right],$$

and $\varepsilon_\text{v}(y, t_i)$ is determined by the formula

$$\varepsilon_\text{v}(y, t_i) = \varepsilon_\text{v}(y, t_{i-1}) + \frac{\sigma(y, t_{i-1}) - \varepsilon_\text{v}(y, t_{i-1}) E^{i-1}_1(y)}{\eta^{i-1}_1(y)} h. \tag{11}$$

Thus, the following scheme is possible for calculating the creep of a beam in pure bending:

1) according to the theory of elasticity, $k(0)$, $\sigma(y, 0)$, $\varepsilon(y, 0)$ are determined;
2) from the history of the preceding deformations, $c^0_{1,2}(y)$ and $\varepsilon_\text{v}(y, 0)$ are found (if the material had not previously been deformed, then $c^0_{1,2}(y) = \varepsilon_\text{v}(y, 0) = 0$);
3) from $\sigma(y, 0)$, with the aid of graphs, $\eta^0_1(y)$, $\eta^0_2(y)$, $E^0_1(y)$ are determined; according to (5) we compute $r^0_{1,2}(y)$;
4) $k(h)$ and $\sigma(y, h)$ are found from formulas (7), (8), and (6), the integrals in (8) being evaluated by any quadrature formula;
5) according to (10) and (9), $c^1_{1,2}(y)$ are found (on subsequent time intervals it is also necessary to use (11));
6) repeat items 3), 4), 5), etc., until the required time interval has been covered.

3°. Let us illustrate the method by the results of a creep calculation for a lead beam of rectangular cross section, whose middle part is under conditions of pure bending (Fig. 3) (for a bending moment $M = 650$ kg · mm, temperature 20°).

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

Figure 1 shows the creep curves of lead under the action of a constant stress. Figure 2 gives the dependences $\eta_1(\sigma)$, $\eta_2(\sigma)$, $E_1(\sigma)$, constructed from the curves of Fig. 1 (with extrapolation).

The calculation of the curvature of the neutral axis and of the stresses was carried out according to the scheme described in item 2°. In Fig. 3, II is the calculated curve; it may be considered to agree satisfactorily with the experimental curve I. Figure 4 shows the stress diagrams in the beam at $t = 0;\ 0.01;\ 1$ hour; the stress curve deforms with time, asymptotically approaching the limiting curve $t = \infty$.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

4°. The method set forth can be considerably simplified if the variable characteristics of the model are replaced by certain averaged (over time and over the coordinate y) values $\overline{\eta}_1$, η_2, and $\overline{E}_1$. Then, for the curvature of the neutral axis, one can obtain the expression

$$\frac{k(t)}{k(0)} = 1 + \frac{E_2}{E_1}\left(1 - e^{-\overline{E}_1 t/\overline{\eta}_1}\right) + \frac{E_2}{\eta_2}\,t. \tag{12}$$

The terms on the right-hand side of (12) give the magnitude of, respectively, the elastic, elastic-plastic, and plastic deformation.

Thus, it becomes possible to use the model given in item 1° as the basis of the creep mechanism in bending. Such an approach makes it possible to carry out the calculation from curves $k = k(t, m)$, where m is a parameter, analo-

stress for creep curves in tension and depending on the bending moment, the shape, and the dimensions of the beam cross section.

Fig. 3

Figure 3: Fig. 3

Fig. 4

Figure 4: Fig. 4

Unfortunately, owing to the lack of sufficient experimental material, it is not now possible to draw a conclusion about the possibilities and limits of applicability of such a point of view; we shall note only that, for the beam of rectangular cross section considered in Sec. 3°, the model with characteristics determined with the aid of Fig. 2, taking $\sigma = \bar{\sigma} = 0.67\,\sigma_{\max}$, gives satisfactory results (curve *III* in Fig. 3).

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CITED LITERATURE

[1] A. R. Rzhanitsyn, *Some Problems in the Mechanics of Systems Deforming in Time*, 1949. [2] L. M. Kachanov, *Foundations of the Theory of Plasticity*, Moscow, 1956.

Note: Figure translations are in progress. See original paper for figures.