SovietRxiv

Soviet-era science, translated into English

V. M. TOLKACHEV

1964

SovietRxiv

View the original and related papers at https://sovietrxiv.org/items/ru-196401.78817

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.

SovietRxiv

Abstract

Full Text

THEORY OF ELASTICITY

V. M. TOLKACHEV

TRANSFER OF LOAD FROM A STRINGER OF FINITE LENGTH TO AN INFINITE AND A SEMI-INFINITE PLATE

(Presented by Academician Yu. N. Rabotnov on 18 VII 1963)

An infinite and a semi-infinite ($\operatorname{Im} z > 0,\ z = x + iy$) elastic plate are considered, to which, on the segment $0 \leq x \leq a$ of the x-axis, a stringer of length a is riveted, loaded at the end $x = 0$ by a force P directed opposite to the x-axis. The distributed load $q(x)$ transmitted from the stringer to the plate is sought.

Using the known solution of the problem of a concentrated force applied to an infinite plane and to the boundary of a half-plane ([1]), we find the deformation of the plate ε_x on the x-axis due to an arbitrary load $q(x)$ directed along the x-axis.

Then, writing the equilibrium equation for a part of the stringer and using the compatibility condition for the deformations of the stringer and the plate, we obtain for both problems the following singular integral equation:

$$\int_0^1 \frac{q(\tau)\,d\tau}{\tau - \tau_0} = \frac{\lambda\pi^2}{4} \int_{\tau_0}^1 q(\tau)\,d\tau, \tag{1}$$

whose solution satisfies the equilibrium condition for the entire stringer

$$\frac{\lambda\pi}{2} \int_0^1 q(\tau)\,d\tau = 1. \tag{2}$$

Here $\tau = x/a,\ \tau_0 = x_0/a,\ 0 \leq x,\ x_0 \leq a.$

Moreover, in the case of an infinite plate

$$\lambda = \frac{16}{\pi(3-\nu)(1+\nu)} \frac{E\delta}{E_{cF}c}\,a, \qquad q(\tau) = \frac{(3-\nu)(1+\nu)E_{cF}c}{8E\delta} \frac{q(x)}{P}, \tag{3}$$

whereas in the case of a semi-infinite plate

SovietRxiv

$$\lambda = \frac{2}{\pi}\frac{E\delta}{E_{cF}c}, \qquad q(\tau) = \frac{E_{cF}c}{E\delta}\frac{q(x)}{P}. \tag{4}$$

In formulas (3) and (4), E, ν, δ are, respectively, the modulus of elasticity, Poisson' s ratio, and thickness of the plate; E_c is the modulus of elasticity, and F_c the cross-sectional area of the stringer.

By the methods developed by L. I. Sedov ([2]), the original equation (1) is reduced to the equivalent Fredholm equation

$$L[q] = A/\sin\theta + B\operatorname{ctg}\theta, \tag{5}$$

where the change of variable has been introduced

$$\tau = \sin^2\frac{\sigma}{2}, \qquad \tau_0 = \sin^2\frac{\theta}{2}, \qquad q(\sigma) = q\Big(\sin\frac{\sigma}{2}\Big). \tag{6}$$

A is an arbitrary constant determined from condition (2),

$$B = \frac{\lambda}{8}\int_0^\pi q(\sigma)\,\sigma\sin\sigma\,d\sigma, \tag{7}$$

$$L[q] = q(\theta) - \frac{\lambda}{8}\int_0^\pi \sin\sigma\ln\left|\frac{\sin\dfrac{\sigma-\theta}{2}}{\sin\dfrac{\sigma+\theta}{2}}\right| q(\sigma)\,d\sigma. \tag{8}$$

The solution of equation (5) has, at the ends of the interval $\theta = 0;\ \pi$, a singularity of the type $\operatorname{ctg}\theta$. With

$$q(\theta) = q_1(\theta) + \frac{A\cos 2\theta}{\sin\theta} + B\operatorname{ctg}\theta \tag{9}$$

we transform equation (5) to the form

$$L(q_1) = f(\theta), \tag{10}$$

where

$$f(\theta) = \frac{A}{4}(8+\lambda)\sin\theta + (A\sin 2\theta + 2B\sin\theta)\,\frac{\lambda}{8}\ln\operatorname{tg}\frac{\theta}{2}. \tag{11}$$

If

SovietRxiv

$$q_1(\theta) = f(\theta) + \sum_{n=1}^{\infty} C_n \sin n\theta, \tag{12}$$

then the solution of equation (10) reduces to the solution of the infinite linear system of equations

$$C'_k - \lambda \sum_{n=1}^{\infty} C'_n \alpha_{kn} = b_k \quad (k = 1, 2, 3, ...). \tag{13}$$

Here

$$\alpha_{kn} = n \left\{ [(k-n)^2 - 1][(k+n)^2 - 1] \right\}^{-1} \tag{14}$$

for $k+n$ even, and $\alpha_{kn} = 0$ for $k+n$ odd,

$$C_n = -\frac{A\lambda^2 C'_n}{4} \quad (n = 1, 3, 5, ...); \qquad C_n = -\frac{B\lambda^2 C'_n}{4} \quad (n = 2, 4, 6, ...), \tag{15}$$

$$b_1 = \frac{8+\lambda}{3\lambda} - \frac{1}{12}; \qquad b_3 = -\frac{8+\lambda}{45\lambda} - \frac{104}{108 \cdot 24};$$

$$b_k = -\frac{8+\lambda}{\lambda k^2(k^2-4)} + \frac{2\left(1 + \frac{1}{3} + \cdots + \frac{1}{k-4}\right)}{(k^2-1)(k^2-9)} - \frac{3k^2(k+1) - 2(k-1)}{2(k^2-4)(k^2-1)k^2(k+3)} \quad (k = 5, 7, ...); \tag{16}$$

$$b_2 = -\frac{1}{12}; \qquad b_k = \frac{2\left(1 + \frac{1}{3} + \cdots + \frac{1}{k+3}\right)}{k^2(k^2-4)} - \frac{3k+2}{2k^2(k^2-1)(k+2)} \quad (k = 4, 6, ...). \tag{17}$$

By virtue of (14), system (13) splits into two independent systems for the even and odd coefficients C'_k and is completely regular for any finite values of the parameters λ.

For large λ, $|C_k| \leqslant 1.756/\lambda$ for odd k, and $|C'_k| \leqslant 1/\lambda$ for even k.

If system (13) is solved, then the solution of the problem is found from formula (9), taking (12) and (15) into account. For the constants A and B we have the formulas

$$A = \frac{32}{\pi^2 \lambda \chi}, \qquad B = AB_1; \tag{18}$$

SovietRxiv

$$B_1 = \frac{\pi^2 \lambda}{128} \frac{\chi}{1 + \frac{\lambda}{4} - \frac{\lambda^2 \omega}{32} - \frac{\lambda^3}{8} \sum_{2,4,\ldots}^{\infty} \frac{C'_n n}{(n^2-1)^2}}; \tag{19}$$

$$\chi = 8 + \lambda - \lambda^2 C'_1 - \frac{\lambda}{3}, \qquad \omega = 0.603599\ldots \tag{20}$$

For the stresses in the stringer $P(\theta)$, referred to the stresses $P(0)$ at the point $\theta = 0$, we have the formula

$$\frac{P(\theta)}{P(0)} = 1 - \frac{\theta}{\pi} - \frac{1}{\pi\chi} \left\{ B_1 (8+\lambda) \sin\theta + \frac{1}{2}(8-\chi)\sin 2\theta + \frac{\lambda}{2}\left(\sin\theta - \frac{1}{3}\sin 3\theta - B_1 \sin 2\theta\right) \ln \operatorname{tg}\frac{\theta}{2} \right.$$

$$\left. + B_1 \lambda \omega_1(\theta) - \lambda^2 \sum_{2,3,4,\ldots}^{\infty} C'_n \delta_n \left[\frac{\sin(n-1)\theta}{n-1} - \frac{\sin(n+1)\theta}{n+1} \right] \right\}; \tag{21}$$

$$\delta_n = 1 \text{ for } n = 3, 5, \ldots; \qquad \delta_n = B_1 \text{ for } n = 2, 4, 6, \ldots;$$

$$\omega_1(\theta) = \int_0^\theta \ln \operatorname{tg}\frac{\theta}{2}\, d\theta = -2 \sum_{1,3,5}^{\infty} \frac{\sin n\theta}{n^2}. \tag{22}$$

Received
9 VII 1963

REFERENCES

[1] N. I. Muskhelishvili, *Some Basic Problems of the Mathematical Theory of Elasticity*, 4th ed., Publishing House of the Academy of Sciences of the USSR, 1954. [2] L. I. Sedov, *Plane Problems of Hydrodynamics and Aerodynamics*, 1950.

Note: Figure translations are in progress. See original paper for figures.

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.