CALCULATION OF THE COLLISION EFFICIENCY OF PARTICLES OF COMPARABLE SIZES

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Abstract

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CALCULATION OF THE COLLISION EFFICIENCY OF PARTICLES OF COMPARABLE SIZES

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The formation of precipitation, in particular the growth rate of particles in clouds, depends to a considerable extent on the efficiency of collision and coalescence of droplets. If it is assumed that every collision leads to coalescence of particles, then the coalescence efficiency is characterized by the collision or capture coefficient—the ratio of the number of particles that have settled on an obstacle to the number of particles whose centers would have intersected the obstacle if they had moved rectilinearly. Thus, determination of the collision coefficient reduces to finding the limiting trajectory of the center of gravity of a particle, i.e., such a trajectory along which the "outermost" particle from the stream still strikes the obstacle.

Many investigators are known to have attempted to estimate the value of the collision coefficient theoretically ([1–10]) or experimentally ([11–14]). Because of the difficulty of the solution, most investigators limited themselves to considering the collision efficiency of particles of very unequal sizes, when the interaction of the hydrodynamic fields of the two drops can be neglected.

The first attempt to calculate the collision coefficient of drops of comparable sizes was made by Pearcey and Hill ([6]), taking into account the interaction forces in Oseen' s approximation, and subsequently by Hocking ([7]), who used the laws of flow around drops in Stokes' approximation. The authors did not take the action of electrical forces into account. In both works ([6,7]), a limiting drop size was established, below which collision is impossible.

In the present work, the collision efficiency of drops of comparable sizes is estimated theoretically in the presence of inertial, hydrodynamic, and electrical forces. The equations of relative motion of charged particles in a homogeneous electric field of intensity E and in the field of gravity are solved. The coordinate system (z, ρ) is associated with the stationary air; the positive direction of the z-axis coincides with the direction of the gravity-force vector (downward), while the electric-field vector is opposite to it (upward).

The equation of motion of drops under the action of the aerodynamic force $\mathbf{F}_a$, the force of gravity $\mathbf{F}_t$, and electrical forces $\mathbf{F}_{\text{el}}$ has the following form*:

Figure 1

Figure 1: Figure 1

$$m_i \frac{d\mathbf{v}_i}{dt} = \mathbf{F}_{ai} + \mathbf{F}_{ti} + \mathbf{F}_{\mathrm{el}i}, \qquad i = 1, 2, \tag{1}$$

where $m_i, \mathbf{v}_i$ are the mass and velocity of the center of gravity of the particles.

Under the conditions of applicability of Stokes' law, the air-resistance force during the motion of drops is expressed as follows:

$$\mathbf{F}_{ai} = -3\pi d_i \eta \mathbf{u}_i(r), \tag{2}$$

where d_i are the diameters of the drop (d_1) and the droplet (d_2); η is the coefficient of viscosity of air; $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1 = z\mathbf{n} + \rho\mathbf{k}$ is the radius vector connecting the centers of the drops ($\mathbf{n}, \mathbf{k}$ are unit vectors).

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* Here and below we assume that the larger particle (drop) corresponds to index 1, and the smaller one (droplet) to index 2.

The mutual interference of the flows of two spheres is taken into account by using the expressions for the flow velocity given by Hocking ([7]):

$$\mathbf{u}_i(\mathbf{r}) = \frac{l_i z - t_i \rho}{r}\,\mathbf{n} + \frac{l_i \rho + t_i z}{r}\,\mathbf{k}, \qquad i = 1, 2. \tag{3}$$

The coefficients l_i and t_i are specified functions of the sizes, coordinates, and velocity components of the particles.

Fig. 1. Dependence of the collision coefficient $K = \rho_0^2$ on the relative sizes of the particles.
1–$d_1 = 50\,\mu$, $E = 0$;
2–$d_1 = 50\,\mu$, $E = -600$ V/cm;
3–$d_1 = 50\,\mu$, $E = -1200$ V/cm;
4–$d_1 = 20\,\mu$, $E = -1200$ V/cm.

The electric forces acting on a drop are composed of the sum of the forces:

$$\mathbf{F}_{\mathrm{el}1} = \mathbf{F}_{q_1 q_2} + \mathbf{F}_{p_1 q_2} + \mathbf{F}_{q_1 p_2} + \mathbf{F}_{p_1 p_2} + \mathbf{F}_{q_1 E}. \tag{4}$$

Similarly, for the forces acting on a droplet:

$$\mathbf{F}_{\mathrm{el}2} = \mathbf{F}_{q_2 q_1} + \mathbf{F}_{q_2 p_1} + \mathbf{F}_{p_2 q_1} + \mathbf{F}_{p_2 p_1} + \mathbf{F}_{q_2 E}, \tag{4'}$$

where $\mathbf{F}_{q_1 q_2} = -\mathbf{F}_{q_2 q_1}$ is the force of interaction of the charges q_1 and q_2:

$$\mathbf{F}_{q_1 q_2} = -q_1 q_2 f(\chi_1, r, a) \frac{\mathbf{r}}{r^3} \tag{5}$$

$$\left(\chi = -\frac{q_2}{q_1}; \qquad a = \frac{\varepsilon_2 c_2^2}{\varepsilon_1 c_1} \right).$$

The expression for the function $f(r, \chi, a)$, which takes the image forces into account, is taken from ([4]).

The dipole moment of droplets polarized in an electric field has the form ([15])

$$\mathbf{p}_i = \frac{d_i^3}{8} \mathbf{E}.$$

The force of interaction of the charge q_2 with the induced dipole p_1 is

$$\mathbf{F}_{p_1 q_2} = -\frac{1}{8} q_2 d_1^3 E \ \mathrm{grad} \frac{\mathbf{nr}}{r^3}; \qquad \mathbf{F}_{p_1 q_2} = -\mathbf{F}_{q_2 p_1}. \tag{6}$$

The force of interaction of the dipole p_2 with the charge q_1 is

$$\mathbf{F}_{q_1 p_2} = -\frac{1}{8} q_1 d_2^3 E \ \mathrm{grad} \frac{\mathbf{nr}}{r^3}; \qquad \mathbf{F}_{q_1 p_2} = -\mathbf{F}_{p_2 q_1}. \tag{7}$$

The force of interaction of the dipoles*

$$\mathbf{F}_{p_1 p_2} = \frac{1}{64} d_1^3 d_2^3 E^2 \ \mathrm{grad} \left(\mathbf{n} \, \mathrm{grad} \frac{\mathbf{nr}}{r^3} \right); \qquad \mathbf{F}_{p_1 p_2} = -\mathbf{F}_{p_2 p_1}. \tag{8}$$

The force of the field acting on a charge is

$$\mathbf{F}_{q_{iE}} = q_i \mathbf{E}. \tag{9}$$

The force of gravity is

$$\mathbf{F}_{\tau_i} = \frac{1}{6} \pi d_i^3 \sigma g \, \mathbf{n}, \tag{10}$$

where σ is the particle density and g is the acceleration of gravity.

Choosing $d_1/2$ as the unit of length, the velocity of the undisturbed flow at infinity u_∞ as the unit of velocity, and $t = d_1/2u_\infty$ as the unit of time, and substituting expressions (5)–(10) into equation (1), we obtain the following system of equations in dimensionless quantities:

$$J\frac{d\mathbf{v}_1(\mathbf{r})}{dt} = 1 - \mathbf{u}_1(\mathbf{r}) + \alpha\left[-\alpha f(\chi, r, a)\frac{\mathbf{r}}{r^3} - (\beta_2 - \beta_1)\,\mathrm{grad}\,\frac{\mathbf{nr}}{r^3} + \frac{\gamma}{3}\,\mathrm{grad}\left(\mathbf{n}\,\mathrm{grad}\,\frac{\mathbf{nr}}{r^3}\right)\right]; \tag{11}$$

* Higher multipoles are not taken into account in the expressions for the electric forces.

$$a^2 J\frac{d\mathbf{v}_2(r)}{dt} = -\mathbf{u}_2(r) + \alpha f(\chi, r, a)\frac{\mathbf{r}}{r^3} + (\beta_2 - \beta_1)\,\mathrm{grad}\,\frac{n\mathbf{r}}{r^3} - \frac{\gamma}{3}\,\mathrm{grad}\left(n\,\mathrm{grad}\,\frac{n\mathbf{r}}{r^3}\right) + (\lambda - \beta_2)\mathbf{n}; \tag{12}$$

$$\frac{d\mathbf{r}}{dt} = \mathbf{v}_2(r) - \mathbf{v}_1(r), \tag{13}$$

where

$$\alpha = \frac{4q_1q_2}{3\pi\eta d_1^2 d_2 u_\infty}; \qquad \beta_1 = \frac{q_1 E d_2^2}{3\pi\eta d_1^3 u_\infty}; \qquad \beta_2 = \frac{q_2 E}{3\pi\eta d_2 u_\infty};$$

$$J = \frac{1}{9\eta}\sigma d_1 u_\infty; \qquad \lambda = \frac{d_2^2\sigma g}{18\eta u_\infty}; \qquad \gamma = \frac{E^2 d_2^2}{4\pi\eta d_1 u_\infty}.$$

From the condition of equality of the forces acting on a drop: $\mathbf{F}_a = \mathbf{F}_{\tau_1} + \mathbf{F}_{q_1E}$, we find the value of the steady velocity

$$u_\infty = \frac{d_1^2\sigma g}{18\eta} - \frac{q_1 E}{3\pi\eta d_1}. \tag{14}$$

Initial conditions:

at $t = 0$

$$\mathbf{v}_1 = \mathbf{n}; \qquad \mathbf{v}_2 = (\lambda - \beta_2)\mathbf{n}; \qquad \mathbf{r} = z_0\mathbf{n} + \rho_0\mathbf{k}; \qquad z_0 = 50.$$

The system of differential equations (11)–(13) was solved on the "Strela" electronic computer by the Adams method with automatic selection of the integration step. The value $z_0 = 50$ was chosen from preliminary calculations at $z_0 = 100;\ 50;\ 25$, on the basis of the requirement of minimal computing time for a specified accuracy of integration. As a result of the calculation, trajectories of the relative motion of the particles and

Table 1

$E = 0,\ q_2 = 0$

		q_1, e.s.u.	q_1, e.s.u.	q_1, e.s.u.	q_1, e.s.u.	q_1, e.s.u.	q_1, e.s.u.	q_1, e.s.u.
		0	$1\cdot10^{-7}$	$1\cdot10^{-6}$	$5\cdot10^{-6}$	$1\cdot10^{-5}$	$5\cdot10^{-5}$	$1\cdot10^{-4}$
$D = 20\,\mu,\ d = 12\,\mu$	K	0,000	0,000	0,1495	1,837	4,367	23,348	44,775
$D = 20\,\mu,\ d = 12\,\mu$	$\tilde{K}$			0,974	3,824	7,235	29,507	53,165
$D = 20\,\mu,\ d = 16\,\mu$	K	0,000	0,000	0,395	3,739	8,333	41,693	76,696
$D = 20\,\mu,\ d = 16\,\mu$	$\tilde{K}$			2,042	7,472	13,591	52,664	93,530

the values of the collision coefficient $K = \rho_0^2$, or, if the "capture effect" is taken into account, $\tilde{K} = (\rho_0 + a)^2$, where ρ_0 is the distance of the limiting trajectory from the central streamline at infinity. Examples of the calculation results are presented in the form of graphs in Figs. 1 and 2 and in Table 1.

A comparison of the graphs of the change in the collision efficiency of uncharged drops in the absence of an electric field and in its presence (Fig. 1) shows that sufficiently large electric fields cause a significant increase in collision efficiency in that range of particle sizes where, under the action of hydrodynamic forces alone, the collision efficiency either decreases or collisions practically do not occur at all.

The smaller the relative velocities of motion of the drops, the greater the impulse of the electric forces; therefore the electric field promotes an increase in the collision coefficient of uncharged drops the more significantly, the closer their sizes are (Fig. 1).

As can be seen from the examples given, when an electric field of the order of 1000 V/cm is applied, the drops, owing to their polarization, are attracted from fairly large horizontal distances (Fig. 2a), and the collision efficiency increases considerably. The collision efficiency-

collisions also increases significantly under the action of charges of the order of 10^{-5} e.s.u. and larger on drops of the same sizes, even in the absence of an electric field (Table 1, Fig. 2b).

The influence of the electric field on the collision efficiency of charged drops depends on the magnitude and sign of the charge, and also on the mutual position of the drops. Depending on the combination of these parameters, the electric field can accelerate or retard the process of coalescence of charged drops in comparison with uncharged ones.

Fig. 2. Trajectories of the relative motion of particles of diameter $d_1 = 20\mu$, $d_2 = 12\mu$. The heavy line shows the limiting trajectory; on the left are trajectories of drops striking the drop, and on the right are trajectories of drops passing by. The dashed line shows the trajectory of uncharged drops of the same sizes that do not strike the drop in the absence of an electric field. *a* $-E = -1200$ V/cm; $q_1 = q_2 = 0$; b–$E = 0$; $q_1 = 5 \cdot 10^{-5}$ e.s.u.; $q_2 = 0$.

Figure 2: Fig. 2. Trajectories of the relative motion of particles of diameter $d_1 = 20\mu$, $d_2 = 12\mu$. The heavy line shows the limiting trajectory; on the left are trajectories of drops striking the drop, and on the right are trajectories of drops passing by. The dashed line shows the trajectory of uncharged drops of the same sizes that do not strike the drop in the absence of an electric field. *a* $-E = -1200$ V/cm; $q_1 = q_2 = 0$; b–$E = 0$; $q_1 = 5 \cdot 10^{-5}$ e.s.u.; $q_2 = 0$.

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The calculation results indicate that weak electric fields (up to 10 V/cm) and small charges on drops (10^{-9}–10^{-7} e.s.u.) do not exert a substantial influence on the collision efficiency. This is especially true for drops of unequal sizes ([4,10]). However, strong electric fields, of thunderstorm magnitude, as well as large charges on particles of comparable sizes, can have a substantial influence on the collision efficiency of drops and, consequently, under appropriate conditions, on the formation of precipitation.

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Note: Figure translations are in progress. See original paper for figures.