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Abstract

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DIFFRACTION OF A SURFACE WAVE BY A PERFECTLY CONDUCTING WEDGE

(Presented by Academician M. A. Leontovich, 14 X 1963)

Consider a rectangular wedge ($x > 0,\ z > 0$), filled with an anisotropic dielectric ($\varepsilon_x = \varepsilon_\perp,\ \varepsilon_z = \varepsilon_\parallel$) and bounded by a perfectly conducting rectangular wedge ($x > 0,\ z < 0$). Along the dielectric, on the side $z > 0$, a surface wave propagates, whose fields $E_x,\ H_y,\ E_z$ in the plane $x = 0$ vary according to the law $\exp[-i(\omega t + \gamma z)]$, and whose wave vector γ is determined by the dispersion equation

$$\Delta(\tau) \equiv v(\tau)\varepsilon_\parallel + \beta(\tau)\big|_{\tau=\pm\gamma} = 0, \tag{1}$$

where $v(\tau) = (\tau^2 - k^2)^{1/2}$; $\beta(\tau) = \left[\frac{\varepsilon_\parallel}{\varepsilon_\perp}(\tau^2 - k^2\varepsilon_\perp)\right]^{1/2}$; $\varepsilon_\parallel < 0,\ \varepsilon_\perp > 1;\ k^2 < \gamma^2 < k^2\varepsilon_\perp;\ k = \omega/c;\ \operatorname{Im} k > 0$.

We shall find the field scattered by the wedge and, in particular, compute the amplitude of the reflected surface wave.

We shall seek the solution for the fields in the following form ($^{1-3}$):

$$\left.\begin{aligned} H_y &= \exp(-i\gamma z + px) + \int_{-\infty}^{+\infty} h(\tau)\exp(i\tau z + vx)\,d\tau, \\ E_z &= \frac{ip}{k}\exp(-i\gamma z + px) + \int_{-\infty}^{+\infty} \frac{iv}{k}\,h(\tau)\exp(i\tau z + vx)\,d\tau, \end{aligned}\right\} \quad -\infty < z < \infty, \quad x < 0;$$

$$\left.\begin{aligned}
H_y &= \exp(-i\gamma z - gx) + \int_{-\infty}^{+\infty} H(\tau) e^{i\tau z - \beta x}\, d\tau + \exp(i\gamma z - gx),\\
E_z &= -\frac{ig}{k\varepsilon_\parallel}\exp(-i\gamma z - gx) - i\int_{-\infty}^{+\infty} \frac{\beta}{k\varepsilon_\parallel} H(\tau) e^{i\tau z - \beta x}\, d\tau\\
&\quad - \frac{ig}{k\varepsilon_\parallel}\exp(i\gamma z - gx),\\
E_x &= \frac{\gamma}{k\varepsilon_\perp}\exp(-i\gamma z - gx) - \int_{-\infty}^{+\infty} \frac{\tau}{k\varepsilon_\perp} H(\tau) e^{i\tau z - \beta x}\, d\tau\\
&\quad - \frac{\gamma}{k\varepsilon_\perp}\exp(i\gamma z - gx),
\end{aligned}\right\} \qquad (2)$$

$$\begin{matrix} z > 0, \\ x > 0, \end{matrix} \qquad p = v(\pm\gamma); \quad g = \beta(\pm\gamma).$$

The first terms in (2) represent the field of the incident wave, and the remaining terms are the scattered fields.

The unknown functions $h(\tau)$ and $H(\tau)$ in (2) must be determined from the boundary conditions. As these conditions we use the equality of the tangential components of the total fields on the surface of the dielectric and the vanishing of the tangential components of the total electric fields on the pow-

surface of a perfectly conducting wedge:

$$\left.H_y(x=+0) = H_y(x=-0), \quad E_z(x=+0) = E_z(x=-0);\right\} z > 0;$$

$$\begin{aligned} E_z(x=-0) &= 0, \qquad z < 0;\\ E_x(z=+0) &= 0, \qquad x > 0. \end{aligned} \qquad (3)$$

From (3), according to ${}^{(2-4)}$, we obtain the following relations for the required functions $h(\tau)$ and $H(\tau)$:

$$\begin{gathered}
H(\tau) - h(\tau) - \frac{1}{2\pi i(\tau - \gamma)} = \varphi^+(\tau); \qquad z_1(\tau) = \frac{\beta(\tau)}{k\varepsilon_\parallel};\\
z_1(\tau)H(\tau) + z_2(\tau)h(\tau) - \frac{q}{2\pi i(\tau - \gamma)} = \psi^+(\tau); \qquad z_2(\tau) = \frac{\nu(\tau)}{k};\\
z_2(\tau)h(\tau) - \frac{q}{2\pi i(\tau + \gamma)} = \xi^-(\tau); \qquad q = \frac{p}{k};\\
H(\tau) - H(-\tau) = 0,
\end{gathered} \qquad (4)$$

where the superscripts + and − denote boundary values on the contour $\operatorname{Im}\tau = 0$ of functions analytic, respectively, in the upper and lower half-planes of the complex variable τ.

Eliminating $h(\tau)$ and $H(\tau)$ from relations (4), we obtain an equation for the function

$$\xi_1(\tau) = \xi^-(-\tau) - \xi^-(\tau) - \frac{q}{\pi i}\frac{\tau}{\tau^2-\gamma^2} \tag{5}$$

$$\begin{aligned}\frac{\Delta(\tau)}{\nu(\tau)\beta(\tau)}\frac{1}{\pi i}\int_{-\infty}^{+\infty}\frac{\xi_1(\tau')\,d\tau'}{\tau'-\tau} + \frac{1}{2\pi i}\int_{-\infty}^{+\infty}\left[\frac{1}{\nu(\tau')}-\frac{1}{\nu(\tau)}\right]\frac{\xi_1(\tau')\,d\tau'}{\tau'-\tau}&\\ = \frac{q\gamma}{\pi i}\frac{\Delta(\tau)}{(\tau^2-\gamma^2)\nu(\tau)\beta(\tau)} + \frac{1}{\pi i}\frac{1}{\beta(\tau)[\beta(\tau)+g]}&.\end{aligned} \tag{5}$$

Since $\operatorname{Im}\tau > 0$, the coefficient of the singular integral in (5) does not vanish on the contour of integration. The index of this equation (5) is zero; therefore it is equivalent to the following Fredholm equation (6):

$$\begin{aligned}\xi_1(\tau) = \frac{q}{\pi i}\frac{\tau}{\tau^2-\gamma^2} + \frac{1}{\pi i}\int_{-\infty}^{+\infty}\frac{\nu(\tau')\,d\tau'}{\Delta(\tau')[\beta(\tau')+g](\tau'-\tau)}+&\\ + \frac{1}{2(\pi i)^2}\int_{-\infty}^{+\infty}\frac{\nu(\tau')\beta(\tau')\,d\tau'}{\Delta(\tau')(\tau'-\tau)}\int_{-\infty}^{+\infty}\left[\frac{1}{\nu(\tau'')}-\frac{1}{\nu(\tau)}\right]\frac{\xi_1(\tau'')\,d\tau''}{\tau''-\tau'}&.\end{aligned} \tag{6}$$

For $|\varepsilon_\parallel| \gg 1$ the kernel in equation (6) contains a small parameter, and therefore it can be solved by the method of iteration. In the zeroth approximation

$$\xi_1^0(\tau) = \frac{q}{\pi i}\frac{\tau}{\tau^2-\gamma^2},$$

and the reflection coefficient, according to (4), (6), is equal to

$$R^a\left(\frac{\pi}{2}\right) = \frac{1}{2}q^2\left[1+\frac{2q}{\pi i}+O(q^2)\right]. \tag{7}$$

In essence, the iteration method for $|\varepsilon_\parallel| \simeq q^{-2} \gg 1$ gives the function $\xi_1(\tau)$ in the form of an expansion in the parameter

$$q = \left(\frac{\varepsilon_\perp - 1}{|\varepsilon_\parallel|\,\varepsilon_\perp}\right)^{1/2}$$

only for small τ ($\tau < k/q$). The asymptotic form of the solution of (6) for large τ can be obtained by using the results of work (7). Assuming that as $\tau \to \infty$ the asymptotic form of $\xi_1^0(\tau)$ is

$$\frac{\text{const}}{\tau^{2\alpha}} \qquad \left(\operatorname{Re}\alpha < \frac{1}{2}\right),$$

and expanding (5) in a series

in decreasing powers of τ, we obtain:

$$(2\sqrt{\varepsilon_\parallel \varepsilon_\perp}+1)\operatorname{ctg}^2 \pi\alpha = 1; \qquad \alpha\,(|\varepsilon_\parallel| \gg 1) = \frac{1}{2} - \frac{1+i}{\sqrt{2}}\,(\varepsilon_\perp |\varepsilon_\parallel|)^{-1/4}. \quad (8)$$

For a wedge with angle $\theta = 0$ (an ideally conducting half-plane $z < 0$ in the anisotropic half-space $x > 0$), the reflection coefficient is equal to

$$R^a(0) = \left(\frac{k\sqrt{\varepsilon_\perp}-\gamma}{k\sqrt{\varepsilon_\perp}+\gamma}\right)^{1/2} R^a\left(\frac{\pi}{2}\right), \qquad q \ll 1. \quad (9)$$

In exactly the same way one can consider the case of an isotropic dielectric ($\varepsilon_\perp = \varepsilon_\parallel = \varepsilon < 0$). As a result, it turns out that for $|\varepsilon| \gg 1$ the reflection coefficients for $\theta = \pi/2$ and $\theta = 0$ differ only in the first approximation in $q \ll 1$

$$\left(q \simeq \frac{1}{\sqrt{|\varepsilon|}} \simeq \frac{\delta}{\lambda}\right):$$

$$R^u\left(\frac{\pi}{2}\right) = \frac{1}{2}q^2\left[1+\frac{2q}{\pi i}+O(q^2)\right]; \quad (10)$$

$$R^u(\theta) = R^u\left(\frac{\pi}{2}\right)\left[1+iq+O(q^2)\right]. \quad (11)$$

It is of interest to compare (7), (9)–(11) with analogous formulas obtained using the boundary conditions of M. A. Leontovich on the surface of the dielectric [2]. The application of these boundary conditions reduces the problem of finding the field in vacuum to an integral equation, which can be solved by the Wiener–Hopf–Fock method. If the reflection coefficient found in this way is expanded in powers of the parameter $q = \delta/\lambda \ll 1$ (the approximate boundary conditions are obtained under the assumption that $q \ll 1$ [2]), then the result turns out to agree, up to quantities of order q^2, with formulas (7) and (10) (for $\theta = \pi/2$). As for the case $\theta = 0$, for an isotropic dielectric the difference is observed in quantities of order q, and in an anisotropic dielectric–even in the zeroth approximation, if $\varepsilon_\perp \ll |\varepsilon_\parallel|$. In the latter case, the spatial dispersion of the surface impedance proves to be substantial.

Thus, for not very acute angles of a conducting wedge ($\theta \sim 1$), the detailed structure of the field near the edge of the wedge has no substantial influence on the field in the far zone [8], so that use of the boundary conditions of M. A. Leontovich makes it possible to obtain the correct expression for the field in this region for both isotropic and anisotropic dielectrics.

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Note: Figure translations are in progress. See original paper for figures.

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