Reports of the Academy of Sciences of the USSR

Corresponding Member of the Academy of Sciences of the USSR A. V. POGORELOV

1964

SovietRxiv

Fig. 1.

Figure 1: Fig. 1.

Abstract

Full Text

Reports of the Academy of Sciences of the USSR

1964. Volume 159, No. 6

THEORY OF ELASTICITY

Corresponding Member of the Academy of Sciences of the USSR A. V. POGORELOV

LOSS OF STABILITY OF SHELLS OF REVOLUTION UNDER INTERNAL PRESSURE

Experience shows that a convex shell of revolution may lose stability and begin to bulge under internal pressure (Fig. 1). In the present note we set forth the results of an investigation of this phenomenon, based essentially on geometrical considerations.

Fig. 1.

In view of the axial symmetry of the surface of the shell, we assume that bulging as a result of loss of stability occurs in a system of small elliptical regions, regularly arranged along some parallel. Similarly to note (1), we assume that the deformation energy of the shell is concentrated on the boundary of the bulging regions and, per unit length of the boundary, is determined by the formula

$$\bar{U} = \frac{2E\delta^2\alpha^2 h}{\sqrt{12(1-\nu^2)}\,\rho}.$$

Here α is the angle between the osculating plane of the boundary γ of the bulging region and the tangent planes of the surface; ρ is the radius of curvature of the curve γ; h is the change in the normal deflection of the shell on crossing the boundary γ into the bulging region; δ is the thickness of the shell; E and ν are elastic constants. Integrating $\bar{U}$ along the arc γ, we find for the deformation energy along the boundary of one bulging region the expression

$$U_1 = \frac{2E\delta^2\pi}{\sqrt{12(1-\nu^2)}\sqrt{R_1R_2}}\,\sigma(\lambda^4+\mu^4+4\lambda^2\mu^2),$$

where R_1 and R_2 are the principal radii of curvature of the shell at the center of the bulge; the parameters λ and μ determine the dimensions of the bulging region, and σ characterizes the magnitude of the bulge. In order to find the total deformation energy over all n regions, the result obtained must be multiplied by n.

The work A performed by the internal pressure during bulging is equal to the product of the pressure magnitude and the change in the volume ΔV bounded by the shell. We conventionally divide the quantity ΔV into two parts, ΔV_i and ΔV_e. Here ΔV_i is the change in volume caused directly by the bulging. It is negative (the shell bulges inward), and for it one obtains the expression

$$\Delta V_i = -\pi\sigma\sqrt{R_1R_2}(\lambda^4+\mu^4+4\lambda^2\mu^2)n.$$

The quantity ΔV_e is connected with the displacement of the planes of the parallels bounding the zone of the bulging regions, and is determined by the formula

$$\Delta V_e = \pi\rho^2\varepsilon,$$

where ρ is the radius of the parallel along which the centers of the bulging regions are located, and ε is the displacement of the indicated planes under deformation of the shell. The determination of the quantity ε is based on the following consideration.

We approximate the shape of the deformed surface everywhere, except in the neighborhood of the boundary of the bulging regions, by an isometric transformation of the initial shape. In this case, since outside the indicated neighborhood

change in the shape of the surface is small, then it seems possible to replace finite bending by infinitesimal bending. The transition to infinitesimal bendings simplifies the study of the shape of the deformed surface. In particular, it is possible to determine the displacement of points of the surface in the axial direction outside the zone occupied by the bulging regions. Accordingly, the magnitude ε is also determined. For it one obtains the expression

$$\varepsilon = \frac{3\pi}{\Delta y}\sqrt{\frac{R_2}{R_1}\lambda\mu(\lambda^2+\mu^2)\sigma\cos\alpha},$$

where Δy is the distance between neighboring centers of bulging; α is the angle between the tangent to the meridian and the axis of the surface; R_1 and R_2 are the principal radii of curvature (R_1 along the meridian). Knowing ε, we find ΔV_e, and with it the work produced by the internal pressure p,

$$A = (\Delta V_i + \Delta V_e)p.$$

From the equilibrium condition

$$d(U - A) = 0,$$

where we differentiate with respect to the parameter σ, which characterizes the magnitude of the bulging, we find the pressure borne by the shell during bulging. It naturally depends on the adopted shape of the bulging region (the parameters λ, μ). Minimizing this value with respect to λ and μ, we obtain the following formula for the magnitude of the critical pressure:

$$p = \frac{2E\delta^2}{\sqrt{3}(1-\nu^2)R_1R_2}\frac{1}{\rho^2/2R_1R_2 - 1}.$$

Recall that here ρ is the radius of the parallel along which the centers of bulging are located; R_1 and R_2 are the principal radii of curvature at the centers of bulging; δ is the shell thickness, and E and ν are elastic constants. The values of λ, μ for which p attains its minimum value correspond to the case of bulging regions strongly elongated along the meridian.

For shells in the shape of an oblate ellipsoid with semiaxes a, b ($a > b$), application of the obtained formula gives the following value of the critical pressure:

$$p = \frac{2E\delta^2}{\sqrt{3}(1-\nu^2)}\frac{1}{a^2/2 - b^2}.$$

The method set forth can be applied to determine the critical external pressure on a strictly convex shell of revolution. In this case, for the critical pressure one obtains the formula

$$p = \frac{2E\delta^2}{\sqrt{3}(1-\nu^2)R_1R_2}\frac{1}{\rho^2/2R_1R_2 + 1}. \qquad (*)$$

Loss of stability is accompanied by the formation of dents compressed along the meridian. For a closed spherical shell of radius R, formula $(*)$ gives

$$p = \frac{2E\delta^2}{\sqrt{3}(1-\nu^2)R^2}\frac{2}{3}.$$

Note that formula (∗), when applied to shallow shells ($\rho^2/2R_1R_2$ is small), gives essentially the same value as in work ([1]).

Physical-Technical Institute
of Low Temperatures
Academy of Sciences of the Ukrainian SSR

Received
26 VI 1964

CITED LITERATURE

[1] A. V. Pogorelov, DAN, 159, No. 5 (1964).

Note: Figure translations are in progress. See original paper for figures.