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Abstract

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V. V. ARSENIN

ON THE BEAM INSTABILITY OF A RAREFIED INHOMOGENEOUS PLASMA

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1. It is known (see, for example, ([1,2])) that in a homogeneous magnetized plasma, in the presence of a beam, perturbations with a nonzero component of the wave vector perpendicular to the magnetic field may be unstable even if the beam velocity is less than the thermal velocity of the particles. In the present paper we shall consider an instability associated with the inhomogeneity of the plasma at the edges of a beam of finite width. It turns out that the beam edges are unstable (including the case when its velocity is less than the thermal velocity) with respect to short-wave perturbations which are not amplified in a homogeneous plasma. We shall not impose any smallness restriction on the density gradient in the transition layer, so that the characteristic size of the inhomogeneity may be comparable with the mean Larmor radius of the ions. We note that the unstable oscillations considered by us are localized at the beam edge, in contrast to oscillations of the beam as a whole, considered by a number of authors (see ([3]) and the works cited there).

We shall start from a system of equations consisting of the linearized kinetic equations with a self-consistent field for electrons and ions and Maxwell' s equations for the field. We neglect collisions. Let the z-axis of a Cartesian coordinate system be directed along the external magnetic field $\mathbf{H}_0$, and let the unperturbed particle density depend on the coordinate x. Assuming the ratio $\beta = 8\pi NT/H_0^2$ (N is the plasma density, T its temperature in energy units) to be small and, accordingly, neglecting the inhomogeneity of the external field, we choose as the unperturbed distribution function of particles of the s-th species the following function of the integrals of motion:

$$f_{0s} = N_0 \left(\frac{\alpha_s^2}{\pi}\right)^{3/2} \exp\{-\alpha_s^2[v_x^2 + v_y^2 + (v_z - V_s)^2]\}\Phi_s\left[\alpha_s|\omega_s|\left(x + \frac{v_y}{\omega_s}\right)\right], \quad (1)$$

where N_0 is a quantity of the dimension of density; $\alpha_s^2 = m_s/2T$; m_s is the mass of the particle; $\omega_s = e_{sH}0/m_{sc}$ is its gyrofrequency; Φ_s is a bounded positive function characterizing the inhomogeneity. In particular, we regard the stationary ions and the beam ions as particles of different species. The Fourier components φ_s of the functions Φ_s are related by the neutrality condition

$$\sum_s e_s R_s \varphi_s(kR_s)\exp(-k^2R_s^2/4) = 0,$$

where $R_s = 1/\alpha_s|\omega_s|$ is the mean Larmor radius. In what follows, for simplicity, we shall restrict ourselves to the case of an ion beam ($V_e = 0$).

It is not difficult to show that for $|\omega| \ll \omega_i$ short-wave ($kR_i \gg 1$) oscillations are potential (the condition of potentiality depends on the magnitude of the component of the wave vector k_z along the magnetic field, but it is certainly satisfied for all k_z if $\beta \ll m_e/m_i$ (^{4})). In this case, for a plasma of arbitrary degree of inhomogeneity (see above), the Fourier representation of the po-

potential

$$\psi(\mathbf{k},\omega) = \frac{1}{(2\pi)^2}\int \psi(\mathbf{r},t)\exp[-i\mathbf{kr} - i\Omega t]\,d\mathbf{r}\,dt$$

satisfies the equation

$$\begin{aligned}(k_x^2+k_y^2+k_z^2)\psi(k_x,k_y,k_z,\omega) = \sum_s \frac{4\pi e^2 N_0}{T}\int_{-\infty}^{\infty}\frac{R_s\varphi_s[(k_x-l)R_i]}{\sqrt{2\pi}}\Bigg\{\frac{2}{\sqrt{\pi}}\,[\omega + k_{zV}s+\\ +i(k_x-l)k_{yR}s^2\frac{\omega_s}{2}\Big]\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\alpha_s\exp(-\alpha_s^2 v_z^2)\,dv_z}{\omega + k_{zV}s + k_{zv}z - n\omega_s}\times\\ \times\exp\left[in\left(\operatorname{arctg}\frac{l}{k_y} - \operatorname{arctg}\frac{k_x}{k_y}\right)\right]\int_0^{\infty} te^{-t^2}J_n\Big(\sqrt{k_x^2+k_y^2}\,R_{st}\Big)\,J_n\Big(\sqrt{l^2+k_y^2}\,R_{st}\Big)\,dt\\ -\exp[-(k_x-l)^2R_s^2/4]\Big\}\,\psi(l,k_y,k_z,\omega)\,dl.\end{aligned} \tag{2}$$

2. Let us proceed to the investigation of the stability of beams of finite width. Let a beam of finite width be formed by a part of the ions, while the density of the immobile electron gas is uniform. In this case the width of the transition layer at the beam boundary may be not only larger than, but also of the order of R_i. For definiteness we shall assume the density gradient of the beam particles in the transition layer to be positive. Consider oscillations of such a plasma with frequencies $|\omega| \ll \omega_i$ in the wavelength range $k^2R_D^2 \gg 1$, where $R_D = (T/4\pi e^2 N_0)^{1/2}$ is the Debye radius. The principal terms in the kernel of the integral equation (2) will be the terms associated with the inhomogeneity in the zeroth ion harmonics, since only such terms (at sufficiently low frequencies) can be of order k^2. Retaining on the right-hand side of (2) only these terms, we obtain for the quantity $\chi = (k_x^2+k_y^2+k_z^2)^{1/2}\psi$ an integral equation with a symmetric kernel:

$$\chi(k_x) = I(\omega, k_z) \int_{-\infty}^{\infty} G(k_x, l)\chi(l)\, dl, \tag{3}$$

where

$$I(\omega, k_z) = \frac{\omega_i}{\sqrt{\pi}} \int_{-\infty}^{\infty} \alpha_i \left(\frac{1}{\omega + k_{zV} + k_{zv}z} - \frac{1}{\omega + k_{zv}z} \right) \exp(-\alpha_i^2 v_z^2)\, dv_z;$$

$$G = i(k_x - l)k_y \frac{R_i^3 \int_0^{\infty} e^{-t^2} J_0\left(\sqrt{k_x^2 + k_y^2}\, R_{it}\right) J_0\left(\sqrt{l^2 + k_y^2}\, R_{it}\right)\, dt}{R_D^2 (k_x^2 + k_y^2 + k_z^2)^{1/2} (l^2 + k_y^2 + k_z^2)^{1/2}} \varphi_i[(k_x - l)R_i],$$

φ_i is the Fourier image of the function Φ_i pertaining to the beam.

To avoid misunderstandings, we note that in (3), in contrast to (2), one can no longer carry out the limiting transition to the case of a homogeneous plasma, since in passing from (2) to (3) we neglected terms proportional to the density. For short-wavelength ($k^2 R_D^2 \gg 1$) undamped oscillations, such neglect is justified up to gradients $\frac{1}{N}\frac{dN}{dx}$ of order $k_{zk}^2 R_D^2$.

To find the oscillation frequencies it is necessary to solve the equations $I(\omega, k_z) = \lambda_n$, where λ_n are the eigenvalues of the equation

$$\chi(k_x) = \lambda \int_{-\infty}^{\infty} G(k_x, l)\chi(l)\, dl. \tag{4}$$

If $\lambda_n \gg 1$ (such values certainly exist, since equation (4) has infinitely many eigenvalues), then $|\omega| \ll \omega_i$, in accordance with the original assumption. Writing out, for the region of instability of interest to us ($\operatorname{Im}\omega < 0$), the real and imaginary parts of the equation $I(\omega, k_z) = \lambda_n$, we find that $\operatorname{Re}\omega = -k_{zV}/2$, while $\operatorname{Im}\omega$ satisfies the equation

$$\frac{2}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{(x + \alpha_i V/2) e^{-x^2}\, dx}{(x + \alpha_i V/2)^2 + (\alpha_i \operatorname{Im}\omega / k_z)^2} = \lambda_n k_z R_i. \tag{5}$$

For this equation to have a solution, it is sufficient that the condition

$$\frac{2}{\sqrt{\pi}} \int_0^{\infty} \left\{ \exp\left[-\left(x - \frac{\alpha_i V}{2} \right)^2 \right] - \exp\left[-\left(x + \frac{\alpha_i V}{2} \right)^2 \right] \right\} \frac{dx}{x} > \lambda_n k_z R_i \tag{6}$$

be satisfied.

For $\alpha_i V \sim 1$ the integral on the left-hand side of (6) is of order unity. Therefore, perturbations localized at the edge of the beam, described by equation (3), are unstable for sufficiently small k_z ($k_z \ll 1/\lambda_n R_i$).

Thus, in a rarefied plasma the edges of an ion beam (including a "slow" one, $\alpha V \sim 1$) are unstable with respect to "almost flute-like" ($k^2 R_D^2 \gg 1$, $k_{zR}i \ll 1$) perturbations. Since the eigenvalues λ_n decrease as the density gradient of the beam particles increases, for given V and k_y the interval of unstable k_z is the wider, the sharper the beam boundary.

3. Let us now consider the case where there is no beam as such ($V = 0$), but, against a background of homogeneous ion density, there is a clump of electrons with a temperature different from that of the remaining plasma electrons. Let the electron density of the clump be $N(x)$, and let their temperature be lower (but not much lower) than the temperature of the remaining electrons. Consider oscillations of such a plasma for $|\omega| \ll \omega_i$ in the wavelength range $k^2 R_D^2 \gg 1$, $R_e^{-1} \gg k \gg R_i^{-1}$. Retaining on the right-hand side of (2) only the terms associated with inhomogeneity in the zeroth electron harmonics and passing to the coordinate representation, we obtain the following equation for determining the frequency ω:

$$\frac{d^2\psi}{dx^2} + \left[-k_y^2 - k_z^2 - I(\omega, k_z) k_y^2 \varkappa(x)\right]\psi = 0, \tag{7}$$

where

$$I(\omega, k_z) = \frac{\omega_i}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\left[\alpha_{1e}\exp(-\alpha_{1e}^2 v_z^2) - \alpha_{2e}\exp(-\alpha_{2e}^2 v_z^2)\right] dv_z}{\omega + k_{zv}z},$$

$$\varkappa = \frac{R_i^2}{2R_D^2 k_{yN}0} \frac{dN}{dx};$$

the indices 1 and 2 refer respectively to the electrons of the clump and of the rest of the plasma. In equation (7) we have passed to ion quantities ω_i, R_i, since it is convenient to measure the coordinate x in ion Larmor radii.

Equation (7) is a Schrödinger equation, in which the role of the energy is played by the quantity $-k_y^2 - k_z^2$, while the potential is $U = I(\omega, k_z)k_y^2\varkappa$; it depends on the parameter ω. It is required to find those values of ω for which the quantity $-k_y^2 - k_z^2$ is an energy level of a particle in the potential well $U(x)$. Up to terms of order $1/k_{yR}i$, these values of ω are equal to the roots of the equation

$$I(\omega, k_z) = -\frac{1}{\varkappa_{\max}}. \tag{8}$$

It is not difficult to see that for $k_z < \omega_i a_e x_{\max}$ this equation has unstable solutions with frequency $|\omega| \sim k_z/a_e$. This instability is related to the drift instability of an inhomogeneous plasma. In this case the role of the ions is played, in our case, by the electrons of the bunch.

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Moscow Engineering Physics Institute
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Note: Figure translations are in progress. See original paper for figures.