Academician I. I. ARTOBOLEVSKII

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Abstract

Full Text

Academician I. I. ARTOBOLEVSKII

MECHANISMS FOR REPRODUCING PARABOLIC AND HYPERBOLIC CURVES

MECHANICS

1°. In the present work we consider the theory of three new six-link mechanisms for generating certain classes of parabolic and hyperbolic curves. Consider the mechanism (Fig. 1), whose link 3 has the form of a right-angled lever ADm; the point A enters into a revolute pair A with slider 2, and the side Dm enters into a prismatic pair with slider 4. Slider 2 slides along the fixed guide Oy, and slider 4 enters into a revolute pair B with the fixed link 1. A crosspiece Ak is rigidly connected with slider 2. The dimensions of the links of the mechanism satisfy the condition $AD = OB = a$. By attaching to links 2 and 3 a two-slider group consisting of sliders 5 and 6, which enter into the revolute pair C, one can reproduce third-order curves of hyperbolic type and hyperbolas of the second order.

Let us consider the curve p—p reproduced by point C. From the equality of triangles BDH and AOH it follows that

$$HD = OH = \frac{a^2 - y^2}{2a}, \tag{1}$$

$$HB = AH = \frac{a^2 + y^2}{2a}, \tag{2}$$

$$OA = DB = y. \tag{3}$$

From the similarity of triangles DCA and DBH we have

$$\frac{HD}{HB} = \frac{AD}{AC}. \tag{4}$$

Taking conditions (1), (2), and (3) into account, we obtain

$$\frac{a^2 - y^2}{a^2 + y^2} = \frac{a}{x}. \tag{5}$$

After obvious transformations we obtain the equation of the curve p—p (Fig. 1), described by point C. We have

$$ay^2 - a^2x + xy^2 + a^3 = 0 \tag{6}$$

or

$$x = a\frac{a^2 + y^2}{a^2 - y^2}. \tag{7}$$

The curve p—p is a third-order curve of hyperbolic type. In Fig. 1 one branch of this curve is shown, reproduced by the mechanism on the interval $A'A''$ of displacement of point A. This branch of the curve has as asymptotes the straight lines $A'T'$ and $A''T''$, parallel to the axis Ox. To reproduce the two other branches of the curve p—p, it is necessary to consider the motion of point A outside the interval $A'A''$.

If the two-slider group is attached in such a way that slider 6 slides along the guide En, whose axis is at a distance b from point D, then sliders 5 and 6 will occupy the positions $5'$ and $6'$, shown in Fig. 1 by dashed lines. The equation of the curve q—q, reproduced by point C', is obtained from consideration of the similar triangles BEG and BDH.

We have

$$\frac{BG}{BE} = \frac{HB}{DB}. \tag{8}$$

Taking conditions (1), (2), and (3) into account, we shall have

$$BG = \frac{a^2 + y^2}{2a} + x_1 - a = \frac{2ax_1 - y^2 - a^2}{2a}, \qquad BE = b - y. \tag{9}$$

Then equation (8) will take the form

$$\frac{2ax_1y - y^2 - a^2}{2a(b - y)} = \frac{a^2 + y^2}{2ay}. \tag{10}$$

After obvious transformations we obtain the equation of the curve q—q (Fig. 1), reproduced by the point C':

$$by^2 - 2ax_1y + a^2b = 0. \tag{11}$$

The curve q—q will be a hyperbola whose asymptotes are the axis Ox and the straight line OQ, the angle of inclination α of which to the axis Ox is determined from the condition

Fig. 1

Figure 1: Fig. 1

$$\operatorname{tg}\alpha = 2\frac{b}{a}. \tag{12}$$

2°. Figure 2 shows a six-link mechanism based on the quadrilateral ADB, consisting of links $1, 2, 3$, and 4, of the same kind as

Fig. 1

that shown in Fig. 1. The two-slider group, consisting of links 5 and 6, is attached not to links 2 and 3, but to links 2 and 4. Slider 5 slides along the guide Ak, and slider 6 along the guide Fs, whose axis is at a distance d from the point D. With this mechanism one can reproduce curves of the 3rd order of parabolic type and parabolas of the 2nd order.

Let us consider the curve p—p, reproduced by the point C. From the similarity of the triangles BDH and BFG we have

$$\frac{BG}{BF} = \frac{HB}{DB}. \tag{13}$$

Taking into account conditions (1), (2), (3), we shall have

$$\frac{2ax - (a^2 + y^2)}{d} = \frac{a^2 + y^2}{y}. \tag{14}$$

The equation of the curve p—p (Fig. 2), described by the point C, is obtained in the form

$$y^3 + dy^2 + a^2 y - 2axy + a^2 d = 0. \tag{15}$$

It is not difficult to see that the curve p—p is a curve of the 3rd order of parabolic type. Figure 2 shows one branch of this curve, described by the point C over the interval $A'A''$ of displacement of the point A.

If $d = 0$, then sliders 5 and 6 will occupy positions $5'$ and $6'$, and point C the position C'. The equation of the curve $q - q$, reproduced by point C', is obtained from equation (15) if we take in it $d = 0$ and $x = x_1$. Then we obtain

$$y^3 + a^2 y - 2ax_1 y = 0 \tag{16}$$

or

Fig. 2

Figure 2: Fig. 2

$$y^2 = a(2x_1 - a), \tag{17}$$

since $y_1 = y$.

The curve $q - q$ will be a parabola of the second order with focus at point B and parameter $2p = 2a$.

3°. A special case of the mechanisms considered will be the mechanism (Fig. 3), in which the angle at point D (Fig. 1) is equal to 180°. The equation of the curve $p - p$

Fig. 2

(Fig. 3), reproduced by point C, is obtained from consideration of the similar triangles AFC and BOA. We have

$$\frac{AC}{AF} = \frac{AB}{OB} \tag{18}$$

or, since $AB = \sqrt{a^2 + y^2}$, $AF = AB + d$, and $AC = x$, then

$$\frac{x}{\sqrt{a^2 + y^2} + d} = \frac{\sqrt{a^2 + y^2}}{a}. \tag{19}$$

After transformations we obtain the equation of the curve $p - p$, reproduced by point C:

$$a^2x^2 - 2ax(a^2 + y^2) + (a^2 + y^2)^2 = d^2(a^2 + y^2). \tag{20}$$

This will be a fourth-order curve of parabolic type.

If $d = 0$, then sliders 5 and 6 will occupy positions $5'$ and $6'$, and point C the position C'. The equation of the curve $t - t$, reproduced by point C', is obtained from equation (20) if we take in it $d = 0$ and $x = x_1$:

$$a^2x_1^2 - 2ax_1(a^2 + y^2) + (a^2 + y^2)^2 = 0 \tag{21}$$

or

$$y^2 = a(x_1 - a), \tag{22}$$

Fig. 3

Figure 3: Fig. 3

i.e., the curve $t-t$ will be a parabola of the second order with parameter $2p = a$.

If the two-link group is attached so that slider 6 slides along the guide En, the axis of which is at a distance b from point A,

Fig. 3

then sliders 5 and 6 will occupy positions $5''$ and $6''$ (Fig. 3). The equation of the curve $q-q$ is obtained from the similarity of triangles AEC'' and BOA. We have

$$\frac{EC''}{AE} = \frac{OA}{OB}. \tag{23}$$

Taking into account that

$$EC'' = \sqrt{x_2^2 - b^2}, \qquad AE = b, \qquad OA = y_2 = y, \qquad OB = a,$$

we obtain

$$\frac{\sqrt{x_2^2 - b^2}}{b} = \frac{y}{a}, \tag{24}$$

whence

$$\frac{x_2^2}{b^2} - \frac{y^2}{a^2} = 1, \tag{25}$$

i.e., the curve $q - q$ will be a hyperbola of the second order.

Structurally, all three mechanisms considered can be made as a single mechanism with adjustable link parameters. For this it is sufficient to make it so that the guides En and Fs can be fixed on links 3 and 4 at any specified distances b and d. In addition, link 3 must have an adjusting device permitting two values of the angle at point D, equal to 90° and 180°. Such a mechanism can reproduce a broad spectrum of parabolic and hyperbolic curves of various orders and types.

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Note: Figure translations are in progress. See original paper for figures.