Reports of the Academy of Sciences of the USSR

1964

SovietRxiv

Abstract

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Reports of the Academy of Sciences of the USSR
1964. Vol. 156, No. 1

THEORY OF ELASTICITY

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ON ONE PARTICULAR SOLUTION OF THE EQUATIONS OF THE THEORY OF IDEAL PLASTICITY

(Presented by Academician L. I. Sedov on 6 I 1964)

A particular solution of the general (three-dimensional) equations of the theory of ideal plasticity is found. Several particular problems correspond to this solution—the pure bending of a rectangular plate, the spatial flow of a plastic material between rough plates, the triaxial compression of a rectangular prism, etc.

The general relations of the theory of ideally plastic flow under the Huber–Mises condition in Cartesian coordinates have the form

$$\frac{\partial \sigma_x}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}+\frac{\partial \tau_{xz}}{\partial z}=0, \qquad \frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \sigma_y}{\partial y}+\frac{\partial \tau_{yz}}{\partial z}=0, \qquad \frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}+\frac{\partial \sigma_z}{\partial z}=0; \tag{1}$$

$$(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+6(\tau_{xy}^2+\tau_{yz}^2+\tau_{xz}^2)=6k^2; \tag{2}$$

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x}=\lambda(2\sigma_x-\sigma_y-\sigma_z),\\ \varepsilon_y &= \frac{\partial v}{\partial y}=\lambda(2\sigma_y-\sigma_z-\sigma_x),\\ \varepsilon_z &= \frac{\partial w}{\partial z}=\lambda(2\sigma_z-\sigma_x-\sigma_y); \end{aligned} \tag{3}$$

$$2\gamma_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}=6\lambda\tau_{xy}, \qquad 2\gamma_{yz}=\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}=6\lambda\tau_{yz}, \qquad 2\gamma_{xz}=\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}=6\lambda\tau_{xz}. \tag{4}$$

From relations (3)-(4) we have

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - \int \frac{\partial w}{\partial x}\, dz + 2\int \gamma_{xz}\, dz, \\ v(x,y,z) &= v_0(x,y) - \int \frac{\partial w}{\partial y}\, dz + 2\int \gamma_{yz}\, dz, \\ w(x,y,z) &= w_0(x,y) - \int (\varepsilon_x + \varepsilon_y)\, dz, \end{aligned} \tag{5}$$

where u_0, v_0, and w_0 are arbitrary functions of x and y.

Assuming that the strain-rate tensor does not depend on x and y, we find

$$\begin{aligned} \varepsilon_x &= \frac{\partial u_0}{\partial x} - \frac{\partial^2 w_0}{\partial x^2} z = A_0 + A_1 z, \\ \varepsilon_y &= \frac{\partial v_0}{\partial y} - \frac{\partial^2 w_0}{\partial y^2} z = B_0 + B_1 z, \\ \gamma_{xy} &= \frac{1}{2}\left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}\right) - \frac{\partial^2 w_0}{\partial x\, \partial y} z = C_0 + C_1 z. \end{aligned} \tag{6}$$

Here $A_0, B_0, C_0, A_1, B_1, C_1$ are arbitrary constants.

Further, from (6) it follows that

$$\begin{aligned} &u_0 = A_0 x + Dy + E, \qquad v_0 = (2C_0 - D)x + B_0 y + F, \\ &w_0 = -\frac{A_1}{2}x^2 - \frac{B_1}{2}y^2 - C_1 xy - Gx - Hy - Q, \end{aligned} \tag{7}$$

where D, E, F, G, H, Q are also arbitrary constants.

From relations (3)–(4) we obtain

$$\sigma_x = \sigma_z + \frac{2\varepsilon_x + \varepsilon_y}{\gamma_{xz}}\tau_{xz}, \qquad \sigma_y = \sigma_z + \frac{\varepsilon_x + 2\varepsilon_y}{\gamma_{xz}}\tau_{xz}, \qquad \tau_{xy} = \frac{\gamma_{xy}}{\gamma_{xz}}\tau_{xz}. \tag{8}$$

Assuming that τ_{xz} does not depend on x and y, and substituting (8) into the equilibrium equations (1), we shall have

$$\frac{\partial \sigma_z}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = 0, \qquad \frac{\partial \sigma_z}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = 0, \qquad \frac{\partial \sigma_z}{\partial z} = 0, \tag{9}$$

and hence

$$\sigma_z = -a_1 x - b_1 y - c_0, \qquad \tau_{xz} = a_1 z + a_0, \qquad \tau_{yz} = b_1 z + b_0. \tag{10}$$

Here a_0, b_0, c_0, a_1, b_1 denote new arbitrary constants.

Determining from relations (2) and (8) the value of γ_{xz} (and consequently also γ_{yz}) and substituting into (8) and (5), we finally obtain

$$\sigma_x = \sigma_z + (2\varepsilon_x + \varepsilon_y)\sqrt{\frac{k^2 - \tau_{xz}^2 - \tau_{yz}^2}{\varepsilon_x^2 + \varepsilon_x\varepsilon_z + \varepsilon_y^2 + \gamma_{xy}^2}}; \tag{11}$$

$$\sigma_y = \sigma_z + (\varepsilon_x + 2\varepsilon_y)\sqrt{\frac{k^2 - \tau_{xz}^2 - \tau_{yz}^2}{\varepsilon_x^2 + \varepsilon_x\varepsilon_y + \varepsilon_y^2 + \gamma_{xy}^2}}; \tag{12}$$

$$\tau_{xy} = \gamma_{xy}\sqrt{\frac{k^2 - \tau_{xz}^2 - \tau_{yz}^2}{\varepsilon_x^2 + \varepsilon_x\varepsilon_y + \varepsilon_y^2 + \gamma_{xy}^2}}; \tag{13}$$

$$u = 2\int\sqrt{\frac{\varepsilon_x^2 + \varepsilon_x\varepsilon_y + \varepsilon_y^2 + \gamma_{xy}^2}{k^2 - \tau_{xz}^2 - \tau_{yz}^2}}\,\tau_{xz}\,dz + A_1xz + C_1yz + A_0x + Dy + Gz + E; \tag{14}$$

$$v = 2\int\sqrt{\frac{\varepsilon_x^2 + \varepsilon_x\varepsilon_y + \varepsilon_y^2 + \gamma_{xy}^2}{k^2 - \tau_{xz}^2 - \tau_{yz}^2}}\,\tau_{yz}\,dz + C_1xz + B_1yz + (2C_0 - D)x + \\ + B_0y + Hz + F; \tag{15}$$

$$w = \frac{A_1}{2}x^2 - \frac{B_1}{2}y^2 - \frac{A_1 + B_1}{2}z^2 - C_1xy - Gx - Hy - (A_0 + B_0)z - Q. \tag{16}$$

The solution obtained, (10)–(16), contains 17 arbitrary constants. When $A_1 \neq 0$ and $B_1 \neq 0$, while all the remaining constants are equal to zero, we have the case of pure bending of a rectangular plate. When only A_0, E, c_0, a_1 are nonzero, we obtain the case of plane deformation of a layer compressed by rough plates (Prandtl' s problem (^{1})). Taking $A_1 = B_1 = C_1 = G = H = Q = a_0 = b_0 = 0$, we shall have the case of spatial flow of material between rough plates (^{11}), etc.

We note that some other particular solutions of the spatial problem of the theory of plasticity have been obtained in works ($^{2-10}$).

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Received
4 I 1964

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