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Abstract

Full Text

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A NEW SOLUTION OF THE PROBLEM OF THE MOTION IN A NEWTONIAN FORCE FIELD OF A BODY HAVING CAVITIES FILLED WITH FLUID

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Assuming that the body with cavities filled with fluid has a fixed point, we write the equations of its motion in the notation of work [1]*

$$A_1 \frac{d\omega_1}{dt} = (A_2 - A_3)(\omega_2\omega_3 - \varepsilon\gamma_2\gamma_3) + \lambda_2\omega_3 - \lambda_3\omega_2 + e_2\gamma_3 - e_3\gamma_2, \qquad (1)$$

$$\frac{d\gamma_1}{dt} = \omega_3\gamma_2 - \omega_2\gamma_3 \qquad (1\ 2\ 3).$$

The known integrals are

$$\gamma_1^2 + \gamma_2^2 + \gamma_3^2 = \Gamma^2, \qquad (2)$$

$$(A_1\omega_1 + \lambda_1)\gamma_1 + (A_2\omega_2 + \lambda_2)\gamma_2 + (A_3\omega_3 + \lambda_3)\gamma_3 = m, \qquad (3)$$

$$A_1\omega_1^2 + A_2\omega_2^2 + A_3\omega_3^2 + \varepsilon(A_1\gamma_1^2 + A_2\gamma_2^2 + A_3\gamma_3^2) -$$

$$- 2(e_1\gamma_1 + e_2\gamma_2 + e_3\gamma_3) = 2h. \qquad (4)$$

Under the conditions

$$A_1 = A_2 + A_3^{**}, \qquad e_1 = 0, \qquad \lambda_1 = 0,$$

$$(A_2^2\lambda_2^2 + A_3^2\lambda_3^2)\varepsilon = A_2^2 e_2^2 + A_3^2 e_3^2$$

the equations (1) have a solution in which

$$\gamma_1 = \frac{(A_2 + A_3)(A_2^2\lambda_2^2 + A_3^2\lambda_3^2)}{(A_2 - A_3)(A_2^2 e_2\lambda_2 - A_3^2 e_3\lambda_3)}(\omega_1 - s), \tag{5}$$

$$\gamma_2 = (A_2^2 e_2^2 + A_3^2 e_3^2)^{-1}\left\{A_2 A_3(e_3\lambda_2 + e_2\lambda_3)\left(\omega_3 - \frac{\lambda_3}{A_2}\right) + \right.$$

$$\left. +(A_2^2 e_2\lambda_2 - A_3^2 e_3\lambda_3)\left(\omega_2 - \frac{\lambda_2}{A_3}\right)\right\}, \tag{6}$$

$$\gamma_3 = (A_3^2 e_3^2 + A_2^2 e_2^2)^{-1}\left\{A_3 A_2(e_2\lambda_3 + e_3\lambda_2)\left(\omega_2 - \frac{\lambda_2}{A_3}\right) + \right.$$

$$\left. +(A_3^2 e_3\lambda_3 - A_2^2 e_2\lambda_2)\left(\omega_3 - \frac{\lambda_3}{A_2}\right)\right\}.$$

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* The quantities μ_i, R_i of work [1] are denoted here by e_i, γ_i, respectively, and the notation of the constant integral (2) has been changed.

** In article [2] an example is given showing that this condition can be realized in a body with cavities filled with fluid.

Substituting (5), (6)$'$ into the integrals (2), (3), we determine ω_2, ω_3 as functions of ω_1:

$$\left(\omega_2 - \frac{\lambda_2}{A_3}\right)^2 + \left(\omega_3 - \frac{\lambda_3}{A_2}\right)^2 =$$

$$= (A_2^2 e_2^2 + A_3^2 e_3^2)\left\{\frac{\Gamma^2}{A_2^2\lambda_2^2 + A_3^2\lambda_3^2} - \frac{(A_2 + A_3)^2(A_2^2\lambda_2^2 + A_3^2\lambda_3^2)}{(A_2 - A_3)^2(A_2^2 e_2\lambda_2 - A_3^2 e_3\lambda_3)}(\omega_1 - s)^2\right\},$$

$$(A_2^2 e_2\lambda_2 - A_3^2 e_3\lambda_3)\left[A_2\left(\omega_2 - \frac{\lambda_2}{A_3}\right)^2 - A_3\left(\omega_3 - \frac{\lambda_3}{A_2}\right)^2\right] + \tag{7}$$

$$+A_2 A_3(A_2 + A_3)(e_3\lambda_2 + e_2\lambda_3)\left(\omega_2 - \frac{\lambda_2}{A_3}\right)\left(\omega_3 - \frac{\lambda_3}{A_2}\right) +$$

$$+(A_2 + A_3)(A_2^2\lambda_2^2 + A_3^2\lambda_3^2)\left[\left(\omega_2 - \frac{\lambda_2}{A_3}\right)\frac{e_2}{A_3} + \left(\omega_3 - \frac{\lambda_3}{A_2}\right)\frac{e_3}{A_2}\right] =$$

$$= (A_2^2 e_2^2 + A_3^2 e_3^2)\left\{m - \frac{(A_2 + A_3)^2(A_2^2\lambda_2^2 + A_3^2\lambda_3^2)}{(A_2 - A_3)(A_2^2 e_2\lambda_2 - A_3^2 e_3\lambda_3)}\omega_1(\omega_1 - s)\right\}.$$

Thus, from (6), γ_2, γ_3 are determined as functions of ω_1, after which the first equation (1) determines, by quadrature, the dependence of ω_1 on t.

The integral (4) in the indicated solution is dependent; it can be composed from the integrals (5), (6), (7). The constants h, m, Γ, s are connected by the relation

$$(A_2 + A_3)(A_2^2\lambda_2^2 + A_3^2\lambda_3^2)^2 \left[\frac{A_2\lambda_2^2}{A_3^2} + \frac{A_3\lambda_3^2}{A_2^2} + (A_2 + A_3)s^2 - 2h\right] +$$

$$+2(A_2 - A_3)(A_2^2 e_2\lambda_2 - A_3^2 e_3\lambda_3)(A_2^2\lambda_2^2 + A_3^2\lambda_3^2)m+$$

$$+ \left[4(A_2^2 e_2\lambda_2 - A_3^2 e_3\lambda_3)^2 + A_2A_3(A_2 + A_3)(e_3\lambda_2 + e_2\lambda_3)^2\right] A_2A_3\Gamma^2 = 0.$$

The indicated solution contains 10 independent parameters:

$$A_2,\ A_3,\ \frac{e_2}{e_3},\ \lambda_2,\ \lambda_3,\ \Gamma,\ s,\ m,\ \omega_1^0,\ \psi_0.$$

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1. P. V. Kharlamov, *Journal of Applied Mechanics and Technical Physics*, No. 4, 17 (1963).

2. E. I. Kharlamova, *Doklady AN SSSR*, **125**, No. 5, 996 (1959).

Note: Figure translations are in progress. See original paper for figures.