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1964

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Abstract

Full Text

CYBERNETICS AND CONTROL THEORY

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UNIVERSAL PULSATING ELEMENTS

(Presented by Academician A. N. Kolmogorov, 27 II 1964)

In the present note we consider certain schemes of automata more general than logical networks ([1]). What is characteristic for these schemes is that the connections between elements change with time and that the behavior of each element of the scheme at the moment $t+1$ depends on the state of some neighborhood of it at the moment t (see § 2°). For such schemes, problems are solved that are related to the problems of the completeness of a basis for ordinary logical networks ([1]).

1°. Elements and networks. As usual, in logical networks each of the elements under consideration has a finite number of states $q_1, \dots, q_N$ and a finite number of input channels $R_1, \dots, R_M$ (henceforth called **inputs**). In what follows the elements will be represented by circles into which arrows, representing inputs, are directed; sometimes the state in which the element is at the given moment will be written inside the circle. Without loss of generality one may consider only such networks as are constructed from copies of one and the same element A. Let a finite numbered collection of copies $\alpha_1, \dots, \alpha_n$ of the element A be given. Suppose that some inputs of some elements of this collection are connected to certain elements of the same collection in such a way that each input of each element is connected with no more than one element. In this case we shall say that a network L over A is given (examples are in Figs. 2 and 3). Those inputs of the elements of the network that are not connected with any elements will be called **free**. If the elements $\alpha_1, \dots, \alpha_n$ of the network L are respectively in the states $q_{\mu,1}, \dots, q_{\mu,n}$, then we shall say that L is in the state $a = (q_{\mu,1}, \dots, q_{\mu,n})$ (abbreviated notation $L(a)$). We shall say that an element $\alpha_{\nu,m} \in L$, relative to an element $\alpha_{\nu,0} \in L$, has a **coordinate** $\alpha_{\nu,0} R_{\pi,1} R_{\pi,2} \cdots R_{\pi,m}$ of length m, if there exist such $\alpha_{\nu,1}, \dots, \alpha_{\nu,(m-1)} \in L$ that the input $R_{\pi,i}$ of the element $\alpha_{\nu,(i-1)}$ is connected to $\alpha_{\nu,i}$ ($i = 1, \dots, m$). The least length of the coordinates of the element $\alpha_{\nu,m}$ relative to $\alpha_{\nu,0}$ will be called the **distance** from $\alpha_{\nu,m}$ to $\alpha_{\nu,0}$. The maximal subnetwork of the network L whose elements are at distance $\leq l$ from α will be called the l-**neighborhood** of α and denoted by χ_α. The l-neighborhoods $\chi_\alpha, \chi_\beta \subset L(a)$ will be called **isomorphic**, for the given state a of the network L, if there exists a one-to-one mapping h of the elements belonging to χ_α onto the elements belonging to χ_β such that: a) $h\alpha = \beta$; b) for every $\alpha_k \in \chi_\alpha$, α_k and $h\alpha_k$ have the same states; c) the input R_j of the element α_k is connected to $\alpha_{k'}$ if and only if the input

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Fig. 1

Figure 1: Fig. 1

R_j of the element $h\alpha_k$ is connected to $h\alpha_{k'}$.

2°. Pulsating elements. Suppose that the rules of functioning of the element A have the following properties: if at the moment t an arbitrary copy α of the element A belongs to some network $L(a)$ over A, and if χ_α is its l-neighborhood in this network, then at the moment $t+1$, depending only on χ_α, considered up to isomorphism: a1) α passes into a definite new state $q(\chi_\alpha)$; a2) α disconnects some of its inputs from the elements to which they were connected at the moment t, and connects some of them, as well as some of its inputs that at the moment t were free, to other elements belonging to χ_α; the remaining inputs of α it leaves free. In this case we shall say that A is a **pulsating element** (PE) with **depth of pulsation** (d.p.) l.

It is important to emphasize that at the moment $t+1$ the actions indicated in a1) and a2) are carried out by all elements of the network $L(a)$ simultaneously. Thus, at the moment $t+1$ the network $L(a)$ passes into another network $L'(a')$. The network into which $L(a)$ passes after T moments will be denoted by $\nabla_A(L(a), T)$.

Example 1. PE A_1 with m.p. $= 2$ has states $0, 1$, inputs R_1, R_2, R_3, and the following rules of operation: 1) if at the moment t, in the neighborhood of an element α (α is an arbitrary instance of PE A_1), there is an element with coordinate $\alpha R_1 R_1$ having state 1 and not coinciding with α, then at the moment $t+1$ α passes into state 1, connects input R_1 to the element $\alpha R_1 R_1$, and disconnects its remaining inputs and leaves them free; 2) in all other cases α does not change its state or commutations. If at the moment t a network $L_1(a)$ over A_1 is given (Fig. 1a), then at the moment $t+1$ it passes into the network of Fig. 1b, and so on.

Fig. 1

3°. Modeling. Let a PE A be given, which has states $q_1, \dots, q_N$ and inputs $R_1, \dots, R_M$. Suppose, along with A, another PE A' is given. In this section it will be defined what it means that PE A can be modeled by blocks constructed from PE A'.

Let B be a network over A' with a fixed numbering of elements. Suppose that in B, in a definite order, M free inputs are distinguished. Denote them by $P_1, \dots, P_M$ and call them the inputs of B. Further, suppose that in B one element is distinguished; denote it by O^* and call it the output of B. Such a network B will be called a block over A' with inputs $P_1, \dots, P_M$ and one output (an example is shown in Fig. 2). In what follows, when speaking of different instances of the block B, we shall assume that they do not contain common instances of the element A'.

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Fig. 2

Figure 2: Fig. 2

Fig. 2

Now suppose that to each state q_j $(j = 1, \dots, N)$ of the element A there is assigned a completely definite nonempty set ψq_j of states of the block B, in such a way that for different q_i, ψq_i do not intersect. Such a correspondence ψ will be called a $(qA \to qB)$-correspondence.

Consider a network L over A with the set of elements $(\alpha_1, \dots, \alpha_n)$. Suppose that to each of its elements α_i there is assigned a particular instance $\varphi\alpha_i$ of the block B, in such a way that different α_i correspond to different instances $\varphi\alpha_i$. Such a correspondence φ will be called an $(L \to B)$-correspondence.

Suppose the network $V(b)$ over A' can be obtained from the network $L(a)$ over A in the following way: a) each element $\alpha_i \in L(a)$ (let q_{α_i} be its state) is replaced by an instance $\varphi\alpha_i$ of the block B in some state belonging to ψq_{α_i}; b) if input R_k of the element α_i is connected to α_j, then input P_k of the block $\varphi\alpha_i$ is connected to the output of the block $\varphi\alpha_j$; if R_k is free, then P_k remains free. In this case we shall say that $V(b)$ is a (φ, ψ)-substitution of the network $L(a)$.

Suppose that a certain $(qA \to qB)$-correspondence ψ and a certain natural number T are given. Define the relation $A' \geq_T^{\psi} A$ as follows: whatever the networks $L(a)$ over A and $V(b)$ over A' may be, if only $V(b)$ is a (φ, ψ)-substitution of $L(a)$ (where φ is some $(L \to B)$-correspondence), then the network $\nabla_{A'}(V(b), T)$ is a (φ', ψ)-substitution of the network $\nabla_A(L(a), 1)$, such that $\varphi'\alpha$ and $\varphi\alpha$ are one and the same instance of the block B.

Example 2. Let A_1 be the PE from Example 1, and $L_1(a)$ the network from Fig. 1a. Let A_1' be another PE, which has states $0, 1, 2, 3$ and inputs R_1', R_2'. In Fig. 2 a block B_1 over A_1' is shown (dashed arrows represent R_1', solid arrows R_2'). Suppose the following are given: a) a $(qA_1 \to qB_1)$-correspondence ψ_1: $\psi_1 0 =$

$= \{(q_1, q_2, q_3)\}$, where $q_1 = 2,\ q_2 = 0, 1,\ q_3 = 0, 1;\ \psi_1^1 = \{(q_1, q_2, q_3)\}$, where $q_1 = 3,\ q_2 = 0, 1,\ q_3 = 0, 1$; b) the $(L_1 \to \dot{B}_1)$-correspondence φ_1 according to Fig. 3. It is easy to see that the network $V_1(b)$ in Fig. 4a is a (φ_1, ψ_1)-substitution of the network $L_1(a)$. It is easy to verify that the rules of operation of the element A_1' can be chosen so that $A_1' \geq_2^{\psi_1} A_1$. In this case, since $L_1(a)$ in one instant passes into $L_1'(a')$ in Fig. 1b, $V_1(b)$ in two instants will pass into $V_1'(b')$ in Fig. 4b. In other words, this means: a) if the element $a_i \in L_1(a)$ at the next instant passes into the state q_{ai}, then the block $\varphi_1 a_i \subset V_1(b)$ after 2 instants passes into a state in $\psi_1 q_{ai}$; b) if the input R_k of the element a_i at the next instant is connected to the element $a_{i'}$, then the input P_k of the block $\varphi_1 a_i$ after 2 instants is connected to the output of the block $\varphi_1 a_{i'}$; if R_k is only disconnected, then P_k is only disconnected.

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Fig. 3

It is clear that further, with $L_1'(a') = \nabla_{A_1}(L_1(a), 1)$ and $V_1'(b') = \nabla^{\pi'}_{A_1'}(V_1(b), 2)$, the same thing is repeated as with $L_1(a)$ and $V_1(b)$, i.e. $\nabla_{A_1'}(V_1'(b'), 2)$ will be a (φ_1, ψ_1)-substitution of the network $\nabla_{A'}(L_1'(a'), 1)$.

Let us say that A' **models** A (abbreviated: $A' \geq A$) if for some $(qA \to qB)$-correspondence ψ and some T the relation $A' \geq^{\psi}_{T} A$ holds.

Fig. 4

From the definition of modeling it follows:

$$(A'' \geq A') \,\&\, (A' \geq A) \;\to\; (A'' \geq A).$$

4°. **Main results.**

Theorem 1. For every PE A there exists a PE A' modeling it, which has only 2 inputs and g.p. $= 2$.

A PE A^0 will be called **universal** for the class $\mathfrak{M}$ of PEs if

$$\forall A\big[(A \in \mathfrak{M}) \to (A^0 \geq A)\big].$$

Theorem 2 (main). There exists a PE A^0, universal for the class of all PEs, and A^0 has 3 states, 3 inputs, and g.p. $= 3$.

From Theorems 1 and 2 it follows that there exists a PE A^*, universal for the class of all PEs, and A^* has only 2 inputs and g.p. $= 2$ (but the number of states > 3).

5°. Let us consider one subclass of the class of all PEs. A PE E will be called a **generalized logical** one if its rules of operation are such that the connections between elements never change.

Theorem 3. No generalized logical element can be universal for the class of all generalized logical elements.

6°. The networks of pulsing elements considered above operate as autonomous automata (i.e., without the action of external signals). However, the concept of pulsing networks with external action can be introduced in a natural way. If the concept of modeling is suitably generalized for such networks, then for them one can prove theorems analogous to Theorems 1, 2, and 3.

The results of the present note may be used in the theory of growing automata.

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Received
27 II 1964

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Note: Figure translations are in progress. See original paper for figures.

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