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Abstract

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BREMSSTRAHLUNG OF ULTRARELATIVISTIC ELECTRONS IN CONDENSED AMORPHOUS BODIES

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Various effects of the influence of a medium on the bremsstrahlung of ultrarelativistic electrons have been considered in a number of works ([1-7]). In particular, in the work of V. M. Galitskii and the author ([7]), a general method was developed for taking into account the influence of the medium on the energy losses of particles in matter, and a quantitative treatment was given of the influence of absorption of quanta on the bremsstrahlung of high-energy electrons for the case in which the quantum frequency ω is much smaller than the initial electron energy E. The purpose of the present work is a quantum-mechanical generalization of the indicated method with the sole condition $\omega, E \gg 1^*$. Further, as A. B. Migdal ([5]) has shown, at sufficiently large quantum energies one must take into account the effect of multiple scattering in pair production. It turns out that the resulting dependence of the pair-production cross section on ω leads to the interval of frequencies in which absorption is substantial being shifted strongly into the classical region $\omega \ll E$, and the mean differential energy losses of the electron acquire an additional dependence on the density and on the frequency.

Let us consider a system consisting of the medium, an electron, and the electromagnetic field. We write the full Hamiltonian of the system in the Schrödinger representation in the form

$$H = H_e + H_r + H^I. \tag{1}$$

Here $H_e = H_{0e} + \sum_m V(\mathbf{r} - \mathbf{r}_m)$ includes the interaction of the electron with all scattering centers; H_r describes the medium, the free electromagnetic field, and their interaction with one another; $H^I = j_i A_i$ represents the interaction of the electron with the electromagnetic field and will be treated as a perturbation. In first order perturbation theory we obtain

$$ic_s^{(1)} = \int_0^t dt_1\,(\psi_s \cdot \varphi_s,\, e^{i(H_e+H_r)t_1} j_k A_k e^{-i(H_e+H_r)t_1} \psi_0 \cdot \varphi_0), \tag{2}$$

where $\psi_s \cdot \varphi_s$ are eigenfunctions of the system in the interaction representation,

$$H_e \psi_s = E_s^e \psi_s, \qquad H_r \varphi_s = E_s^r \varphi_s.$$

The total energy losses of the electron per unit time are determined by the transition probability $\left|c_s^{(1)}\right|^2$ per unit time, summed over

* In this work the system of units $h = m = c = 1$ is used.

over all final states of the system with $E_s^e > 0$:

$$W = \sum_{E_s^e > 0} \frac{d}{dt} \left|c_s^{(1)}\right|^2 = 2\,\mathrm{Re} \int_0^t dt_1\, (\psi_0\varphi_0,\ e^{iH_e t_1} j_k \widetilde{A}_k K e^{iH_e(t-t_1)} j_i A_i e^{-iH_e t} \psi_0\varphi_0). \tag{3}$$

where

$$\widetilde{A}_k = e^{iH_r t} A_k e^{-iH_r t}$$

and K is the projection energy operator. Expanding $\widetilde{A}_i$ in a Fourier integral over the coordinates,

$$\widetilde{A}_i(\mathbf{r}, t) = \int dk\, \widetilde{A}_i(\mathbf{k}, t) e^{i\mathbf{kr}}, \qquad dk = \frac{d^3k}{(2\pi)^3}, \tag{4}$$

and introducing the notation

$$I_{ki} = \frac{1}{e^2} (\psi_0,\ e^{iH_e t_1} j_k e^{i\mathbf{kr}} K e^{iH_e(t-t_1)} j_i e^{-i\mathbf{k}_1\mathbf{r}} e^{-iH_e t} \psi_0), \tag{5}$$

we find

$$W = e^2 \cdot 2\,\mathrm{Re} \int_0^t dt_1 \int dk\, dk_1\, (\varphi_0,\ \widetilde{A}_k(t_1) \widetilde{A}_i^+(t) \varphi_0) \cdot I_{ki}. \tag{6}$$

The integrand $(\varphi_0, \widetilde{A}_k \widetilde{A}_i^+ \varphi_0)$ is directly related to the Green' s function of the electromagnetic field in the medium,

$$(\varphi_0,\ \widetilde{A}_k(t_1) \widetilde{A}_i^+(t) \varphi_0) = -iD_{ik}^+(\mathbf{k}, \mathbf{k}_1; t, t_1), \qquad t_1 < t.$$

In what follows we shall restrict ourselves to the case of a homogeneous medium. Then, passing in (6) to integration over $\tau = t - t_1$ and further expanding the D-function in a Fourier integral with respect to time, after simple transformations we obtain

$$W = -e^2 \cdot 4\,\mathrm{Re} \int_0^\infty \frac{d\omega}{2\pi} \int_0^t d\tau\, e^{i\omega\tau} \int dk\; \mathrm{Im}\, D_{ik}(\mathbf{k},\omega) \cdot I_{ki}. \tag{7}$$

Since large wavelengths are essential in the problem, the influence of the substance on the electromagnetic field may be taken into account phenomenologically by introducing the dielectric constant of the medium $\varepsilon = \varepsilon^I + i\varepsilon^{\mathrm{II}}$:

$$\varepsilon^I = 1 - \frac{\omega_0^2}{\omega^2}, \qquad \varepsilon^{\mathrm{II}} = \frac{n\sigma}{\omega} = \frac{1}{L\omega}, \tag{8}$$

where $\omega_0^2 = 4\pi n e^2$, n is the nuclear density, σ is the integral cross section for pair production, and L is the radiation length. Therefore, choosing a gauge with scalar potential equal to zero, we can write (see, for example, ([8])):

$$\mathrm{Im}\, D_{ik} = -\frac{4\pi\varepsilon^{\mathrm{II}}}{\omega^2|\varepsilon|^2} \left\{ \frac{k_i k_k}{k^2} + |\varepsilon|^2 \omega^4 \frac{\delta_{ik} - k_i k_k/k^2}{|k^2 - \varepsilon\omega^2|^2} \right\}. \tag{9}$$

For the energies and frequencies under consideration, the first term in (9) proves to be inessential. Taking into account that $j_i = e\alpha_i$ $(i = 1, 2, 3)$, after averaging over the coordinates of the scattering centers we arrive at the following expression for the mean differential energy losses of the electron per unit time:

$$\dot{Q}_\omega = \frac{e^2\omega^3}{2\pi^3} \mathrm{Re} \int_0^t dt\, e^{i\omega\tau} \int \frac{\varepsilon^{\mathrm{II}}\, d^3k}{|k^2 - \varepsilon\omega^2|^2} \left(\delta_{ik} - \frac{k_i k_k}{k^2} \right) \cdot \overline{I}_{ki}, \tag{10}$$

$$I_{ki} = (\psi_0,\; e^{iH_e t_1} \alpha_k e^{i\mathbf{kr}} K e^{iH_e\tau} \alpha_i e^{-i\mathbf{kr}} e^{-iH_e\tau} e^{-iH_e t_1} \psi_0). \tag{11}$$

Averaging of the matrix element (11) can be carried out by the method developed by Migdal (see ([5])), with the difference that now ω and k are not connected by the usual relation (see (7)). The result obtained is conveniently represented as two terms–the losses to bremsstrahlung and to pair creation by an electron in the medium:

$$\dot{Q}_\omega = \dot{Q}_\omega^T + \dot{Q}_\omega^{\Pi}; \tag{12}$$

$$\dot{Q}_\omega^T = \frac{e^2\omega^3}{\pi^2} \int \frac{\varepsilon^{\mathrm{II}}(k)\, dk}{|k^2 - \varepsilon\omega^2|^2} \left\{ 2\left[1 + \left(1 - \frac{\omega}{E}\right)^2\right] \left[\frac{Bk}{E(E-\omega)}\right]^{1/2} F(s) + \frac{k^3}{E^3(E-\omega)} G(s) \right\} \tag{12'}$$

$$\dot{Q}^{\Pi}_{\omega} = \frac{e^2\omega^3}{\pi}\int \frac{\varepsilon^{\Pi}(k)\,dk}{|k^2-\varepsilon\omega^2|^2}\left\{-8s\left[1+\left(1-\frac{\omega}{E}\right)^2\right]\left[\frac{Bk}{E(E-\omega)}\right]^{1/2}\theta(-s-1)+\frac{k^3}{E^3(E-\omega)}\theta\left(-s-\frac{1}{2}\right)\right\}, \quad (12")$$

where

$$F(s) = \frac{1}{6s}\left[\theta(s+1)\Phi(s)+\theta(-s-1)\Phi(-s)\right];$$

$$G(s) = \frac{1}{48s^2}\left[\theta\left(s+\frac{1}{2}\right)G(s)+\theta\left(-s-\frac{1}{2}\right)G(-s)\right];$$

$\theta(s)$ is the unit function; $\Phi(s)$ and $G(s)$ are the functions introduced by Migdal in ([4,5]);

$$s = \frac{1}{4}\left[\omega-k\left(1-\frac{1}{2E(E-\omega)}\right)\right]\left[\frac{E(E-\omega)}{Bk}\right]^{1/2}, \qquad B = 4\pi NZ^2e^4\ln(191Z^{-1/3}).$$

In the classical region $\omega \ll E$, $(12')$, $(12'')$ obviously pass over into the formulas found in ([7]). On the other hand, in the absence of absorption ($\varepsilon^{\Pi}\to 0$), $\dot{Q}^{\Pi}_{\omega}$ vanishes, while in $(12')$ the function $\delta(k-\omega\sqrt{\varepsilon^I})$ appears, and $\dot{Q}_{\omega}$ passes over into the expression for bremsstrahlung obtained in Migdal' s work ([5]).

Absorption of quanta affects bremsstrahlung when the width of the resonance denominator in $(12')$ becomes greater than the width of the functions $F(s)$ and $G(s)$. This is realized in the frequency interval $L\omega_0^2 \ll \omega \ll E^2/(64BL^2+E)$. The radiation length L does not depend on ω and is equal to $L_0 = 9E_s^2/28B$* up to quantum energies $\lesssim 7L_0/18E_s^2$ ($E_s = (4\pi/e^2)^{1/2}$ is the radiation energy). In the general case the function $L(\omega)$ has a complicated form. It is substantially simplified in the high-frequency limit

$$L(\omega) = \frac{E_s^2}{3\pi}\sqrt{\frac{\omega}{B}}, \qquad \omega \gg \frac{7}{18}\frac{L_0}{E_s^2} = \frac{1}{8B}. \quad (13)$$

Consequently, taking into account the effect of multiple scattering in pair creation by a quantum leads to the fact that the indicated interval is strongly shifted into the classical region and, for $E \gg 28L_0/27\pi$, is described by the inequalities

$$L_0\omega_0^2 \ll \omega \ll \frac{3\pi}{8}\frac{E}{E_s^2}, \qquad E \gg \frac{28}{27\pi}L_0, \quad (14)$$

where

$$\dot{Q}^T_\omega = \frac{16\xi e^2}{\pi^2}\frac{B\omega}{E^2}L(\omega), \qquad \xi = 2\ln(2\operatorname{sh}\pi) \simeq 2\pi. \tag{15}$$

The previous result for the mean losses ([7]) is retained when the conditions

$$L_0\omega_0^2 \ll \omega \ll \frac{7}{18}\frac{L_0}{E_s^2}, \qquad \dot{Q}^T_\omega = \frac{144\xi}{7\pi^2}\frac{\omega}{E^2}. \tag{16}$$

are satisfied.

* For lead $\omega_0 \simeq 60$ eV, $L_0\omega_0^2 \simeq 1.2 \cdot 10^8$ eV.

Formula (15) again takes a simple form in the frequency interval

$$\frac{7}{18}\frac{L_0}{E_s^2} \ll \omega \ll \frac{3\pi}{8}\frac{E}{E_s^2}, \qquad \dot{Q}^{\mathrm{T}}_\omega = \frac{64\xi}{3\pi^2}\frac{\omega}{E^2}\sqrt{B\omega}. \tag{17}$$

Compared with (16), this expression contains an additional dependence on density and frequency, $(n\omega)^{1/2}$. In the final part of the spectrum, $3\pi E/8E_s^2 \ll \omega \ll E$, the radiation intensity is determined by Migdal' s formula [5]. To obtain the magnitude of the losses for pair production by an electron in the quantum region $\omega \sim E$, it is necessary to take into account the spatial dispersion of the dielectric constant $\varepsilon^{\mathrm{II}}(k)$.

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Note: Figure translations are in progress. See original paper for figures.