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Abstract

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Thermal Conductivity and Moisture Exchange in a Turbulent Atmosphere in the Presence of Phase Transitions of Moisture

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1. The processes of formation and evolution of cloudiness and fogs belong among the most complex and, at the same time, the most important atmospheric phenomena for practice. Clarifying their regularities is a necessary condition for the further development of numerical methods of weather forecasting, the theory of active influences on atmospheric processes, etc.

In recent times, substantial results in the development of the theory of the phenomena under consideration have been obtained mainly in the works of L. T. Matveev, M. E. Shvets, and their students. However, the physical concepts underlying these and other works require refinement. In fact, in the literature up to now there has been no physically rigorous formulation of the general problem of the transfer of heat and moisture in a turbulent atmosphere in the presence of liquid or solid water in the form of aerosols. The aim of the present work is to formulate the corresponding equations and to obtain the conditions for the "matching" of solutions on the surface bounding the region of fog or cloud. Analysis of these general relations makes it possible to draw a number of conclusions about the mechanism of the processes under study, and also to outline ways of solving certain applied problems.

2. For points of the atmosphere lying outside a cloud, the equations of transfer of heat and water vapor, as is known, have the form:

$$\frac{\partial T}{\partial t} + (\mathbf{u}, \vec{\nabla})T + u_3\gamma_a = -\left(\vec{\nabla}, \frac{\mathbf{P}}{c_p\rho}\right) - \frac{1}{c_p\rho}(\vec{\nabla}, \mathbf{D}), \tag{1}$$

$$\frac{\partial q}{\partial t} + (\mathbf{u}, \vec{\nabla})q = -\left(\vec{\nabla}, \frac{\mathbf{Q}}{\rho}\right). \tag{2}$$

Here t is time, T is absolute temperature, q is specific humidity, $\mathbf{u}$ is the mean wind velocity, u_3 is its vertical component, γ_a is the dry-adiabatic temperature gradient, c_p is the heat capacity of air at constant pressure, ρ is the density of air, **D** is the radiation flux, and **P** and **Q** are the turbulent fluxes of heat and water vapor.

The components of the vectors **P** and **Q** in a Cartesian coordinate system x_1, x_2, x_3 (x_3 vertical) are determined by the equalities

$$\frac{P_j}{c_p \rho} = -k_j \left(\frac{\partial T}{\partial x_j} + \gamma_a \delta_{j3} \right), \qquad \frac{Q_j}{\rho} = -k_j \frac{\partial q}{\partial x_j}, \tag{3}$$

where k is the coefficient of turbulent exchange, and δ_{j3} is the Kronecker symbol.

3. We shall assume that water in the atmosphere occurs only in the gaseous and liquid phases. In this case, when studying the processes of heat and moisture exchange inside a cloud, along with temperature and specific humidity one should introduce into consideration the specific water content b, representing the mass of liquid water per unit mass of air. As shown in ([1]), under natural conditions water droplets suspended in a cloud are practically completely carried along by the motions of air particles. Thus, the transfer of water content in the atmosphere occurs in exactly the same way as the transfer of heat and water ⋯

vapor*. The equations that the distributions of the quantities under consideration must satisfy, in the present case, have the form

$$\frac{\partial T}{\partial t} + (\mathbf{u}, \vec{\nabla})T + u_3 \gamma_a = -\left(\vec{\nabla}, \frac{\mathbf{P}}{c_p \rho} \right) - \frac{1}{c_p \rho} (\vec{\nabla}, \mathbf{D}) + \frac{L}{c_p} m, \tag{4}$$

$$\frac{\partial q}{\partial t} + (\mathbf{u}, \vec{\nabla})q = -\left(\vec{\nabla}, \frac{\mathbf{Q}}{\rho} \right) - m, \tag{5}$$

$$\frac{\partial b}{\partial t} + (\mathbf{u}, \vec{\nabla})b = -\left(\vec{\nabla}, \frac{\mathbf{B}}{\rho} \right) + m, \tag{6}$$

where m is the mass of moisture condensing per unit time in a unit mass of air, L is the latent heat of vaporization; **B** is the turbulent flux of liquid water.

Since water vapor in a cloud is in the saturated state, the specific humidity is uniquely expressed in terms of pressure p and temperature:

$$q = q_m(p, T) = \frac{R}{R_w} \frac{E(T)}{p}, \tag{7}$$

where R and R_w are the gas constants of air and water vapor, and $E(T)$ is the maximum water-vapor elasticity at temperature T.

Taking into account (in accordance with (7)) the effect of phase transitions of moisture during vertical displacements of turbulent eddies, we obtain, in the case considered, for the components of the quantities **P**, **Q**, and **B** the expressions

$$\frac{P_j}{c_p\rho} = -k_j\left(\frac{\partial T}{\partial x_j} + \gamma_B\delta_{j3}\right), \qquad \frac{Q_j}{\rho} = -k_j\left(\frac{\partial q}{\partial x_j} + \beta\delta_{j3}\right),$$

$$\frac{B_j}{\rho} = -k_j\left(\frac{\partial b}{\partial x_j} - \beta\delta_{j3}\right). \tag{8}$$

Here γ_B is the moist-adiabatic temperature gradient, and β is a quantity determined by the equality

$$\beta = \frac{c_p}{L}(\gamma_a - \gamma_B). \tag{9}$$

In physical meaning, β is the value of the equilibrium humidity gradient in a cloud. Table 1 gives the values of this quantity as a function of temperature τ and pressure p.

Let us note that, up to now, the equilibrium humidity gradient in formulas of type (8) has not been taken into account. According to experimental data ([2]), the characteristic value of the actual humidity gradient in a cloud is approximately 0.1 $\frac{\text{g}}{\text{kg}}$/100 m. As is seen from the table, β has the same order. Thus, it is clear that neglecting the quantity β (as is done everywhere in the modern literature) must lead to errors of the order of 100% in determining the turbulent fluxes under consideration.

4. Denote the region occupied by the cloud at time t by Ω. Denote the boundary of the cloud by Σ. Introduce the specific moisture content S, setting

$$S = \begin{cases} q, & \text{for } M \notin \Omega, \\ q + b, & \text{for } M \in \Omega, \end{cases} \tag{10}$$

where M is an arbitrary point of space. Form the difference

$$s - q_m = f(M, t). \tag{11}$$

The function f is defined for all t in the whole atmospheric space, and outside the cloud $f \leq 0$, while in the cloud $f > 0$. Thus, the surface Σ

* The case of rain is not considered here.

is determined by the set-theoretic equality*

$$\Sigma = \lim_{\varepsilon \to 0} \{M;\ 0 < f(M,t) < \varepsilon\}. \tag{12}$$

From physical considerations it is clear that the quantities T, q, and b, considered as functions of the point of space and of time, are continuous everywhere in their domain of definition, including at the cloud boundary. Assuming that the surface Σ is sufficiently smooth, one can show that the derivatives of the functions under consideration in directions tangent to Σ also do not undergo discontinuities when crossing this surface. For derivatives along the normal $\mathbf{n}$, in this actually general case, the following relations hold; for greater clarity we give them in terms of the components, normal to Σ, of the corresponding turbulent fluxes:

$$P_n^{(i)} = P_n^{(e)} - \left(D_n^{(i)} - D_n^{(e)}\right) + LB_n^{(i)}, \tag{13}$$

$$Q_n^{(i)} = Q_n^{(e)} - B_n^{(i)}, \tag{14}$$

$$B_n^{(i)} = \frac{1}{1+c}\left[cQ_n^{(e)} - \frac{1}{L}P_n^{(e)} + \frac{1}{L}\left(D_n^{(i)} - D_n^{(e)}\right)\right], \tag{15}$$

where

$$c = \frac{R_w c_p \rho}{RLE'(T)}. \tag{16}$$

The indices (i) and (e) in (13)–(15) denote, respectively, the internal and external (with respect to Ω) values of the quantities considered on Σ. In their physical meaning, (13) and (14) are the equations of the heat balance and moisture balance at the cloud boundary. Equation (15) relates the water-content flux normal to Σ (equal to the rate of evaporation of moisture from the cloud surface) to the values of the parameters external with respect to the cloud. It should be emphasized that the jump in the normal component of the radiation flux at the cloud boundary, $D_n^i - D_n^e$, may be very substantial, since a known fraction of the incident radiation is reflected by the cloud.

Table 1

Values of the equilibrium moisture gradient β $\left(\text{in } \frac{\text{g}}{\text{kg}}/100\,\text{m}\right)$

p, in mb	$\tau = 30°C$	$\tau = 20°C$	$\tau = 10°C$	$\tau = 0°C$	$\tau = -10°C$
1000	0.26	0.23	0.19	0.14	0.09
900	0.27	0.24	0.19	0.15	0.10
800	0.27	0.25	0.20	0.16	0.10

The boundary conditions (13)–(15) have not been obtained until now. The only work on this question belongs to M. E. Shvets ([3]), who proposed writing the equations of the heat balance and moisture balance of the cloud boundary by analogy with similar equations for a surface separating ice and liquid water. Such a hypothesis, however, cannot be strictly justified; in particular, this is evident from the fact that at the cloud boundary the water content vanishes.

5. Let us consider a cloud layer $h(x_1, x_2, t) < x_3 < H(x_1, x_2, t)$ of great horizontal extent. In this case, when studying the evolution of cloudiness, it is possible, as is well known, to neglect the effects of horizontal diffusion and horizontal transport of radiation. We shall assume, for simplicity, that the components of the velocity of the mean flow in the layer (h, H) do not vary with height. Then, as a result of the corresponding transformations of the relations obtained above, we find

$$\left(\frac{\partial}{\partial t} + u_1 \frac{\partial}{\partial x_1} + u_2 \frac{\partial}{\partial x_2}\right) \int_h^H b\, dx_3 = \beta u_2 (H - h) + \frac{1}{1 + cL} \left[\frac{\frac{1}{L}(P_3 + D_3) - cQ_3}{\rho} \right]_{h-0}^{H+0}. \tag{17}$$

* Usually the expression $\Sigma = \{M;\ f(M, t) = 0\}$ that is used is valid only in those cases in which its right-han

(17) is naturally called the equation of cloud evolution. This equation relates the change in integral water content (characterizing the total store of liquid water in the cloud) to the values of the parameters that determine the external conditions.

It follows from (17) that there exists the possibility of influencing cloudiness by means of purposeful changes in external parameters (for example, u_3 or D_3). The effect of such influences can then be calculated.

Another area of application of the cloud-evolution equation is connected with numerical weather forecasting. Indeed, since the principal terms on the right-hand side of (17) can be determined, this equation makes it possible to forecast the change in integral water content.

Starting from the general relations (1)–(16), one can obtain an equation of fog evolution analogous to (17) in its physical meaning. This opens up possibilities for studying the effects of influences on fogs, and also for developing methods for their forecasting.

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named after A. I. Voeikov

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Note: Figure translations are in progress. See original paper for figures.