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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

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HYDROMECHANICS

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A METHOD FOR CALCULATING THREE-DIMENSIONAL FLOW PAST BODIES WITH A DETACHED SHOCK WAVE

(Presented by Academician G. I. Petrov, 10 XI 1963)

At the end of 1961 the authors proposed a numerical method for integrating the equations of gas dynamics as applied to the calculation of supersonic flow past bodies with a detached shock wave. The high efficiency of the method made it possible in 1962 and 1963 to carry out a series of studies in which, over a wide range of variation of the determining parameters, axisymmetric flow past blunt bodies of various shapes by a perfect gas was investigated, as well as with allowance for equilibrium physico-chemical transformations.

Fig. 1

In the present work a generalization of the method to the case of three-dimensional flow with a detached shock wave is given, and some calculation results are presented.

Let, in the subsonic and transonic region, the body be bounded by a smooth (analytic) convex surface and have a plane of symmetry in which the velocity vector of the oncoming flow is located. We introduce a spherical coordinate system r, θ, ψ, whose axis ($\theta = 0$) lies in the plane of symmetry and is directed toward the oncoming flow. Instead of r it is convenient to introduce the variable

$$\xi = \frac{r - r_{\mathrm{T}}(\theta, \psi)}{r_{\mathrm{c}}(\theta, \psi) - r_{\mathrm{T}}(\theta, \psi)},$$

Fig. 2

Figure 2: Fig. 2

Fig. 3

Figure 3: Fig. 3

where $r = r_{\text{T}}(\theta, \psi)$ is the equation of the body surface, and $r = r_{\text{c}}(\theta, \psi)$ is the equation of the unknown surface of the compression shock.

Let us draw $4k+4$ meridional half-planes $\psi = \pm\psi_0, \dots, \pm\psi_k, \pm\pi\mp\psi_k, \dots, \pm\pi\mp\psi_0$ and n conical surfaces $\theta = \theta_1, \theta_2, \dots, \theta_n$. Their intersections form $n(4k+4)+1$ rays issuing from the origin of coordinates (including the axis $\theta = 0$). For a fixed value of ξ, all gas-dynamic parameters can be approximated from their values at the $n(4k+4)+1$ points of intersection of the rays with the surface $\xi = \text{const}$. In each meridional plane $\psi = \psi_j(-\pi + \psi_j)$ the parameters are approximated by Lagrange polynomials through $(2n+1)$ points. Taking into account the periodicity of all parameters in ψ, for fixed values of θ they are approximated by trigonometric interpolation polynomials from the values at $(4k+4)$ points. The presence of a plane of symmetry makes it possible to express all approximations in terms of the values of the parameters on $n(2k+2)+1$ rays located in the half-space $0 \le \psi \le \pi$. In this case, functions even in ψ are approximated by polynomials in cosines, and odd functions by polynomials in sines.

Computing the derivatives with respect to θ and ψ by means of approximations, we substitute them into the system of gas-dynamic equations and require that the system be satisfied on all $n(2k+2)+1$ rays. We obtain an approximating system of ordinary differential equations with respect to the values of the parameters on these rays. The sought shock surface ($\xi = 1$) is approximated by the same scheme and contains $n(2k+2)+1$ arbitrary parameters (constants). In order that the boundary condition on the body surface ($\xi = 0$) be satisfied at the nodes, for the approximating system of ordinary differential equations one has to solve a boundary-value problem, selecting the corresponding values of $n(2k+2)+1$ parameters in the shock equation.

Writing out further the derivatives with respect to ξ in the form of difference relations, we obtain an explicit difference scheme with an arbitrary arrangement of nodes on the surfaces $\xi = \text{const}$, which was used in the work.

Fig. 2

Fig. 3

According to this scheme, a program was compiled for a high-speed computer with grid parameters $\psi_0 = 0$, $k = 1$, $n = 3$. In this case the polynomials in ψ for even functions contain terms up to $\cos 3\psi$, and for odd functions up to $\sin 2\psi$, while the derivatives with respect to θ are approximated by 7 points. The accuracy of the approximation corresponds to 19 nodes on the surface $\xi = \text{const}$.

The presence of a plane of symmetry made it possible to reduce the problem of solving the approximating system to integration along 13 rays. The selection of the 13 arbitrary parameters determining the shape of the shock surface is performed automatically. Increasing the number of arbitrary parameters of the boundary-value problem from 3-5 in the calculation of axisymmetric flows to 13 causes no difficulties.

As an example, Figs. 1, 2, and 3 present some results of calculations of the flow past ellipsoids of revolution with axis ratios $\delta = 1.5$ and 3.07 and with the minor axis directed toward the flow, for $M = 3$ and $\gamma = 1.4$.

In Fig. 1, for $\delta = 1.5$, the shape is shown of the lines of intersection of the shock and sonic surfaces with a number of meridional ($\psi =$ const) planes at angles of attack $\alpha = 0$ (dashed lines) and $\alpha = 0.2618$ (15°) (solid lines) ($\psi > \pi/2$ corresponds to the windward side, and $\psi < \pi/2$ to the leeward side).

Figure 2 gives the pressure distribution over the body surface in the plane of symmetry. Here y is the coordinate of the Cartesian system with origin at the center

of the ellipsoid and the unit vector lying in the plane of symmetry and coinciding with the major semiaxis. The upper curves represent the pressure distribution for $\delta = 1.5$ and various angles of attack; on the lower curve for $\delta = 3.07$ and $\alpha = 0.1745$ (10°), the experimental data of O. Ya. Karpeiskii are plotted as points.

Figure 3 shows the shape of the shock wave and of the sonic lines in the plane of symmetry for $\delta = 3.07$ and $\alpha = 0.1745$ (10°). The experimental results are plotted as points. On the windward side, the change in pressure in the transonic region becomes sharper, and inflection points appear on the lines of intersection of the sonic surface with meridional planes, as in axisymmetric flow past more blunt bodies.

Up to $\alpha = 0.1745$, in the cases investigated, the dependence of the corresponding parameters on the angle of attack can be approximated linearly with an accuracy of up to 1-2%.

The estimates carried out show that the maximum relative error of the results is of the order of 1%.

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Note: Figure translations are in progress. See original paper for figures.