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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

THEORY OF ELASTICITY

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ON THE CRITICAL EXTERNAL PRESSURE ON A CONVEX SHALLOW SHELL

In the present note we consider the question of the loss of stability of a shallow strictly convex shell under the action of a uniform external pressure. It is known that, at a certain value of the pressure p_e, such a shell loses stability and begins to buckle. We wish to determine the magnitude of this critical pressure.

1. At the moment when the shell loses stability, the load perceived by it is stationary and practically does not change under appreciable buckling. Under considerable buckling it is admissible to apply the general considerations by means of which essentially supercritical deformations are investigated in work ([1]). In the present case these considerations reduce to the following. We assume that the form of the deformed shell with buckling is naturally divided into three regions (Fig. 1). The region G_1, adjacent to the edge of the shell, is close to the initial form; the region G_2 is close to the initial form shifted by the amount h determined by the buckling; the narrow annular region G_{12} separates G_1 and G_2. The shell undergoes the essential deformations–bending and the accompanying stretching (compression) of the middle surface–in the region G_{12}, where the deformation energy is mainly concentrated.

Fig. 1

2. By a construction similar to that used in work ([1]), for the deformation energy per unit length of a curve γ, conventionally separating the regions G_1 and G_2, one obtains the expression

$$\overline{U} = \frac{\delta E}{2(1-\mu^2)} \int_{-\varepsilon^*}^{\varepsilon^*} \left(\frac{\delta^2 v''^2}{12} + \frac{u^2}{\rho^2} \right) ds.$$

Here u is the displacement of a point of the surface in the tangent plane; v is the displacement along the normal; ρ is the radius of curvature of the curve γ; δ

is the thickness of the shell; E is the modulus of elasticity; μ is Poisson' s ratio. The integration is carried out over the conditional width $2\varepsilon^*$ of the region G_{12} of essential deformations.

We determine the form of the shell in the transition zone G_{12} from the condition of a minimum of the functional $\overline{U}$ for a given total deformation determined by the deflection h over the buckling region G_2. Under known conditions relating to the dimensions of the buckling region, and for small shell thickness, the solution of the variational problem for the functional $\overline{U}$ leads to the following expression for the deformation energy per unit length of γ:

$$\overline{U} = \frac{2E\delta^2\alpha^2 h}{\sqrt{12}(1-\mu^2)\rho},$$

where α is the angle between the osculating plane of the curve γ and the tangent planes of the surface. The same expression for the deformation energy is also obtained under a more general assumption concerning the character of the buckling, when the deflection h is not constant over the region G_2. Then h is the change

deflection in passing from the region G_1 to G_2 at the given point of the contour γ. We draw attention to this because we intend to use this result in solving other problems connected with the stability of shells.

3. With sufficient regularity of the shape of the shell and a small region of bulging, under the assumptions of item 1 concerning the character of the supercritical deformation, the region G_2 may be regarded as similar to the indicatrix of curvature of the surface at the center of bulging. In this case the total strain energy, without the work done, is found by integrating $\overline{U}$ along the arc of the curve γ, and for it one obtains the expression

$$U = \frac{4\pi E\delta^2 h\lambda^2}{\sqrt{12}\,(1-\mu^2)\,\sqrt{R_1 R_2}}.$$

Here R_1 and R_2 are the principal radii of curvature of the shell at the center of bulging; λ is a parameter characterizing the dimensions of the region of bulging, while the remaining quantities have their previous meaning.

4. We determine the magnitude of the critical pressure p from the condition of equilibrium of the shell during bulging,

$$d(U-A) = 0,$$

where U is the strain energy of the shell; A is the work performed by the external pressure; differentiation is carried out with respect to the variable $h\lambda^2$, characterizing the bulging.

The work A performed by the external pressure is measured by the product of the pressure p and the magnitude of the change in the volume bounded by the shell. For it one obtains the expression

$$A = \pi p \sqrt{R_1 R_2}\, h \lambda^2.$$

Substituting the values of U and A into the equilibrium condition of the shell, we find the critical pressure

$$p_e = \frac{2E\delta^2}{\sqrt{3}\,(1-\mu^2)\,R_1 R_2}.$$

5. The method described was applied to determine the external critical pressure on a three-layer shell. In this case the formula obtained was

$$p_e = \frac{2E\delta^2}{\sqrt{3}\,(1-\mu^2)\,R_1 R_2} + 2Gt\,\frac{R_1 + R_2}{R_1 R_2}.$$

Here δ is the thickness of the outer layers; t is the thickness of the filler; E is the modulus of elasticity of the outer layers; G is the shear modulus of the filler.

Remark. The results obtained pertain to strictly convex shells. Strict convexity usually presupposes positivity of the normal curvature. However, in the present consideration this condition must be strengthened by requiring that the maximum and minimum of the normal curvatures $1/R_1$ and $1/R_2$ be of the same order.

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Note: Figure translations are in progress. See original paper for figures.