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Abstract

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ON THE PHYSICAL NATURE OF STEADY-STATE MOTIONS IN NONLINEAR PULSE SYSTEMS

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1. In pulse systems ([1]), periodic motions are defined not on a continuum of frequencies

$$\bar{\omega} > 0, \quad \text{real}, \tag{1}$$

as is the case in ordinary, continuous systems, but on a countable set, generally speaking, of arbitrarily chosen frequencies

$$\bar{\omega} = 2\pi/N, \tag{2}$$

where

$$N \geq 2, \quad \text{integer}, \tag{3}$$

is the relative period of the motion. Arguing formally (see, for example, ([2]), p. 288), to the set (2), under condition (3), there can correspond no sufficiently broad class of motions, and, consequently, it would seem that there are grounds to believe that in nonlinear pulse systems proper periodic motions are an exceptional phenomenon.

Meanwhile, periodic, in particular simple periodic, motions are observed in nonlinear pulse systems comparatively often. The question arises: how can this fact be reconciled with the formal arguments, contradicting it, mentioned above?

The method developed on the basis of ([3]), ([4,5]), for determining steady-state motions in pulse systems has made it possible to find an answer to the question posed. The results obtained serve, in a certain sense, as a substantiation of the legitimacy of the widespread attempts to estimate the steady-state motions of pulse systems by determining simple periodic motions, and give, at least, a rough idea of the limits of the expedient use of such an approach.

In what follows the notation of ([4,5]) is retained.

2. Let us consider a nonlinear pulse system whose steady-state motions admit an approximation by the predominant harmonic ([5]). Such motions are approximately determined, similarly to the determination of simple periodic motions by the harmonic-balance method ([3]), with the aid of the frequency characteristic $K(j\bar{\omega})$ of the linear part (LP) and the equivalent complex gain coefficient, with respect to the predominant harmonic, $J^*_{M_1}$ of the nonlinear pulse element (NPE).

The equivalent gain coefficient $J^*_{M_1}$, like the frequency characteristic $K(j\bar{\omega})$, is a function of frequency ([1]). Accordingly, the frequency of the predominant harmonic is determined not only by the form and scale of the frequency characteristic of the LP, but also by the **frequency properties of the NPE**.

3. The dependence of the quantity $J^*_{M_1}$ on frequency

$$J^*_{M_1} = J^*_{M_1}(\bar{\omega} \mid C,\ \psi,\ F,\ \Phi_\gamma,\ T_0) \tag{4}$$

is very complicated (see ([4,5])), and its analytical investigation is difficult. Direct computations of this dependence (performed on the "Ural" digital computer for a series of typical pulse ele–

···of elements of real automatic systems) show that the equivalent complex gain coefficient at the predominant harmonic of a nonlinear pulse element is a peculiar complex function of frequency. The form of this function depends substantially on the amplitude and phase of the input signal, on the form of the modulation characteristics, and on the pulse shape, and does not depend on the magnitude of the repetition period. Except for certain ("nonrough") cases, at amplitudes exceeding a certain level (whose value depends on the phase of the input signal and on the parameters of the element), the modulus of the equivalent complex gain coefficient at the predominant harmonic of pulse elements of real automatic systems has a clearly expressed resonance character, with an even number of "ideal" narrow-band peaks at frequencies $\bar{\omega} = 2\pi/N$. In other words, with increasing amplitude the pulse element reveals the properties of a high-Q, multiresonance oscillatory system with resonant frequencies (2). In the general case, the relative magnitude of the resonance peaks increases with increasing frequency, amplitude, and absolute value of the phase of the input signal and, all other conditions being equal, is the larger the richer the spectrum generated by the nonlinear transformation proper. The growth of the peak magnitudes with frequency is nonmonotonic: adjacent peaks corresponding to frequencies with an even relative period are, all other conditions being equal, larger than the peaks at frequencies with an odd period. As the frequency decreases, the peak magnitudes and the difference between the peak magnitudes for even and odd periods decrease, tending to zero.

Fig. 1

For sufficiently large amplitudes, the majorant and minorant of the sequence of

Fig. 1

Figure 1: Fig. 1

resonance peaks are characterized by the dependences

$$O_{\max}(\bar{\omega}) = \frac{2}{\pi}\frac{K_n}{T_0 C}\frac{\bar{\omega}}{\sin\bar{\omega}/2}\left|\Phi_\gamma(j\omega)\right|, \tag{5}$$

$$O_{\min}(\bar{\omega}) = \frac{4}{\pi}\frac{K_n}{T_0 C}\left|\Phi_\gamma(j\omega)\right| \tag{6}$$

respectively (Fig. 1).

4. The resonant nature of nonlinear pulse transformation explains the origin in pulse systems of seemingly quite exceptional simple periodic motions with frequencies $\bar{\omega} = 2\pi/N$. It follows from the preceding that the existence of such motions, all other conditions being equal, is the more probable (and, correspondingly, the existence of complex steady-state motions there is less probable), the higher the gain coefficient of the open loop of the system, the "stronger" the nonlinearity of the nonlinear pulse element, the smaller the oscillatory character of the linear part, and the smaller the principal time constant of the latter in comparison with the repetition period of the pulse element. At the same time, motions with an even relative period are, generally speaking, more probable than motions with an odd period. The comparative generality, for real systems, of conditions favoring the existence of simple periodic motions makes it possible to speak of a certain typicality of these motions in pulse systems. In a computational sense, the indicated conditions determine physical factors favorable to the determination of steady-state motions of pulse systems by means of methods for determining simple periodic motions.

5. The distinctive character of the physical nature of the intrinsic steady-state motions of pulse systems testifies to the complexity of the phenomena in these systems and to the necessity of a cautious approach to their formal analytical investigation. The discovered properties of the SIE confirm the expediency of studying the steady-state motions of pulse systems by determining simple periodic motions.

In light of the results obtained, the semi-intuitive formal arguments [2] concerning the peculiarities of the existence of periodic motions in nonlinear pulse systems cannot be regarded as satisfactory, objectively reflecting the physical essence of the phenomenon.

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CITED LITERATURE

1. Ya. Z. Tsypkin, *Theory of Pulse Systems*, 1958.

2. R. E. Kalman, Proc. Symposium on Nonlinear Circuit Analysis, **6**, 273 (1956).

3. L. S. Goldfarb, *Automation and Telemechanics*, **8**, 5, 349 (1947).

4. M. M. Simkin, DAN, **131**, No. 6, 1323 (1960).

5. M. M. Simkin, DAN, **149**, No. 3, 586 (1963).

Note: Figure translations are in progress. See original paper for figures.