ON THE CALCULATION OF THE BOUNDARY LAYER OF A COMPRESSIBLE FLUID WITH SLIP BOUNDARY CONDITIONS

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Abstract

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ON THE CALCULATION OF THE BOUNDARY LAYER OF A COMPRESSIBLE FLUID WITH SLIP BOUNDARY CONDITIONS

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In a number of works ([1-3] and others), investigations were carried out of the influence of wall jumps in velocity and temperature on the flow of a compressible fluid in the boundary layer on flat and axisymmetric bodies under conditions when the influence of the interaction of the boundary layer with the inviscid flow, the influence of transverse curvature, etc., may be considered independently of slip effects. In other words, the classical boundary-layer problem was solved with modified boundary conditions. In doing so, a linearization of the solutions with respect to known self-similar solutions of boundary-layer theory with no-slip boundary conditions was used, with respect to a small parameter entering into the slip boundary conditions.

Below it will be shown that, under certain assumptions regarding the boundary condition for the temperature, such a problem is very simply solved for an arbitrarily prescribed value of the pressure gradient and arbitrary laws of dependence of the Prandtl number Pr and of the product $N = \rho\mu$ on the enthalpy h. Here ρ is the density, μ is the viscosity coefficient.

Let the axis x_1 be directed along the surface of a flat or axisymmetric body, and let the axis y be measured from the surface of the body along the normal to it.

Instead of these variables, introduce Dorodnitsyn' s variables

$$x = \int_0^{x_1} \rho(x_1)\, dx_1, \qquad \eta = \int_0^{y} \rho\, dy.$$

Then the slip boundary conditions take the form

$$u_w \equiv u(x,0) = \beta \left.\frac{\partial u}{\partial \eta}\right|_{\eta=0}, \qquad v(x,0) = 0,$$

$$\Delta h \equiv h(x,0) - h_w = C\beta \left.\frac{\partial h}{\partial \eta}\right|_{\eta=0}, \tag{1}$$

$$\beta = \sqrt{\frac{\pi\chi}{2}}\,\frac{\mu_w}{a_w}\,\frac{2-\sigma}{\sigma}, \qquad C = \frac{2\chi}{\chi+1}\,\frac{2-\alpha}{\alpha\,\mathrm{Pr}}\,\frac{\sigma}{2-\sigma}.$$

Here σ, α are the coefficients of reflection and accommodation, respectively; χ is the ratio of specific heats; a is the speed of sound; the velocity components u, v are directed along the axes x, y.

We construct the solution of the problem, assuming that β does not depend on x and (as is often done [2]) $C = 1$. Let $u_0(x, \eta), v_0(x, \eta), h_0(x, \eta)$ be a solution of the boundary-layer equations satisfying the no-slip conditions at $\eta = 0$:

$$u_0(x, 0) = 0, \qquad v_0(x, 0) = 0, \qquad h_0(x, 0) = h_w. \tag{2}$$

In view of the fact that the boundary-layer equations are not changed under the replacement of the variable η by $\eta + \beta$, the functions

$$u(x, \eta) = u_0(x, \eta + \beta), \qquad v(x, \eta) = v_0(x, \eta + \beta), \qquad h(x, \eta) = h_0(x, \eta + \beta)$$

give a solution of the boundary-layer equations satisfying the no-slip conditions at $\eta = -\beta$. It is easy to see that at $\eta = 0$ these functions satisfy conditions (1), i.e., they give a solution of the problem posed.

To the article by G. A. Razuvaev, K. S. Minsker, Yu. A. Sangalov, p. 1298

Fig. 1. Electron diffraction and X-ray diffraction patterns of polyolefin and polyvinyl monomers. X-ray diffraction patterns of polymethyl methacrylate (*a*), polystyrene (*b*), polyethylene (*v*). Electron diffraction pattern of polypropylene (*g*), polyvinyl chloride (*d*),

Indeed, expanding $u_0(x, \eta + \beta)$, $v_0(x, \eta + \beta)$, $h_0(x, \eta + \beta)$ in a series in β and neglecting quantities of order β^2, we obtain the relations

$$u(x, \eta) = u_0(x, \eta) + \beta \frac{\partial u_0(x, \eta)}{\partial \eta}, \qquad v(x, \eta) = v_0(x, \eta) + \beta \frac{\partial v_0(x, \eta)}{\partial \eta},$$

$$h(x, \eta) = h_0(x, \eta) + \beta \frac{\partial h_0(x, \eta)}{\partial \eta}, \tag{3}$$

which, at $\eta = 0$, taking (2) into account, reduce to the relations (1). (It follows from the continuity equation that $\partial v_0(x, \eta)/\partial \eta = 0$ at the wall.) The boundary conditions at $\eta = \infty$ are also satisfied.

Let us note that in the case of a thermally insulated surface, when $\partial h_0/\partial \eta = 0$ at $\eta = 0$, the solution does not depend on C. Consequently, within the range of applicability of boundary-layer theory with slip boundary conditions, the solution (3) obtained by us in this case will be exact, provided, of course, that the requirement that β be independent of x is satisfied. As is known, this requirement is satisfied under the following conditions: either for $\mathrm{Pr} = 1$ and an arbitrary value of the pressure gradient, or for $dp/dx = 0$ and an arbitrary value of Pr, or for $u_w \sim \sqrt{T_w}$ in the case of an ideal gas, etc.

With the aid of the solution (3) it is easy to draw conclusions about the influence of u_w, Δh on boundary-layer separation, on the wall shear stress τ, on the heat flux from the gas to the wall q, etc. Using the momentum and energy equations, we obtain:

$$\tau \equiv N \frac{\partial u}{\partial \eta} = N_0 \frac{\partial u_0}{\partial \eta} + \frac{\beta}{\rho_0} \frac{\partial p}{\partial x_1}, \qquad q = \frac{N}{\mathrm{Pr}} \frac{\partial h}{\partial \eta} + u\tau = \frac{N_0}{\mathrm{Pr}_0} \frac{\partial h_0}{\partial \eta}.$$

Here $u_0 = u_0(x, \eta)$, etc.; N and Pr depend arbitrarily on h. In the formula for q, the term $u\tau$ characterizes the work of the friction force in the presence of the slip velocity (3). Thus, u_w and Δh change τ if $dp/dx \neq 0$, but do not affect q.

It is necessary to note that the method proposed here is a generalization of a method known from the theory of incompressible viscous flows, where the effect of u_w is taken into account by an analogous displacement of the origin of the coordinate y by an amount proportional to the mean free path of the gas molecules.

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Note: Figure translations are in progress. See original paper for figures.