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Abstract

Full Text

THEORY OF ELASTICITY

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On a Refinement of the Hypothesis of Plane Reflection

The exact solution of problems of the interaction of acoustic shock waves with elastic structures is associated with great computational difficulties. It has been possible to obtain exact results relatively simply only for estimating the overall displacement of a body as a rigid whole ([1]). To solve the problem of the interaction of an acoustic wave with an elastic infinitely long cylindrical shell, the authors of ([2]) introduced the hypothesis of plane reflection. This hypothesis has also been used in a number of other works. A more exact solution of the problem of the interaction of a wave with a cylindrical shell was obtained in ([3]). A refinement of the previously used hypothesis of plane reflection was found in ([3]) with the aid of asymptotic representations of Bessel functions. In the present note it is shown that this refinement is of a general nature and is a correction that takes into account the local mean curvature of the surface of the body.

Let some body be immersed in an elastic medium. We refer the points of the medium to a system of orthogonal curvilinear coordinates α, β, γ, associated with the surface of the body. The coordinate lines α, β coincide with the lines of principal curvature, and the coordinate γ is measured along the outward normal to the surface. The equation of motion of the medium surrounding the body can be written in the form

$$\frac{1}{H_1 H_2 H_3}\left[\frac{\partial}{\partial\alpha}\left(\frac{H_2 H_3}{H_1}\frac{\partial}{\partial\alpha}\right)+\frac{\partial}{\partial\beta}\left(\frac{H_3 H_1}{H_2}\frac{\partial}{\partial\beta}\right)+\frac{\partial}{\partial\gamma}\left(\frac{H_1 H_2}{H_3}\frac{\partial}{\partial\gamma}\right)\right]\psi=\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}, \qquad \gamma>0. \tag{1}$$

Here H_j are the Lamé parameters, ψ is the velocity potential of the radiated waves, c is the speed of sound, and t is time.

In the coordinate system under consideration we have

$$H_1 = A(1+k_1\gamma), \qquad H_2 = B(1+k_2\gamma), \qquad H_3 = 1,$$

where k_1, k_2, A, B are, respectively, the principal curvatures and the Lamé parameters of the surface.

Considering the motion of the medium in a thin layer near the surface of the body, we assume γ to be small. Equation (1) takes the form

$$\nabla^2\psi + \frac{\partial^2\psi}{\partial\gamma^2} + (k_1 + k_2)\frac{\partial\psi}{\partial\gamma} = \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}, \tag{2}$$

where

$$\nabla^2 = \frac{1}{AB}\left[\frac{\partial}{\partial\alpha}\left(\frac{B}{A}\frac{\partial}{\partial\alpha}\right) + \frac{\partial}{\partial\beta}\left(\frac{A}{B}\frac{\partial}{\partial\beta}\right)\right].$$

Suppose that the velocity field of the points of the surface $V(\alpha,\beta,t)$ in the normal directions is prescribed, and that at the initial instant of time the surface was at rest: $V(\alpha,\beta,0) = 0$. From the condition of compatible motion of the elements of the surface of the body and of the medium we have

$$\frac{\partial\psi}{\partial\gamma} = V(\alpha,\beta,t), \qquad \gamma = 0. \tag{3}$$

Let us establish a relation between the prescribed velocity field $V(\alpha,\beta,t)$ and the pressure of the medium on the surface. Considering the initial moments of motion, we shall assume that, in the layer of fluid near the surface of the body, the change of velocities along the coordinate lines α and β is considerably smaller than in the direction of the normal to the surface. Then in equation (2) one may omit the term $\nabla^2\psi$. We obtain

$$\frac{\partial^2\psi}{\partial\gamma^2} + (k_1 + k_2)\frac{\partial\psi}{\partial\gamma} = \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}. \tag{4}$$

We shall obtain the solution of equation (4) using the Laplace transform with respect to t.

Taking into account the initial condition and the radiation condition, we find

$$L(\psi) = -\frac{1}{s}e^{-s\gamma}L(V), \qquad s = k + \sqrt{k^2 + p^2/c^2}. \tag{5}$$

Here

$$L(f) = \int_0^\infty f(t)e^{-pt}\,dt,$$

and k is the mean curvature

$$k = (k_1 + k_2)/2. \tag{6}$$

For the pressure on the surface at the point α, β we have

$$P = -\rho \left[\frac{\partial \psi}{\partial t}\right]_{\gamma=0}. \tag{7}$$

For the initial moments of time, let us consider the behavior of the solution (5) in a neighborhood of $p = \infty$. Taking (7) into account, we have

$$P = \rho c \left[V - kc \int_0^t V\,dt\right]. \tag{8}$$

This relation, represented in terms of the velocity potential ψ, has the form

$$-\frac{\partial \psi}{\partial t} = c\left[\frac{\partial \psi}{\partial \gamma} - kc\int_0^t \frac{\partial \psi}{\partial \gamma}\,dt\right] \quad \text{for } \gamma = 0. \tag{9}$$

In the case of radiation of acoustic waves by a cylinder we have $k = 1/2R$, where R is the radius of the cylinder. Formula (9) in this case coincides with the formula obtained in paper ([3]). For a sphere, in formula (9) one should put $k = 1/R$ (R is the radius of the sphere). For $k = 0$ we obtain the exact relation for waves radiated by an infinite plate, which was used in paper ([2]) for an approximate calculation of a cylindrical shell. Thus, relation (9) is a generalization of the plane-reflection hypothesis and contains an additional term that takes into account the local mean curvature of the radiating surface.

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Note: Figure translations are in progress. See original paper for figures.