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Abstract

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ASTRONOMY

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ESTIMATE OF THE MASS OF A SUPERSTAR

An analysis of observational data on superstars of the 3C 273 type shows ([1]) that the continuous spectrum of these objects in the optical region is emitted by a central body with dimensions of the order of $2 \cdot 10^{16}$ cm, while the emission lines arise in an outer envelope with dimensions of several parsecs or more. The generation of the luminous flux probably takes place in the innermost central region of the central body, which we shall call the core. We shall conventionally call the plasma surrounding the core the atmosphere. To estimate the mass of a superstar, let us consider the forces acting on the plasma of the atmosphere. Suppose that the force of gravity is balanced by the force of light pressure. This assumption, a certain justification for which will be given below, leads, as we shall show, to a value of the mass of the superstar of the order of $10^8 M_\odot$.

The initial quantity in this case is the total radiation flux of the object. For 3C 273, according to ([2]), the energy flux at the surface of the Earth is $q_e = 3.5 \cdot 10^{25}$ erg/cm$^2 \cdot$ sec. The redshift $\lambda_1/\lambda_2 = 1.158$, with $H = 100$ km/sec $\cdot$ Mpc, corresponds to a distance $R = 5 \cdot 10^8$ Mpc, which gives the total flux of the object *

$$Q = 4\pi R^2 q_e = 2 \cdot 10^{46} \text{ erg/sec.} \tag{1}$$

For other objects the total flux has the same order of magnitude.

At a distance r from the center the light flux is $q = Q/4\pi r^2$. In the plasma, Compton scattering of quanta by electrons occurs. In this case the mean (in time) force acting on one electron is

$$\overline{F}_1 = \sigma q / c, \tag{2}$$

where the cross section $\sigma = 6.7 \cdot 10^{-25}$ cm^2.

Let us make the following obvious remarks.

This expression does not depend on the frequency of the quanta so long as $\hbar\omega \ll mc^2$, and does not depend on the angular distribution of the quanta. Indeed, expression (2) can be obtained by considering one electron on which quanta from a point central source are incident. Let the density of quanta be denoted by n_ω. The energy flux is $q = n_\omega c\hbar\omega$, the number of scattering events per unit time is equal to $\nu = n_\omega c\sigma$, in each event a quantum gives up its momentum $p = \hbar\omega/c$, the mean momentum of the scattered quantum is zero, and the force $F_1 = p\nu$, whence (2) follows.

Let us consider the opposite case of radiative heat conduction in a medium whose optical thickness is greater than unity. In this case the flux is

$$q = -D\, d\varepsilon/dr,$$

where ε is the density of radiant energy, and the diffusion coefficient is

$$D = {}^1/_3\, lc = {}^1/_3\, c/n_e\sigma,$$

where n_e is the electron density.

The force acting on one electron is

$$F_1 = -\frac{1}{n_e}\frac{dP}{dr}.$$

Substituting, for nearly isotropic radiation, $P = \varepsilon/3$, we again obtain expression (2). In a strongly ionized plasma, Compton scattering

* Cosmological corrections in this case are small, but they are significant for distant objects.

is the main process responsible for radiative friction*. By virtue of electroneutrality, per electron there is a mass μ/A, $A = 6 \cdot 10^{23}$.

Equating the force of gravity and the force of radiative friction per electron,

$$GM\mu/Ar^2 = \sigma q/c = \sigma Q/4\pi cr^2,$$

we obtain the mass of the body $M = 3 \cdot 10^{41}$ g $= 10^8 M_\odot$ (for $\mu = 1$).

In essence we have used the usual equation of hydrostatic equilibrium for a star in which the pressure of light exceeds the pressure of matter. As is clear from the foregoing, for its applicability it is not in fact necessary to deal with a large optical thickness and an almost isotropic radiation field, for which Pascal' s law holds. Furthermore, it is quite unnecessary that the star as a whole be in a state of hydrostatic equilibrium. In the approximation in which the material pressure of the matter can be neglected, the plasma is in a state of neutral equilibrium for any distribution of it over the radius $n = n(r)$. The equilibrium condition does not predetermine the density of matter in the vicinity of the center; the condition includes the actually observed energy flux, irrespective of the mechanism generating the flux.

The known fact of variability of the light flux of 3C 273 indicates that an optical thickness of order 1 is reached at a comparatively small distance from the center. For further estimates we shall take the radius to be equal to 1 light-week, i.e. $r = 2 \cdot 10^{16}$ cm. At this distance $q = 4 \cdot 10^{12}$ erg/cm^2·sec, i.e. it corresponds to $T_{\text{eff}} = 1.6 \cdot 10^4$ °K $= 1.5$ eV.

We estimate the density of matter at r_0 from the condition that a substantial increase of the distance gives an optical thickness of order unity:

$$n_e \sigma \Delta r = 1. \tag{3}$$

Substituting $\Delta r = r_0$, we find $n_e = 10^8$ cm^{-3}.

We shall assume that the density of matter changes substantially over a distance of order r. The gradient of the material pressure is of order

$$\frac{dP_m}{dr} \approx \frac{P_m}{r} = n_e kT(1 + 1/\mu)/r.$$

The force referred to one electron is equal to

$$F_g = kT(1 + 1/\mu)/r.$$

Substituting $r = 2 \cdot 10^{16}$ cm and $T_{\text{eff}} = 1.6 \cdot 10^4$, we arrive at the conclusion that this force is a million times smaller than the light pressure. As the radius decreases it grows as $r^{-5/4}$ for $\rho =$ const, or as $r^{-7/4}$ for $\rho \sim r^{-2}$, i.e. more slowly than gravity and radiative friction. We shall show that under such conditions one may neglect other processes of interaction of matter with light (apart from Compton scattering).

Consider a hydrogen plasma. Compton scattering will play the determining role if

$$\sigma n_e / \bar{\sigma}_\phi n_1 > 1, \tag{4}$$

where $\bar{\sigma}_\phi$ is the effective value of the photoionization cross section of the hydrogen atom under the given conditions; n_1 is the concentration of neutral atoms. Since the spectrophotometric temperature ([3],[4]) (see also ([5])) corresponds to the effective one at the distance r_0, for estimating n_e/n_1 one may use the Saha formula**. The effective cross section $\bar{\sigma}_\phi$, by definition, is

$$\bar{\sigma}_\phi = \int_0^\infty q_\nu \sigma_{\phi\nu}\, d\nu \bigg/ \int_0^\infty q_\nu\, d\nu.$$

* We call this force radiative friction in order to emphasize that it is caused by the presence of an energy flux relative to the matter.

** According to the estimate of I. S. Shklovskii ([6]) and the data of ([5]), the spectrum has a nonthermal character; this changes the estimates of n_e/n_1 and $\bar{\sigma}_\phi$. However, the final inequality (5) is satisfied with a large margin, and no reasonable variations of n_e/n_1 and $\bar{\sigma}_\phi$ change the conclusion.

For $kT \ll I$ (I is the ionization potential of hydrogen)

$$\bar{\sigma}_\phi \approx 10^{-18}(I/kT)^3 e^{-I/kT};$$

for $kT \gg I$:

$$\bar{\sigma}_\phi \approx 10^{-18}(I/kT)^3.$$

As a result, condition (4) is rewritten in the form

$$n_e < 10^{12}\Theta^{9/2}\ \mathrm{cm}^{-3}, \tag{5}$$

where Θ is the temperature in electron volts. Allowance for the admixture of heavy elements and for free-free transitions somewhat reduces the numerical coefficient on the right-hand side of inequality (5). We see that for n_e found from condition (3), inequality (5) is satisfied with an enormous margin.

Let us now consider regions with $r < r_0$. The following estimates are illustrative in character and are made under certain assumptions. The inequalities obtained are satisfied with a large margin, and therefore the final conclusion is not in doubt. Here the optical thickness is large and the temperature distribution is determined by the radiation-diffusion formula

$$\frac{d\left(\frac{1}{4}acT^4\right)}{dr} = \frac{3}{4}\frac{\sigma\mu}{A}\rho\frac{\Theta}{4\pi r^2},$$

where a is Stefan' s constant.

In a strictly stationary state, at constant velocity, the mass flux is constant and $\rho \sim r^{-2}$. Let us assume such a density distribution. Then from the diffusion equation it follows that for $r < r_0$, instead of (5), the condition obtained is:

$$n_e < 10^{12}\cdot\Theta_0^{9/2}\left(\frac{r_0}{r}\right)^{27/8}\ \mathrm{cm}^{-3},$$

where Θ is the temperature at r_0.

Since the left-hand side of the inequality is proportional to r^{-2}, this condition is satisfied with a large margin for all $r < r_0$. Consequently, everywhere for $r < 2 \cdot 10^{16}$ cm Compton scattering is the determining process in the interaction of radiation with matter.

Let us now turn to regions with radius greater than a light week. Here the degree of ionization is determined by the formula with the dilution factor of the radiation $W \approx r_0^2/4r^2$:

$$\frac{n_e}{n_1} = \left(\frac{n_e}{n_1}\right)_0 \frac{(n_e)_0 W}{n_e} e^{-\tau}, \tag{6}$$

where $(n_e/n_1)_0$ is the degree of ionization at r_0, and τ is the optical thickness. For the effective cross section $\bar{\sigma}_\phi$ we have $\bar{\sigma}_\phi = \bar{\sigma}_\phi(r_0) \approx 10^{-19}$ cm^2. If for $r > r_0$, $\rho \sim r^{-2}$, then the right-hand side of (6) is constant (τ is small) and condition (4) is satisfied, i.e. here too Compton scattering is predominant. If, however, the plasma distribution is such that, beginning from some r, condition (4) is not satisfied, then light pressure becomes much greater than gravity and the matter is ejected outward. For example, let $n_e = \text{const} \approx 10^5$ cm^{-3}, as follows from Shklovsky' s estimate for the outer envelope (1)*. Then

$$\frac{n_e}{n_1} \approx \frac{2.5 \cdot 10^{44}}{r^2} e^{-\tau}.$$

Substituting this value into (4), we find that at $r \approx 10^{19}$ cm photoionization begins to predominate, and the matter is ejected.

It may be supposed that the source of the radiated energy is accretion, i.e. the fall of matter located in the plasma into a small neighborhood

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* According to (5), $n_e \approx 10^7$ cm^{-3}. In this case the outer envelope cannot be continuous and is a thin hollow sphere with $R = 10^{19}$ cm (6). Our estimates refer to the atmosphere, i.e. to a deeper region, and not to this envelope, and, as emphasized, are illustrative in character.

of the heavy collapsed core ([7]). Let us recall that for $M = 10^8 M_\odot$ the gravitational (Schwarzschild) radius is $3 \cdot 10^{13}$ cm, i.e., it constitutes a small fraction of the assumed outer radius of the plasma. Assuming that during accretion up to 10% of the rest energy of the infalling matter can be radiated, we obtain the rate of consumption of matter necessary to cover the energy losses:

$$\frac{dm}{dt} = \frac{Q}{c^2}\frac{1}{0.1} = 2 \cdot 10^{26}\ \frac{\text{g}}{\text{s}} = \frac{3M_\odot}{\text{yr}}.$$

If the plasma is suspended, supported by radiation pressure, then one can imagine that a decrease in the light flux will lead to a fall of the plasma, i.e., to an increase in accretion. In this case the light flux will again increase, which will reduce the accretion: the system possesses a self-regulating mechanism. This mechanism may lead to oscillations. For a rough estimate of their period one should take the period of revolution of a particle in the field of a mass $10^8 M_\odot$ at a distance r_0, which gives $t \simeq 3$ yr. It is interesting to compare this estimate with the period ~ 3 yr observed in 3C 273 and 3C 48 ([8],[9]). Let us note, finally, that the plasma has an azimuthal instability, one that does not depend on the gravitational interaction of its separate parts.

Let us imagine that $n_e = n_e(r) f(\Theta, \varphi)$ depends on angle. Then the light flux will also turn out to depend on angle, $q = \dfrac{Q}{4\pi r^2}\psi(\Theta, \varphi)$, with ψ being larger where f is smaller. Equilibrium will be disturbed; the plasma will be squeezed out where there is less of it and will fall where there is more of it. Material pressure and magnetic forces tend to even out the plasma.

The main results may be summarized as follows:

1. The assumption that the radiation pressure in the atmosphere of a super-star is balanced by the force of gravity leads to an estimate of the mass of the core of the super-star as $\simeq 10^8 M_\odot$.
2. In the outer parts of the atmosphere, where the light pressure on matter is already determined mainly by photoionization processes rather than by scattering by electrons, the radiation forces increase sharply, the equilibrium is disturbed, and the outer parts of the atmosphere are ejected outward.
3. A possible mechanism for the generation of the energy of a super-star is the accretion of atmospheric matter into the small vicinity of the collapsed core. The amount of infalling matter needed to provide the observed light flux is $\simeq 3M_\odot$/yr. This mechanism possesses self-regulation.

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Note: Figure translations are in progress. See original paper for figures.