GEOPHYSICS

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1964

SovietRxiv

Abstract

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GEOPHYSICS

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STRUCTURE OF THE TEMPERATURE FIELD IN CUMULUS CLOUDS

(Presented by Academician E. K. Fedorov on 24 VI 1964)

1. It is known that the principal source of energy in cumulus clouds is the heat of condensation, whose release leads to the development of intense convective motions inside the clouds. Since these motions occur at sufficiently large Reynolds numbers ($\mathrm{Re} = 10^7$—10^9), the convective flows are strongly turbulent; they produce disturbances of smaller scales, which in turn are transformed into still smaller ones, and so on (until the energy of the finest disturbances is converted directly into heat).

It is obvious that the influence of Archimedean forces should decrease as the scale of the disturbances decreases, and that there exists some size of inhomogeneities L_*, below which the influence of buoyancy forces is insignificant. The value L_* ([1]) depends, in particular, on the dissipation rate of turbulent energy ε and on the analogue ε for the temperature field, N (the measure of inhomogeneities of the temperature field that disappear per unit time due to molecular diffusion). However, while there are some estimates for the values of ε in cumulus clouds ([2]), for N there are no data even on the order of magnitude. The present work is an attempt to estimate the values of N and L_* from measurements of temperature pulsations in cumulus clouds ([3]).

2. As already indicated above, turbulent motions for inhomogeneities of scale $L < L_*$ are determined only by dynamical causes. According to ([1]), the value L_* can be estimated by the method of dimensions:

$$L_* = \beta^{-3/2}\varepsilon^{5/4}N^{-3/4}, \tag{1}$$

where the convection parameter $\beta = g/T$, and the temperature structure function for the indicated scale of inhomogeneities ($r < L_*$) has the form

$$D_T(r) = \overline{[T(r_1) - T(r_1 + r)]^2} = G^2 r^{2/3}, \tag{2}$$

where

Fig. 1. Structure functions of temperature fluctuations in a cloud. 1 —height 750 m; 2 —height 2250 m; the cross determines the value $\lambda^{2/3}$

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Fig. 2. Rate of dissipation of turbulent energy ε in a powerful cumulonimbus cloud according to measurements ([2]) (line) and calculated by formula (7) (points)

Figure 2: Fig. 2. Rate of dissipation of turbulent energy ε in a powerful cumulonimbus cloud according to measurements ([2]) (line) and calculated by formula (7) (points)

$$G^2 = cN\varepsilon^{-1/3}. \tag{3}$$

Thus, the statistical characteristics of the temperature field, as well as the value L_*, are determined by two unknown parameters ε and N (the quantities c ([4]) and β may be regarded as known). Since the order of magnitude of ε in cumulus clouds is known ([2]), with the aid of the structure function (2) and (3) it is not difficult to determine the value of N. In doing so, errors in determining ε (see (3)) do not lead to substantial errors in estimating the value of N.

It is also necessary to note that, in calculating the structure functions, the inertia of the temperature sensor should be taken into account. According to ([5], [6]), when using a sensor with spatial averaging $\lambda = \bar{u}\tau$ ($\bar{u}$ is the mean velocity of motion of the sensor in the flow, τ is the time constant of the sens—

···sensor) the structure function for $\lambda \ll r \ll 20\lambda$ is transformed into the form

$$D_T(r) \simeq c\varepsilon^{-1/3}Nr^{2/3} + c^*\varepsilon^{-1/3}N\lambda^{2/3}, \tag{4}$$

where c^*, like c, is a constant.

3. To calculate the structure functions, oscillograms recording the temperature in a cumulonimbus cloud (with a thickness of approximately 3000 m), obtained when the cloud was crossed by an aircraft at various altitudes for the purpose of studying convective motions in clouds ([3]), were used. The measurements were made on 31 VIII 1955 in the Kiev region during the period 13 h 20 min—14 h 30 min mean local time. The apparatus used, the measurement procedure, the conditions and character of the flight are described in ([3]) (the external appearance of the cloud and samples of oscillograms are given, respectively, in Figs. 71-a and 82).

Fig. 1. Structure functions of temperature fluctuations in a cloud. 1 —height 750 m; 2 —height 2250 m; the cross determines the value $\lambda^{2/3}$.

Fig. 2. Rate of dissipation of turbulent energy ε in a powerful cumulonimbus cloud according to measurements ([2]) (line) and calculated by formula (7) (points).

The structure functions were calculated for heights of 250; 750; 1750; 2250 and 2750 m from the cloud base. In the calculation by (4) it was assumed that $c = 2.4$ ([4]); $c^* = 1.14$ ([6]); $\beta = 0.33$ m/sec$^2 \cdot$ deg; $\bar{u} = 70$ m/sec. An estimate of the time constant of the sensor from experimentally obtained structure functions gives $\tau = 0.03$ sec, which agrees with the data given in ([3]).

With the indicated values of τ, all the calculated structure functions agree well with the "two-thirds law"(4). As an example, Fig. 1 shows structure functions (in coordinates $D_T(r), r^{2/3}$) for heights of 750 and 2250 m from the cloud base. As is seen from the data presented, up to a certain value r_* the structure formulae agree well with formula (4), i.e., for $r < r_*$ the "two-thirds law" is fulfilled.

It should also be noted that the slopes of the straight lines increase substantially with height; at a height of 2250 m the slope is approximately two orders of magnitude greater than at a height of 750 m (Fig. 1).

Table 1

Height above the cloud base, m	G^2	N
250	10^{-5}	$5 \cdot 10^{-7}$
750	$6 \cdot 10^{-4}$	$3 \cdot 10^{-5}$
1750	$8 \cdot 10^{-3}$	$4 \cdot 10^{-4}$
2250	$5 \cdot 10^{-3}$	$3 \cdot 10^{-4}$
2750	$3 \cdot 10^{-4}$	$2 \cdot 10^{-4}$

4. Table 1 gives the values of G^2 and N, calculated for different heights, for $\varepsilon = 500$ cm^2/sec^3. According to the results of measurements ([2]) and others, the values of ε for cumulonimbus clouds range from 200 to 800 cm^2/sec^3.

(Fig. 2); the values of G^2 were determined from measurements of temperature pulsations.

As can be seen from the data in the table, the quantity N varies within considerable limits and, at the same time, increases substantially with height up to approximately one half or two thirds of the cloud depth, thereafter remaining approximately constant. The estimates given for the quantity N are apparently sufficiently objective, since the adopted value of ε enters into (3) to the power 1/3; a change in ε by an order of magnitude leads to a change in N by only approximately a factor of two.

Knowing the order of magnitude of N, one can also estimate the order of magnitude of the turbulent-exchange coefficient K in the cloud for the scales of perturbations under consideration. According to (4),

$$N = K(\text{grad}\, T)^2. \tag{5}$$

Taking the value $\text{grad}\, T \approx 0.7$ deg/100 m, we find that in the cumulonimbus clouds in which the measurements were made, the value of the turbulent-exchange coefficient is of the order of 10^2 near the cloud base to 10^4–10^5 cm^2/sec in the central part.

The structural functions obtained make it possible to estimate not only the quantity N (for known ε), but also the order of magnitude of L_*. An estimate of L_* may be given by that limiting value $r = r_*$, beginning with which a deviation from the "two-thirds law" is observed. Examination of graphs similar to those shown in Fig. 1 has shown that the indicated values of r_* are of the order of 50–100 m. Thus, for inhomogeneities of smaller scale, temperature fluctuations are caused by dynamical factors. This agrees with the estimate of the dimensions of convective formations; the mean sizes of the zones crossed by the aircraft that were warmer than the surrounding cloudy air, which may be interpreted as the horizontal dimensions of ascending convective currents, are in the same cloud 100–250 m, with the dimensions increasing continuously with height.

5. Since the quantity L_* was derived by dimensional analysis, the quantities r_* and L_* cannot be regarded as equal. However, if one assumes, as is done in similarity theory, that they are related by some constant of order unity, i.e., that

$$r_* \approx L_*, \tag{6}$$

then this makes it possible to carry out certain estimates.

Under condition (6), from the experimentally determined parameters of the structural functions r_* and G^2, one can determine ε and N. Using (3) and (1), we obtain:

$$\varepsilon = c^{-3/4}\beta^{3/2}G^{3/2}L_*, \tag{7}$$

$$N = c^{-5/4}\beta^{1/2}G^{5/2}L_*^{1/3}. \tag{8}$$

A check on the validity of assumption (6) may be provided by comparing the values of ε obtained by two different methods: from formula (7) and from measurement data ([2]). The results of the comparison, presented in Fig. 2, show that the values of ε, estimated by the different methods, are close to one another. This makes it possible to regard assumption (6) as justified and, consequently, the use of relations (7) and (8) for estimating the values of ε and N in convective clouds as possible.

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Received
17 VI 1964

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Note: Figure translations are in progress. See original paper for figures.