# CRYSTALLOGRAPHY

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**Abstract**

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# ON THE GENERATION CONDITION FOR A TWO-LAYER CRYSTAL

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In papers ([1], [2]) the conditions for the generation of electromagnetic waves by an ideally homogeneous crystalline layer were considered. In reality, however, the crystals used for quantum generators are not optically homogeneous, but have inhomogeneity in the refractive index and absorption coefficient. In particular, crystals may consist of separate regions oriented arbitrarily with respect to one another. In the case of uniaxial crystals (only these are considered in the present paper), this means that the optical axes of these regions do not coincide in direction. In connection with this, it becomes necessary to consider the conditions for self-excitation (generation) of a plane-parallel layer having optical inhomogeneities. The simplest model of such a layer may be a plane-parallel crystalline plate consisting of two layers of equal thickness $d/2$ (see Fig. 1), but with arbitrary orientation of the optical axis.

**Fig. 1**

To determine the generation conditions it is necessary to solve Maxwell' s equations with account of the boundary relations on the three planes of separation. We shall represent the electromagnetic radiation in the form of plane monochromatic waves

$$\mathbf{E}_i = \mathbf{E}_i^{(0)} e^{i(\omega t - k\mathbf{m}_i\mathbf{r})}, \qquad \mathbf{H}_i = \mathbf{H}_i^{(0)} e^{i(\omega t - k\mathbf{m}_i\mathbf{r})} \tag{1}$$

($i = 1, 2, \dots, 10$, see Fig. 1), where $k$ is the wave number; $\mathbf{m}$ is the refraction vector, equal for homogeneous waves to the product of the wave-normal vector $\mathbf{n}$ and the refractive index $n$, i.e. $\mathbf{m} = n\mathbf{n}$. We assume that no external radiation is incident on the plate. In other words, the system operates in the generation regime, and therefore only radiates energy. In such a system, for an arbitrary orientation of the optical axis in both layers, 10 waves will propagate. The refraction vectors of these waves in our case are equal to:

$$\mathbf{m}_1 = -n\mathbf{q}, \qquad \mathbf{m}_2 = n_o\mathbf{q}, \qquad \mathbf{m}_3 = -n_o\mathbf{q}, \qquad \mathbf{m}_4 = n_e\mathbf{q}, \qquad \mathbf{m}_5 = -n_e\mathbf{q}, \tag{2}$$

$$\mathbf{m}_6 = n_o\mathbf{q}, \qquad \mathbf{m}_7 = -n_o\mathbf{q}, \qquad \mathbf{m}_8 = n'_e\mathbf{q}, \qquad \mathbf{m}_9 = -n'_e\mathbf{q}, \qquad \mathbf{m}_{10} = n\mathbf{q}.$$

Here $\mathbf{q}$ is the unit vector normal to the surface of the plate (see Fig. 1), $n$ is the refractive index of the surrounding medium, $n_o$ is the refractive index of the ordinary wave, and $n_e$, $n'_e$ are the refractive indices of the extraordinary wave of the upper and lower layers.

Using Maxwell' s equations

$$\mathbf{H} = [\mathbf{mE}], \qquad \varepsilon\mathbf{E} = -[\mathbf{mH}] \tag{3}$$

and the relations (2), the boundary conditions for the interfaces can be written in the form of the following system of vector equations:

$$\begin{gathered}
[\mathbf{E}_1 - \mathbf{E}_2 - \mathbf{E}_3 - \mathbf{E}_4 - \mathbf{E}_5, \mathbf{q}] = 0,\\
[n\mathbf{E}_1 + n_o\mathbf{E}_2 - n_o\mathbf{E}_3 + n_e\mathbf{E}_4 - n_e\mathbf{E}_5, \mathbf{q}] = 0,\\
[\mathbf{E}_2 + \mathbf{E}_3 + \mathbf{E}_4 + \mathbf{E}_5 - \mathbf{E}_6 - \mathbf{E}_7 - \mathbf{E}_8 - \mathbf{E}_9, \mathbf{q}] = 0,\\
[n_o\mathbf{E}_2 - n_o\mathbf{E}_3 + n_e\mathbf{E}_4 - n_e\mathbf{E}_5 - n_o\mathbf{E}_6 + n_o\mathbf{E}_7 - n'_e\mathbf{E}_8 + n'_e\mathbf{E}_9, \mathbf{q}] = 0,\\
[\mathbf{E}_6 + \mathbf{E}_7 + \mathbf{E}_8 + \mathbf{E}_9 - \mathbf{E}_{10}, \mathbf{q}] = 0,\\
[n_o\mathbf{E}_6 - n_o\mathbf{E}_7 + n'_e\mathbf{E}_8 - n'_e\mathbf{E}_9 - n\mathbf{E}_{10}, \mathbf{q}] = 0.
\end{gathered} \tag{4}$$

The wave amplitudes $\mathbf{E}_i^{(0)}$, according to (3), shall be represented as follows*:

$$\begin{gathered}
\mathbf{E}_1^{(0)} = \alpha_1[\mathbf{qc}_1] + \alpha_2[\mathbf{q}[\mathbf{qc}_1]], \qquad \mathbf{E}_2^{(0)} = A_2 n_o[\mathbf{qc}_1], \qquad \mathbf{E}_3^{(0)} = A_3 n_o[\mathbf{qc}_1],\\
\mathbf{E}_4^{(0)} = A_4\left(n_o^2 - n_e^2\mathbf{q}\cdot\mathbf{q}\right)\mathbf{c}_1, \qquad \mathbf{E}_5^{(0)} = A_5\left(n_o^2 - n_e^2\mathbf{q}\cdot\mathbf{q}\right)\mathbf{c}_1,\\
\mathbf{E}_6^{(0)} = A_6 n_o[\mathbf{qc}_2], \qquad \mathbf{E}_7^{(0)} = A_7 n_o[\mathbf{qc}_2], \qquad \mathbf{E}_8^{(0)} = A_8\left(n_o^2 - n_{e'}^2\mathbf{q}\cdot\mathbf{q}\right)\mathbf{c}_2,\\
\mathbf{E}_9^{(0)} = A_9\left(n_o^2 - n_{e'}^2\mathbf{q}\cdot\mathbf{q}\right)\mathbf{c}_2, \qquad \mathbf{E}_{10}^{(0)} = \beta_1[\mathbf{qc}_2] + \beta_2[\mathbf{q}[\mathbf{qc}_2]],
\end{gathered} \tag{5}$$

where $\mathbf{c}_1$ and $\mathbf{c}_2$ are the vectors of the optical axes of the upper and lower layers, respectively. Substituting further (1) into (4), taking (5) into account and assuming that for the first interface (I) $\mathbf{qr} = -d/2$, for the second (II) $\mathbf{qr} = 0$, and for the third (III) $\mathbf{qr} = d/2$, we obtain a system of 6 vector, or 12 scalar, equations with respect to 12 unknown amplitudes. The requirement that the system have a nonzero solution gives the following condition for self-excitation (generation) of the active layer under consideration:

$$2n_o^2\gamma_1^2(M_1 - M_2)\{n'_e(1 - M_4)(1 + M_2) - n_e(1 + M_4)(1 - M_2)\} -$$

$$-\gamma_2^2\{n_e'(1+M_1)(1-M_4)-n_o(1+M_4)(1-M_1)\}\{n_e(1+M_3)(1-M_2)-$$
$$-n_o(1-M_3)(1+M_2)\}=0, \qquad (6)$$

where

$$\gamma_1=\frac{[\mathbf{qc}_1][\mathbf{qc}_2]}{[\mathbf{qc}_1]^2}, \qquad \gamma_2=\frac{\mathbf{q}[\mathbf{c}_1\mathbf{c}_2]}{[\mathbf{qc}_1]^2}, \qquad (7)$$

$$M_1=\frac{n_o+n}{n_o-n}e^{ikn_{od}}, \qquad M_2=\frac{n_e+n}{n_e-n}e^{ikn_{ed}},$$
$$M_3=\frac{n_o-n}{n_o+n}e^{-ikn_{od}}, \qquad M_4=\frac{n_e'-n}{n_e'+n}e^{-ikn_{ed}'}. \qquad (8)$$

From relations (6)–(8) we see that the generation condition for a double active layer is considerably more complicated than for a homogeneous one (see (1)). Since it contains a comparatively large number of parameters related to one another, it is evidently somewhat more difficult to achieve its fulfillment. At the same time, however, it should be noted that the parameter $\gamma_2$, closely connected with the orientation of both layers, enters the generation condition (6) quadratically. This means that, for small angles between the optical axes $\mathbf{c}_1$, $\mathbf{c}_2$, one may neglect in (6) the terms proportional to $\gamma_2$, and the generation condition will differ very little from that in the case of a homogeneous crystal. More precisely, small deviations from the homogeneity of the crystal should not affect the generation of the ordinary wave, while the generation condition for the extraordinary wave undergoes certain changes (since $n_e' \neq n_e$). Therefore it is always easier to obtain generation of the ordinary wave than of the extraordinary one, which is in fact observed in experiments (see, for example, (4,5)).

Let us now consider relation (6) for some particular orientations of the optical axes.

* It is assumed that the normal $\mathbf{q}$ is not parallel to $\mathbf{c}_1$, $\mathbf{c}_2$. In the case where $\mathbf{q} \parallel \mathbf{c}_1$ or $\mathbf{q} \parallel \mathbf{c}_2$, the corresponding layer behaves as an isotropic medium.

1) $\mathbf{c}_1 \parallel \mathbf{c}_2$, i.e., the orientation of the axes coincides, which is equivalent to homogeneity of the plate. In this case $\gamma_1 = 1$, $\gamma_2 = 0$. From (6) it follows that $(M_1 - M_3)(M_2 - M_4) = 0$, or, according to (8), we obtain

$$\left(\frac{n_o-n}{n_o+n}\right)^2 \exp\{-2ikn_od\}=1, \qquad \left(\frac{n_e-n}{n_e+n}\right)^2 e^{-2ikn_ed}=1. \qquad (9)$$

Relations (9) are the usual self-excitation conditions for the ordinary and extraordinary waves.

2) $\mathbf{q}[\mathbf{c}_1\mathbf{c}_2] = 0$, i.e., the optical axes of both layers and the normal lie in one plane. In this case $\gamma_2 = 0$, $\gamma_1 \neq 0$. From (6), (8) we obtain

$$\left(\frac{n_o - n}{n_o + n}\right)^2 e^{-2ikn_o d} = 1, \tag{10}$$

$$n_e'(1 - M_4)(1 + M_2) - n_e(1 + M_4)(1 - M_2) = 0. \tag{11}$$

Since relation (10) is the usual generation condition for the ordinary wave, this means that for it the generation condition does not depend on the presence of inhomogeneity. This is quite understandable, since the refractive index of this wave does not change in passing from one layer to the other. Relation (11), which determines the generation condition for the extraordinary wave, is more complicated. If, however, the optical axes $\mathbf{c}_1$ and $\mathbf{c}_2$ are oriented symmetrically with respect to the normal $\mathbf{q}$, then $n_e' = n_e$ and (11) reduces to the usual generation condition for the extraordinary wave. Thus, in this case the inhomogeneity of the crystal plate is not manifested at all (for waves propagating normally to its surface), and the result does not depend on the magnitude of the angle between the axes $\mathbf{c}_1$ and $\mathbf{c}_2$.

3) $\mathbf{c}_1\mathbf{q} = 0$, $\mathbf{c}_2\mathbf{q} = 0$. The optical axes of both layers are parallel to the plane of the plate. From (7) it follows that $\gamma_1^2 + \gamma_2^2 = 1$, $\gamma_1 = \mathbf{c}_1\mathbf{c}_2$. In this case the generation condition (6) is simplified only slightly. This means that the inhomogeneity of the crystal plate has a more substantial effect than in the preceding cases. Relation (6) is simplified somewhat if the orientation of the layers is mutually perpendicular, i.e., if $\mathbf{c}_1 \perp \mathbf{c}_2$. In this case $\gamma_1 = 0$, $\gamma_2 = 1$, and instead of (6) we have

$$\begin{aligned} n_e(1 + M_1)(1 - M_4) - n_o(1 + M_4)(1 - M_1) &= 0, \\ n_e(1 + M_3)(1 - M_2) - n_o(1 + M_2)(1 - M_3) &= 0. \end{aligned} \tag{12}$$

From (12) we see that in this case too the inhomogeneity of the crystal plate has a very substantial effect. It is interesting to note that, when either of the conditions (12) is satisfied, both types of waves will be generated at once.

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*Note: Figure translations are in progress. See original paper for figures.*