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Abstract

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On the Strength of Model Specimens of Unidirectional Structures

The strength of such a heterogeneous system as fiberglass is determined to a considerable extent by the nature of the distribution of stresses among the individual elements of its structure. Attempts at an experimental study of this question have been undertaken by a number of authors ([1–4]). Some of them used an optical method to obtain a qualitative picture of the stress distribution ([1–3]); others tried to use model specimens to determine stresses by a purely mechanical route ([5,6]). None of these works makes it possible to indicate a method for evaluating the influence of various physicochemical parameters of the components of a heterogeneous material on its strength.

The present work contains an attempt to evaluate the stress distribution in a certain model, with the aim of identifying the principal factors determining the strength of a heterogeneous material of the oriented-fiberglass type*.

For the experimental investigation of the stressed state, a polarization-optical method was used. A qualitative picture was obtained by cinematographic filming ([7]) of the specimen during loading up to failure.

The specimens used were cross-linked polymers employed as binders in fiberglass —polyesters and polyepoxide—and had the form of a double "dog-bone" ; along the axis of the specimen there passed a reinforcing element, which was a glass rod with a diameter from 50 to 1000 μ (Fig. 1*a*).

In Fig. 1*b* one can clearly see the concentration of stresses appearing near breaks in the glass rod; the number of these breaks gradually increases as the specimen is stretched (1*b*). In the overwhelming majority of cases the character of failure corresponded to Fig. 1*c*, where the plane of rupture of the polymer does not coincide with the place of failure of the rod; the exposed portion of the glass reinforcing element, protruding beyond the fracture surface of the polymer, is clearly visible.

Measurements showed a high degree of correlation between the length of the protruding portion l_k and the diameter of the rod d_v, as well as a dependence of this length on the type of polymer; the measurement results are given below (Fig. 3).

Figure 1. Frames from a motion picture of the stretching of a specimen made of PN-1 with single reinforcing elements: a —before the destruction of the glass rod, b —the glass rod broke in one place, c —the rod broke in three places, d — the specimen broke; a section of the rod protruding beyond the fracture surface of the lower half of the specimen is visible.

Figure 1: Figure 1. Frames from a motion picture of the stretching of a specimen made of PN-1 with single reinforcing elements: a —before the destruction of the glass rod, b —the glass rod broke in one place, c —the rod broke in three places, d —the specimen broke; a section of the rod protruding beyond the fracture surface of the lower half of the specimen is visible.

As can be seen from Fig. 1, in a small region adjacent to the reinforcing element, a nonuniform field of stresses and deformations arises. The region of high stresses in the polymer has a clearly pronounced local character—they develop in the layer adjoining the glass rod. For this reason, for an analytical description of the distribution of stresses arising at the polymer—glass contact surface, in the very first approximation one may use the solution of the problem of defor-

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* At the initial stage of the work, the experimental physicochemical part of the investigation (V. A. Kargin, Yu. M. Malinskii, B. Yu. Trifel) and the theoretical-calculational part (A. L. Rabinovich) were carried out independently of one another. After mutual familiarization with the results of the work already completed, the further investigation was conducted jointly by the authors of the article.

Fig. 1. Frames from a motion picture of the stretching of a specimen made of PN-1 with single reinforcing elements: **a** —before the destruction of the glass rod, **b** —the glass rod broke in one place, **c** —the rod broke in three places, **d** — the specimen broke; a section of the rod protruding beyond the fracture surface of the lower half of the specimen is visible.

shear deformation of a thin polymer layer located between two elastic ones.

In cross-linked polymers one may approximately neglect residual deformations and take into account only elastic deformations together with highly elastic ones ([8,9]). In this case, as the constitutive equation, the generalized Maxwell equation proposed by G. I. Gurevich ([10]), which has received satisfactory confirmation in the uniaxial case ([9]), is applicable. Here it is used in the form ([11]).

The problem mentioned above is approximately reduced to three equations:

$$\frac{\partial T_2}{\partial x} = -\tau_3, \qquad T_2 + (G_3 l^2/4\nu^2)\,\frac{\partial \varepsilon_3}{\partial x} = (\beta/b)P(t),$$

$$\frac{\partial \varepsilon_3}{\partial t} = \frac{1}{G_3}\frac{\partial \tau_3}{\partial t} + \frac{3}{\eta_3^*} f(\tau, \varepsilon), \tag{1}$$

where x is the abscissa along the rod, l is its length, t is time, T_2 is the force in the elastic element, $P(t)$ is the external load specified as a function of time t, τ_3 and ε_3 are, respectively, the tangential stress and shear strain in the polymer layer near the contact surface, G_3 is the shear modulus of the polymer (Hookean), β and ν are parameters expressible only through the elastic constants and relative dimensions of the transverse sections of the elements or, equivalently, their volume contents. The function $f(\tau, \varepsilon)$ and the viscosity coefficient of the polymer η_3^* have the following expressions:

$$f(\tau, \varepsilon) = \left(1 + G_{\infty,3}/G_3\right)\tau_3 - G_{\infty,3}\varepsilon_3,$$

$$1/\eta_3^* = (1/\eta_{0,3}^*)\exp\left\{\frac{3}{2m_3^*}\left|f(\tau, \varepsilon)\right|\right\}, \tag{2}$$

where, in turn, the notation used is that of the parameters of highly elastic deformation of polymers ([8,9,11,12]): $G_{\infty,3}$ is the modulus of high elasticity in shear, m_3^* is the rate modulus, and $\eta_{0,3}^*$ is the initial-viscosity coefficient. Under isothermal conditions these quantities may be taken as constants; in general, however, they are functions of temperature and, as some preliminary data show, apparently depend substantially on the supramolecular structure of the polymer.

The system of equations (1) makes it possible to determine T_2, τ_3, and ε_3 as functions of (x, t) for given initial and boundary conditions. The former are always taken to be homogeneous; the latter depend on the loading and fixing scheme. If $P(t)$ is specified*, then in the case considered one may take as homogeneous also the boundary conditions for T_2 $(x = \pm l/2)$, as well as for $\tau_3, \varepsilon_3 (x = 0)$.

Owing to the presence of the exponential factor in expression (2) for the viscosity coefficient η_3^*, equations (1) prove to be nonlinear. In the general case, the solution of the boundary-value problem is found numerically by approximating (1) with finite-difference equations. However, in a number of cases, depending on the values of the constants, a sufficient approximation may be obtained on the basis of the linearized equations $(m_3^* \to \infty)$. In this case, from (1) there follows a third-order equation with respect to T_2:

$$\frac{\partial}{\partial t}\left[\frac{\partial^2 T_2}{\partial x^2} - \alpha^2 T_2\right] + a^*\left[\left(1 + \frac{G_3}{G_{\infty,3}}\right)\frac{\partial^2 T_2}{\partial x^2} - \alpha^2 T_2\right] = -\beta\alpha^2\left[\frac{dP}{dt} + a^* P\right](1/b); \tag{3}$$

here $\alpha = 2\nu/l$, $a^* = a(G_{\infty,3}/G_3)$, $a = 3G_3/\eta^*_{0,3}$. In the case of homogeneous boundary conditions for T_2, the solution of (3) is readily found by Fourier' s method:

$$T_2(x,t) = \frac{P_0\beta}{b}\sum\left[\overline{P} - \alpha c_n e^{-c_n^* at}\int_0^t \overline{P}e^{c_n^* at}\,dt\right]c_{0,n}\cos\frac{n\pi x}{l}, \qquad (4)$$

* We note that in the case where the displacement regime of the ends of the elastic elements is prescribed, in

where $\overline{P} = P(t)/P_0$, $c_n = (\pi n/2\nu)^2/[1+(\pi n/2\nu)^2]$, $c_n^* = c_n + (G_{\infty,3}/l_3)$, $c_{0n} = -4\sin(n\pi/2)/\pi n[1+(\pi n/2\nu)^2]$.

The results of solving equations (1) and (3) show that the character of the distribution of stresses along the length of the rod–$\tau_3(x)$ and $T_2(x)$–is determined mainly by three dimensionless parameters: ν, $(G_{\infty,3}/G_3)$, and at_0. In the last of these, t_0 is a certain quantity having the dimension of time and characterizing the loading regime. For example, at a constant rate of load increase this quantity is the reciprocal of the value of the indicated rate, while in creep t_0 is the time over which the load increases to a constant value.

From an analysis of the solutions obtained it follows that, beginning with a certain stiffness of the polymer layer, characterized by the quantity

$$\nu^* = \nu\sqrt{(G_{\infty,3}/G_3)/[1+(G_{\infty,3}/G_3)]}, \qquad (5)$$

irrespective of the conditions, the distribution of shear stresses τ_3 on the contact surface along the length has a sharply expressed character of concentration at the end of the reinforcing element. A peculiar edge effect appears. An approximate allowance for the two-dimensional stress state in the polymer leads to an epure of shear stresses, schematically shown in Fig. 2, which qualitatively corresponds to the isochrome pattern in Fig. 1 , .

The attenuation of the edge effect is determined mainly by the quantity a_0^*:

$$(\alpha_0^*)^2 = (2\nu^* d_B/l)^2 = (G_3/E_2)(d_B/\delta_3)/[1+(G_3/G_{\infty,3})](1-\beta), \qquad (6)$$

in which δ_3 is the thickness of the polymer layer, and E_2 is the modulus of elasticity of the reinforcing element. With increasing a_0^*, the attenuation becomes more rapid. As is seen from (6), a_0^* does not depend on the length l, but only on the physical constants and the dimensions of the cross sections of the elements of the system.

Fig. 2

Fig. 2

Figure 2: Fig. 2

Fig. 3

Figure 3: Fig. 3

As the external load increases, the stresses and strains in the region of the edge effect increase. They may reach values corresponding to the adhesive or cohesive strength of the polymer, or else to the limiting strain of one of the components of the reinforced system, and then failure of the latter occurs.

In this case the dimensions of the failure region, the length of the freed section of the glass rod in the experimental specimens (Fig. 1), naturally depend on the length of the zone of the edge effect and on the rate of its attenuation. Use of the results of the solution of the above problem, taking into account the real conditions of failure, leads to a simple approximate relation between the length of the zone of adhesive failure l_k and the parameters of the system:

$$(l_k/d_\text{B})\alpha_0^* = \ln\kappa \quad \text{or} \quad (l_k/d_\text{B}) = C\sqrt{(\delta_3/d_\text{B})}, \tag{7}$$

where $C = \ln\kappa[(1 + G_3/G_{\infty,3})(E_2/G_3)(1-\beta)]^{1/2}$; here $\kappa = \tau_\text{max}/\tau_k$ is the ratio of τ_max to a certain characteristic stress; this ratio depends, in particular, also on the statistical scatter in the values of adhesive strength. If, as was done in our experiments, d_B is varied while all the other parameters are left unchanged, then according to formula (7) there should be a linear relation between (l_k/d_B) and $\sqrt{(\delta_3/d_\text{B})}$. The experimental points in Fig. 3 are indeed located near a straight line in the indicated coordinates.

The influence of the edge effect described above on strength can also be illustrated by the following facts. The strength of PN-1 specimens in our experiments was $\sim$ 420 kg/cm^2; the strength of similar specimens containing a reinforcing glass rod (with failure proceeding according to the type shown in Fig. 1) was 108–155 kg/cm^2, depending on the rod diameter. The strength of a specimen made of ED-5 epoxy resin decreased upon introduction of a reinforcing element by only 15–20%; in these specimens, fracture of the polymer and of the glass rod occurred almost simultaneously because, in magnitude of elongation at rupture, ED-5 is close to glass. If, however, in preparing a specimen from ED-5, a reinforcing rod previously broken into two parts was embedded in it, the strength of such a specimen was 35% lower than the strength of an unreinforced specimen. In all these cases, the cause of the reduction in strength is undoubtedly the edge effect near the places where the continuity of the reinforcing element is disrupted.

Fig. 3

Thus, in the present work, by means of experimental study and theoretical calculation, it has been shown that near places where the continuity of reinforcing elements is disrupted, in the boundary layer of the binder directly adjacent to the fiber, a sharply pronounced concentration of tangential stresses arises—a zone of edge effect—which can create a site of failure of the binder; approximate expressions are given for the principal parameters determining the character of the distribution of stresses and strains, the magnitude, and the attenuation of the edge effect.

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Note: Figure translations are in progress. See original paper for figures.