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Abstract

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ENERGIES OF OCTUPOLE COLLECTIVE STATES WITH $I\pi K = 1 - 0$ OF EVEN-EVEN STRONGLY DEFORMED NUCLEI

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On the basis of the method of approximate second quantization (^{1}), a number of authors (^{2}) have carried out general studies of the collective properties of atomic nuclei. The greatest progress has been achieved in the region of spherical nuclei, where the energies of collective states and the probabilities of electromagnetic transitions have been calculated ($^{3-5}$). In the region of strongly deformed nuclei the studies (^{6}) are limited: the basic equations are given and the question of excluding the spurious state is studied. Only in (^{7}) were the energies of γ vibrations and the probabilities of $E2$-transitions calculated. In the present work, on the basis of the method of approximate second quantization within the framework of the superconducting model of the nucleus, the energies of octupole collective states with $I\pi K = 1 - 0$ of even-even strongly deformed nuclei in the regions $152 \leq A \leq 186$ and $228 \leq A \leq 254$ have been calculated.

The secular equation determining the frequencies of octupole vibrations has the form

$$\begin{aligned}
1 = {} & 2\chi_n^{(3)} \sum_{ss'} \frac{f^{n^2}(ss')\left(U_{ss'}^n\right)^2}{\varepsilon_n(s) + \varepsilon_n(s') - \dfrac{\omega^2}{\varepsilon_n(s) + \varepsilon_n(s')}} \\
& + 2\chi_p^{(3)} \sum_{\nu\nu'} \frac{f^{p^2}(\nu\nu')\left(U_{\nu\nu'}^p\right)^2}{\varepsilon_p(\nu) + \varepsilon_p(\nu') - \dfrac{\omega^2}{\varepsilon_p(\nu) + \varepsilon_p(\nu')}} \\
& + 4\left(\chi_{np}^{(3)^2} - \chi_n\chi_p\right) \sum_{ss'} \frac{f^{n^2}(ss')\left(U_{ss'}^n\right)^2}{\varepsilon_n(s) + \varepsilon_n(s') - \dfrac{\omega^2}{\varepsilon_n(s) + \varepsilon_n(s')}} \\
& \times \sum_{\nu\nu'} \frac{f^{p^2}(\nu\nu')\left(U_{\nu\nu'}^p\right)^2}{\varepsilon_p(\nu) + \varepsilon_p(\nu') - \dfrac{\omega^2}{\varepsilon_p(\nu) + \varepsilon_p(\nu')}},
\end{aligned} \tag{1}$$

Fig. 1

Figure 1: Fig. 1

where $\chi_n^{(3)}$, $\chi_p^{(3)}$, $\chi_{np}^{(3)}$ are constants of the octupole-octupole interactions; $f(s,s')$ is the matrix element of the octupole-moment operator; the index n refers to the neutron system and p to the proton system; the summation over ss' ($\nu\nu'$) is carried out over the single-particle levels of the mean field of the neutron (proton) system. Pair correlations of the superconducting type are considered as in (^{8}); $\varepsilon(s) = \sqrt{C^2 + \{E(s) - \lambda\}^2}$, $U_{ss'} = u_s v_{s'} + v_s u_{s'}$. The values of the correlation functions C, the chemical potentials λ, and the schemes of single-particle levels of the mean field are taken from (^{9,10}).

The frequencies of octupole vibrations ω are found by numerically solving the secular equation (1) on an electronic computer. The first root ω is sought in the interval

$$0 < \omega < \min_{ss',\,\nu\nu'} \left(\varepsilon(s) + \varepsilon(s'),\ \varepsilon(\nu) + \varepsilon(\nu')\right) \tag{2}$$

by successive bisection of the interval. In the case $\chi_n^{(3)} = \chi_p^{(3)} = \chi_{np}^{(3)} \equiv \chi^{(3)}$, there is no root in the interval (2) if $\chi^{(3)} > \chi_{\max}^{(3)}$. The second and subsequent roots of (1) are located between successive poles on the right-hand side of (1); they exist for any values of $\chi^{(3)}$.

The values of ω, calculated from (1), depend on the wave functions and eigenvalues of the mean-field potential, and also on which mean-field levels were taken into account in (1). The terms in (1) with s and s' corresponding to particle and hole states in all nuclei with $|E(s) - \lambda| \gg C$ and $|E(s') - \lambda| \gg C$ lead only to a renormalization of the constant $\chi^{(3)}$, just as in $^{(4)}$. Those terms in (1) which in some nuclei correspond to particle and hole states, while in others only to particle (hole) states, lead not only to a renormalization of $\chi^{(3)}$, but also to a change of ω in some nuclei in comparison with others.

Fig. 1. Energies of octupole collective states with $K\pi = 0-$ in the region $228 \leq A \leq 254$. 1—experimental data from $^{(13)}$; 2—calculated energy values for $\chi^{(3)} = 0.00081\,\hbar\omega_0$ (connected by straight lines for clarity); 3—energies for Th at $\chi^{(3)} = 0.00081\,\hbar\omega_0$ and for the remaining nuclei at $\chi^{(3)} = 0.00085\,\hbar\omega_0$.

The correctness of the placement of the levels of the Nilsson scheme in the regions $61 \leq Z \leq 79,\ 89 \leq N \leq 115$ and $87 \leq Z \leq 99,\ 137 \leq N \leq 155$ is confirmed by experimental data on one-quasiparticle levels of odd-A nuclei. With respect to the behavior and selection of the remaining levels there is some arbitrariness. To reduce this arbitrariness, we included all orbitals of those subshells for which the position of at least one one-particle level had been experimentally confirmed. The calculations were carried out at deformation $\delta = 0.3$ for nuclei in the region $152 \leq A \leq 186$ and at $\delta = 0.2$ for nuclei in the region $228 \leq A \leq 254$, using the wave functions given in ⁽¹¹⁾. In each region, for all nuclei, identical one-particle mean-field energies $E(s)$ were used, and changes in the equilibrium deformation for different nuclei were not taken into account. To determine how strongly the results of the calculations depend on the wave functions in the region $228 \leq A \leq 254$, ω was calculated with wave functions at $\delta = 0.3$, but with unchanged values of $E(s)$. The values of ω obtained in this case differ little from the values of ω for $\delta = 0.2$. The constant of the octupole-octupole interaction $\chi^{(3)}$ was taken equal to $\chi^{(3)} = 1.3/A$ MeV, i.e., the values $\chi^{(3)}A$ are the same in both regions of strongly deformed nuclei.

The results of calculations of the energies of octupole collective states with $K\pi = 0-$ in the region $228 \leq A \leq 254$ at $\delta = 0.2$, in the case $\chi_n^{(3)} = \chi_p^{(3)} = \chi_{np}^{(3)} = \chi^{(3)}$, are shown in Fig. 1. The best agreement with all experimental data is obtained for $\chi^{(3)} = 0.00083\,\hbar\omega_0 \approx 0.0055$ MeV $\approx 1.3/A$ MeV. However, for Th isotopes the best value is $\chi^{(3)} = 0.00081\,\hbar\omega_0$, and for U isotopes $\chi^{(3)} = 0.00085\,\hbar\omega_0$. As was noted in ⁽¹²⁾, the calculated smallest values of ω are in satisfactory agreement with the experimental energy values of states with $K\pi = 0-$. The tendency toward lowering of the energies of these states in the light Th and U isotopes is correctly reproduced. In the isotopes Th, U, and Pu the states

with $K\pi = 0-$ are collective to a high degree; the values of ω are 0.8-1.0 MeV smaller than the energy values of the nearest poles in (1). As for Fm^{254}, the contribution to the state with $K\pi = 0-$ of the proton two-quasiparticle state with configuration $633\uparrow -514\downarrow$ is 96%, so that this state is, to good accuracy, two-quasiparticle. Its energy, equal to 1.4 MeV, is only 26 keV less than the energy of the nearest pole.

Fig. 2. Energies of octupole collective states with $K\pi = 0-$ in the region $152 \leqslant A \leqslant 186$.
1 –experimental data from ⁽¹⁴⁾; 2 –calculated energy values for $\chi^{(13)} = 0.00170\,\hbar\omega_0$ (connected by straight lines for clarity)

pole. The energy of the two-quasiparticle proton state $633\uparrow -514\downarrow$ in Fm^{254}, according to the calculations ⁽¹⁰⁾ with allowance for the blocking effect, but at $\chi^{(3)} = 0$, is 1.08 MeV. If, in the study of collective effects, we took the blocking effect into account, we would obtain the energy of this state equal to 1 MeV.

Thus, some states with $K\pi = 0-$ possess collective properties, while others are two-quasiparticle. Whether the structure of a given state is collective or quasiparticle is determined by the mean field of the nucleus.

The results of calculations of the energies of states with $K\pi = 0-$ in the region $152 \leqslant A \leqslant 186$ at $\delta = 0.3$, in the case $\chi_n^{(3)} = \chi_p^{(3)} = \chi_{np}^{(3)} \equiv \chi^{(3)}$, are presented in Fig. 2. As is seen from the figure, satisfactory agreement has been obtained between the calculated energy values of the states with $K\pi = 0-$ and the corresponding experimental data. In this region of strongly deformed nuclei, all the lowest states with $K\pi = 0-$ are collective. Thus, in Er^{166}, one two-quasiparticle state contributes 30% to the 0– state with energy 1.66 MeV, 2 states contribute 16% each, and 9 states contribute 1–5% each. The energy ω in Er^{166} is 0.56 MeV smaller than the energy value of the nearest pole and 0.34 MeV smaller than the energy of the two-quasiparticle state with allowance for the blocking effect.

Calculations have been carried out for more general cases $\chi_n^{(3)} \neq \chi_p^{(3)} \neq \chi_{np}^{(3)}$, which, however, differ little from calculations with $\chi_n^{(3)} = \chi_p^{(3)} = \chi_{np}^{(3)} \equiv \chi^{(3)}$. In solving (1), conservation of the number of particles on average was monitored; for this purpose the quantity

$$\Delta n = \left\langle Q \sum_{s\sigma} a_{s\sigma}^{+} a_{s\sigma} Q^{+} \right\rangle - \left\langle \sum_{s\sigma} a_{s\sigma}^{+} a_{s\sigma} \right\rangle , \tag{3}$$

was calculated, i.e., the difference between the average number of neutrons (protons) in the excited collective state and in the ground state. In the overwhelming majority of cases, $\Delta n < 0.3$ was obtained; however, states with $\Delta n = 0.3$-0.6 also occur. The reduced probabilities of $E3$- and $E1$-transitions from states with $K\pi = 0-$ to the ground state were calculated. The reduced probabilities of $E3$-transitions, calculated with $e_p = e + e_{\text{eff}},\ e_n = e_{\text{eff}},\ e_{\text{eff}} = 0.5e$ as in (4), are 1.2–3.6 times greater than single-

single-particle ones, except for the isotopes Cm, Cf, and Fm, where they are somewhat smaller than the single-particle ones. The quoted probabilities of $E1$-transitions with $e_p = \frac{N}{A}e,\ e_n = -\frac{Z}{A}e$ are approximately 10^2 times smaller than the single-particle ones.

Thus, the behavior of the energies of collective octupole states with $K\pi = 0$ has been explained by introducing one new constant $\chi^{(3)}$, all the remaining parameters having been fixed earlier in (8,9). It should be noted that the microscopic interpretation of the states on the basis of the superfluid model of the nucleus differs strongly from the phenomenological interpretation of the generalized nuclear model. Thus, according to the interpretation of the superfluid model of the nucleus, octupole states in some nuclei lie comparatively low (below the β- and γ-vibrational states) and possess pronounced collective properties, whereas in other nuclei such states have large energy and, in their nature, are close to two-quasiparticle excited states.

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