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Abstract

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ON THE FREQUENCY SHIFT AND DAMPING OF INTERACTING LANGMUIR OSCILLATIONS

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1. As is known, the interaction between elementary excitations in a condensed medium leads to a shift of their energy and to damping, caused by the transfer of energy over the excitation spectrum from some harmonics to others. L. P. Pitaevskii ([1]) considered the properties of the excitation spectrum in helium near the threshold above which an excitation decays into two excitations of lower energy. V. L. Pokrovskii and A. M. Dykhne ([2]) studied the properties of the spectrum of acoustic excitations near decay thresholds in crystals. The damping of magnons interacting with phonons was investigated by E. N. Yakovlev ([3]). For phonons and magnons, the frequency shifts and damping caused by the interactions phonon–phonon, phonon–magnon, and magnon–magnon were considered in detail by V. N. Kashcheev and M. A. Krivoglaz ([4]) and by M. A. Krivoglaz ([5]). It is natural to expect that interactions between oscillations in a plasma will also lead to a shift of their frequency and to damping. Nevertheless, to our knowledge, this question has not been discussed by anyone. The aim of the present work is to find the interaction-induced frequency shift and damping of Langmuir oscillations (l-plasmons).

2. We shall consider l-plasmons whose wave vector $k \ll R_D^{-1}$, where $R_D = \sqrt{T/4\pi e^2 n}$ is the Debye radius. According to the laws of conservation of energy and momentum, for l-plasmons the only allowed process is a four-plasmon process, in which one l-plasmon is scattered by another l-plasmon. The complete Hamiltonian of Langmuir oscillations, taking into account the indicated four-plasmon process, in the second-quantization representation has the form ([6,7])

$$H = H_0 + \varepsilon H' = \sum_k \Omega_k a_k^+ a_k + \varepsilon \sum_{k_1 k_2 k_3 k_4} \Psi_{k_1 k_2 k_3 k_4} a_{k_1}^+ a_{k_2}^+ a_{k_3} a_{k_4} + \text{conj.}, \quad (1)$$

where

$$\Omega_k^2 = \Omega_0^2 (1 + 3k^2 R_D^2);$$

$$\Psi_{k_1k_2k_3k_4} = \frac{\pi}{V}\,\Omega_0^{-2}\left(\frac{e}{m}\right)^2\frac{1}{k_1k_2k_3k_4}\,[(\mathbf{k}_1\mathbf{k}_4)(\mathbf{k}_2\mathbf{k}_4)(\mathbf{k}_3\mathbf{k}_4)+(\mathbf{k}_1\mathbf{k}_3)(\mathbf{k}_2\mathbf{k}_3)(\mathbf{k}_4\mathbf{k}_3)+$$

$$+(\mathbf{k}_1\mathbf{k}_2)(\mathbf{k}_3\mathbf{k}_2)(\mathbf{k}_4\mathbf{k}_2)+(\mathbf{k}_4\mathbf{k}_1)(\mathbf{k}_2\mathbf{k}_1)(\mathbf{k}_3\mathbf{k}_1)]\,; \quad (2)$$

a_k^+, a_k are, respectively, the creation and annihilation operators of a plasmon with energy Ω_k and momentum $\mathbf{k}$. The parameter ε has been introduced before H' for convenience. Below, perturbation theory is applied and an expansion in powers of ε is carried out.

Introduce the one-particle two-time retarded Green function [8,9]

$$G_k(t,t') = -i\theta(t-t')\langle[a_k(t);a_k^+(t')]\rangle, \quad (3)$$

where

$$\langle ...\rangle = \mathrm{Sp}(e^{-H/T}\,...)\{\mathrm{Sp}(e^{-H/T})\}^{-1},$$

$$\theta(t-t') = \begin{cases}1, & t>t',\\ 0, & t<t',\end{cases}$$

$$[a_k(t);a_k^+(t')] = a_k(t)a_k^+(t') - a_k^+(t')a_k(t).$$

Let us write the equation of motion for the function (3):

$$i\frac{dG_k(t,t')}{dt} = \delta(t-t') - i\theta(t-t')\left\langle\left[i\frac{da_k(t)}{dt};a_k^+(t')\right]\right\rangle.$$

Expanding this equation with the aid of the Hamiltonian (1), we obtain

$$\begin{aligned} i\frac{dG_k(t',t)}{dt} = \delta(t-t') + \Omega_{kG}k(t,t') + \varepsilon\sum_{k_2k_3k_4}\Psi_{kk_2k_3k_4}F_{k_2k_3k_4;k} \\ +\varepsilon\sum_{k_1k_3k_4}\Psi_{k_1kk_3k_4}F_{k_1k_3k_4;k} + \varepsilon\sum_{k_1k_2k_4}\Psi_{k_1k_2kk_4}F_{k_4k_1k_2;k} \\ +\varepsilon\sum_{k_1k_2k_3}\Psi_{k_1k_2k_3k}F_{k_3k_1k_2;k}, \end{aligned} \quad (4)$$

where

$$F_{k_{ik_{jk}}l;k} = -i\theta(t-t')\langle[a_{k_i}^+a_{k_j}a_{k_l};a_k^+(t')]\rangle. \quad (5)$$

The equations of motion for the functions (5) will contain higher-order Green functions; for these Green functions, in turn, one can write equations of motion. Continuing this process, we obtain an infinite system of coupled equations. We shall decouple this chain, isolating and retaining in the equations of motion for the functions $F_{k_{ik_{jk}}l;k}$ only terms containing $F_{k_{ik_{jk}}l;k}$ and G_k. Such a simple interpolation is justified by the smallness of the interaction of l-plasmons. Thus, we have

$$i\frac{dF_{k_{ik_{jk}}l;k}}{dt} = (-\Omega_{k_i} + \Omega_{k_j} + \Omega_{k_l})F_{k_{ik_{jk}}l;k} + 2\varepsilon B_{k_{ik_{jk}}l;k}G_k, \tag{6}$$

where

$$B_{k_{ik_{jk}}l;k} = (\Psi_{k_{jk_{lkk}}i} + \Psi_{k_{jk_{lk_{ik}}}} + \Psi_{kk_{ik_{jk}}l} + \Psi_{kk_{ik_{lk}}j})(N_{k_l}N_{k_l} + N_{k_i} + N_{k_j}N_{k_i} - \tag{7}$$

$$-N_{k_j}N_{k_l}), \qquad N_k = \langle a_k^+ a_k\rangle.$$

Passing to the Fourier representation $A = \int A(\omega)e^{-i\omega t}\,d\omega$, from equations (4) and (6) we obtain:

$$\Big[\omega - \Omega_k - \varepsilon^2\Big(\sum_{k_2k_3k_4}\Psi_{kk_2k_3k_4}\Gamma_{k_2k_3k_4;k} + \sum_{k_1k_3k_4}\Psi_{k_1kk_3k_4}\Gamma_{k_1k_3k_4;k}$$
$$+ \sum_{k_1k_2k_4}\Psi_{k_1k_2kk_4}\Gamma_{k_4k_1k_2;k} + \sum_{k_1k_2k_3}\Psi_{k_1k_2k_3;k}\Gamma_{k_3k_1k_2;k}\Big)\Big]G_k(\omega) = \frac{1}{2\pi}, \tag{8}$$

where

$$\Gamma_{k_{ik_{jk}}l;k} = \frac{2B_{k_{ik_{jk}}l;k}}{\omega + \Omega_{k_i} - \Omega_{k_j} - \Omega_{k_l}}. \tag{9}$$

We continue the function $G_k(\omega)$ into the upper half-plane, $\omega \to \omega + i\alpha$. For $\alpha \to 0$ we have

$$[\omega - \Omega_k - \varepsilon^2(Q_k^{(1)}(\omega) - i\pi Q_k^{(2)}(\omega))]G_k(\omega) = \frac{1}{2\pi},$$

where

$$Q_k^{(1)}(\Omega_k) = P\Bigg(\sum_{k_2k_3k_4} \Psi_{kk_2k_3k_4}\Gamma_{k_2k_3k_4;k}(\Omega_k) + \sum_{k_1k_3k_4} \Psi_{k_1kk_3k_4}\Gamma_{k_1k_3k_4;k}(\Omega_k) + \sum_{k_1k_2k_4} \Psi_{k_1k_2kk_4}\Gamma_{k_4k_1k_2;k}(\Omega_k) + \sum_{k_1k_2k_3} \Psi_{k_1k_2k_3k}\Gamma_{k_3k_1k_2;k}(\Omega_k)\Bigg); \tag{10}$$

$$\begin{aligned} Q_k^{(2)}(\Omega_k) = &\sum_{k_2k_3k_4} \Psi_{kk_2k_3k_4}B_{k_2k_3k_4;k}\delta(\Omega_k+\Omega_{k_2}-\Omega_{k_3}-\Omega_{k_4})+ \\ &+\sum_{k_1k_3k_4} \Psi_{k_1kk_3k_4}B_{k_1k_3k_4;k}\delta(\Omega_k+\Omega_{k_1}-\Omega_{k_3}-\Omega_{k_4})+ \\ &+\sum_{k_1k_2k_4} \Psi_{k_1k_2kk_4}B_{k_4k_1k_2;k}\delta(\Omega_k+\Omega_{k_4}-\Omega_{k_1}-\Omega_{k_2})+ \\ &+\sum_{k_1k_2k_3} \Psi_{k_1k_2k_3k}B_{k_3k_1k_2;k}\delta(\Omega_k+\Omega_{k_3}-\Omega_{k_1}-\Omega_{k_2}). \end{aligned} \tag{11}$$

P in (10) means that the integral is taken in the sense of the principal value. $Q_k^{(1)}(\Omega_k)$ and $Q_k^{(2)}(\Omega_k)$ express, respectively, the frequency shift and the damping of the l-plasmons caused by the interaction.

3. Let us first estimate the order of magnitude of the damping. In (11) all terms are of the same order; therefore, for an estimate it is sufficient to restrict ourselves to one, the first, term. Using expression (2) and substituting for N_k the Rayleigh–Jeans distribution function $N_k = T/\Omega_k$, we obtain

$$Q_k^{(2)}(\Omega_k) \simeq \frac{1}{V^2}\frac{T}{m}\frac{R_D^2}{n^2}\sum_{k_2k_3}\Phi(k,k_2,k_3)\,\delta(\Omega_k+\Omega_{k_2}-\Omega_{k_3}-\Omega_{k_4}). \tag{12}$$

Here

$$\Phi(k,k_2,k_3) = \frac{[k^2+(\mathbf{kk}_2)-(\mathbf{kk}_3)]^2[(\mathbf{k}_3\mathbf{k})+k_2^2-(\mathbf{k}_2\mathbf{k}_3)]^2[(\mathbf{k}_3\mathbf{k})+(\mathbf{k}_3\mathbf{k}_2)-k_3^2]^2}{k^2k_2^2k_3^2(k^2+k_2^2+k_3^2+2(\mathbf{kk}_2)-2(\mathbf{kk}_3)-2(\mathbf{k}_2\mathbf{k}_3))}.$$

Taking into account that $k \ll R_D^{-1}$, from (12) it is not hard to find the order of magnitude of the damping:

$$Q_k^{(2)}(\Omega_k) \sim \frac{\Omega_0}{N_D^2}(kD_D)^2, \tag{13}$$

where N_D is the number of particles in the Debye sphere.

For the magnitude of the frequency shift, from (10) we find:

$$Q_k^{(1)} \sim Q_k^{(2)}. \tag{14}$$

The corresponding frequency broadening is also of order $Q_k^{(2)}$ (this follows from the uncertainty principle).

In an ideal plasma $N_D \gg 1$; therefore, as a consequence of (13) and (14), the natural conditions are satisfied:

$$Q_k^{(1)} \ll \Omega_k, \qquad Q_k^{(2)} \ll \Omega_k.$$

Because of collisions of electrons with ions, the l-plasmons damp with the decrement

$$\gamma_{\rm st} \sim \frac{\Omega_0}{N_D} \ln N_D;$$

the ratio

$$\frac{Q_k^{(2)}}{\gamma_{\rm st}} \sim \frac{(kR_D)^2}{N_D \ln N_D} \ll 1.$$

The Landau damping decrement

$$\gamma_L \sim \frac{\Omega_0}{(kR_D)^3} e^{-1/(kR_D)^2},$$

therefore

$$\frac{Q_k^{(2)}}{\gamma_L} \sim \frac{(kR_D)^5}{N_D^2} e^{1/(kR_D)^2}.$$

It follows from the last expression that $Q_k^{(2)} \gg \gamma_L$ for waves with wave vectors $k \gg k_0$, where k_0 is found from the equation

$$e^{1/(k_0 R_D)^2} = N_D^2 \frac{1}{(k_0 R_D)^5}.$$

The frequency shifts and damping of other branches of plasma oscillations due to interaction can be found in an analogous way, knowing the Hamiltonian of the interaction between these oscillations.

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Note: Figure translations are in progress. See original paper for figures.